Critical Configurations For Radial Distortion Self-Calibration

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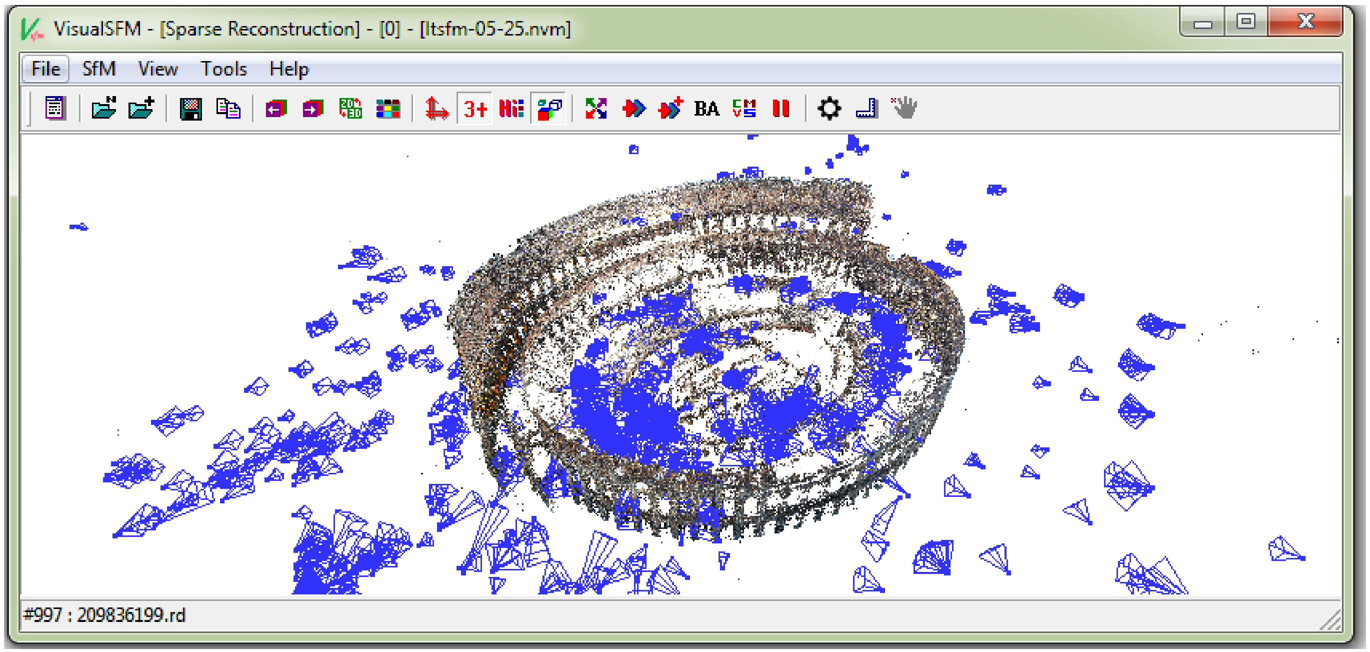
VisualSFM

- A Visual Structure from Motion System, 2011
  - SiftGPU + Multicore BA + Fast SfM + GUI

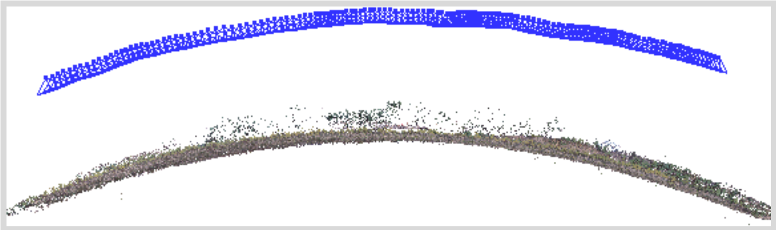


  - Used in aerial survey, geology, archaeology, VFX, 3D printing, etc.
  - Reconstruction failures are often well understood.

Question from Thomas Gröninger

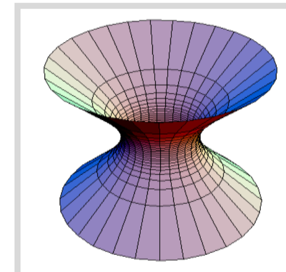
- A typical aerial image capture
  - UAV flies at roughly constant height
  - Camera pointing downward (nadir)
  - Un-calibrated GoPro camera

- Distorted reconstruction, why?
  - Ground should be roughly **FLAT**
  - Incorrect radial distortion estimation

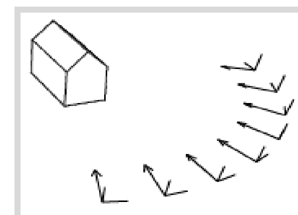


Ambiguities in 3D Reconstruction

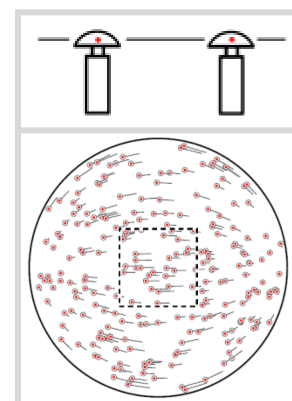
- Extensive studies for perspective cameras
  - For calibrated reconstruction from image velocity or two views, critical surfaces are *ruled quadrics [Horn 1987, Maybank 1993].*



  - Critical motions exist for self-calibration, for example, planar motion and orbital motion*[Sturm 1997, Sturm 1999, Kahl et al. 2000, etc.]*.

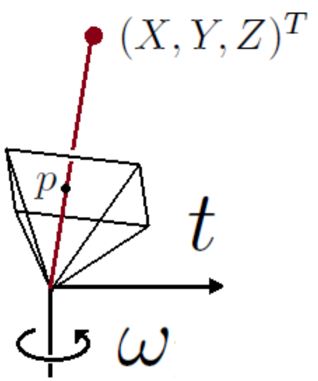


- Little study for radial distortion self-calibration
  - Parallel feature displacements and camera motion under pure translation. *[Mičušík et al. 2006]*



Critical Surfaces

- Horn, *Motion fields are hardly ever ambiguous*, 1987



  - Given a translational speed t and rotational speed ω, the image velocity is a function of p and Z.

$$p' = V(t, \omega, p, Z)$$

  - For two motion $\{t_1, \omega_1\}$ and $\{t_2, \omega_2\}$, the surface pair $\{Z_1, Z_2\}$ that produce the same image velocity satisfy:

$$V(t_1, \omega_1, p, Z_1) = V(t_2, \omega_2, p, Z_2)$$

  - These ***critical*** surfaces are ruled quadrics.

The Problem

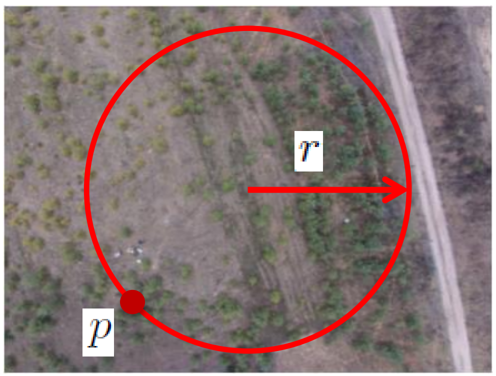
Given two cameras with

- **Different radial distortions** and
- Possibly different motions,

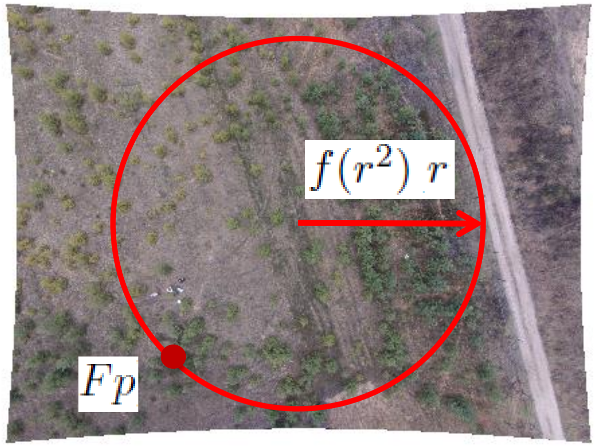
What surfaces can produce the same motion field?

Radial Distortion

- Central and centered radial distortion



Original image



Undistorted image

  - Using an implicit radial distortion function $f(r^2)$
  - Not limited to specific radial distortion parameterization
  - Works for central omni-directional cameras

Critical Surfaces

- Image velocity in the undistorted image

$$(Fp)' = (F + 2F'pp^T)\, p'$$

undistorted image — original image

- Consider the following two configurations:
  - 1st camera with motion $\{t_1, \omega_1\}$ without radial distortion
  - 2nd camera with motion $\{t_2, \omega_2\}$ and distortion function f

Solve for the critical surface pair Z_1 and Z_2 :

$$V(t_2, \omega_2, Fp, Z_2) = (F + 2F'pp^T)\; V(t_1, \omega_1, p, Z_1)$$

Undistorted 2nd image — **1st image**

Critical Surface Pair

- The two corresponding surfaces

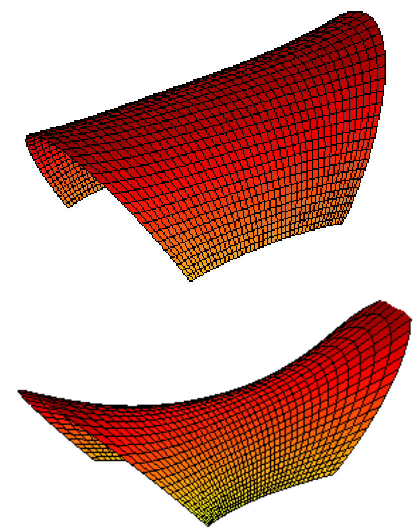
$$Z_1 = \frac{\begin{pmatrix} -2f'(t_1 \cdot \hat{z})(p^T p)(t_2 \times \hat{z})^T p + \\ 2f' p^T t_1 (t_2 \times \hat{z})^T p - (t_2 \times F t_1)^T F p \end{pmatrix}}{\begin{pmatrix} ((Fp) \times \omega_2 - F(p \times \omega_1)) \cdot (t_2 \times Fp) \\ +2f'(p^T p) p^T (\omega_1 \times \hat{z})(t_2 \times \hat{z})^T p \end{pmatrix}}$$

$$Z_2 = \frac{Z_1 \, t_2 \cdot (\hat{z} \times p)}{(F t_1 - ((Fp) \times \omega_2 - F(p \times \omega_1)) Z_1) \cdot (\hat{z} \times p)}$$

  - The critical surfaces in Horn's paper can be obtained by using $f = 1$ and $f' = 0$;

  - Complicated surfaces due to $f' \neq 0$;
  - Often resembles the ruled quadrics.

Example:

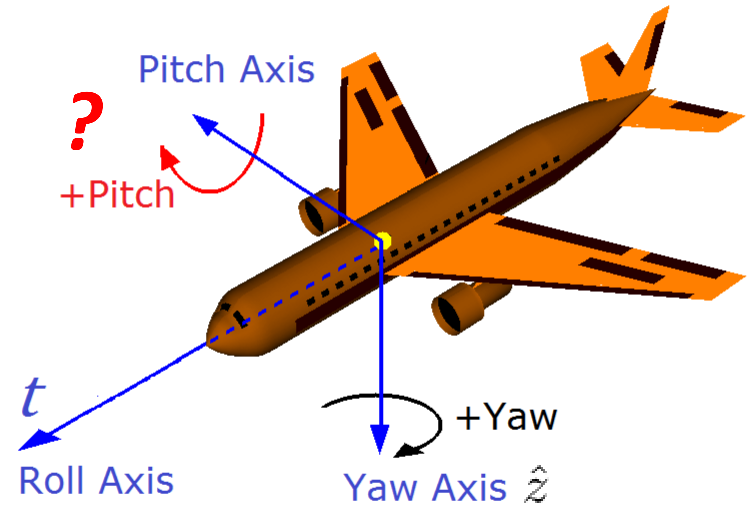


Gröninger's Case

- A special instantaneous motion:
  - **Camera points downward, no roll**

$$t \perp \hat{z},\ \omega \perp t$$

  - Moving on a sphere while pointing to the center, or moving on a plane while pointing perpendicularly



- A special configuration of two such motions:
  - Known translation $t_1 \parallel t_2$
  - Known yaw speed $(\omega_1 - \omega_2) \cdot \hat{z} = 0$
  - **Different pitch speed – the unknown**

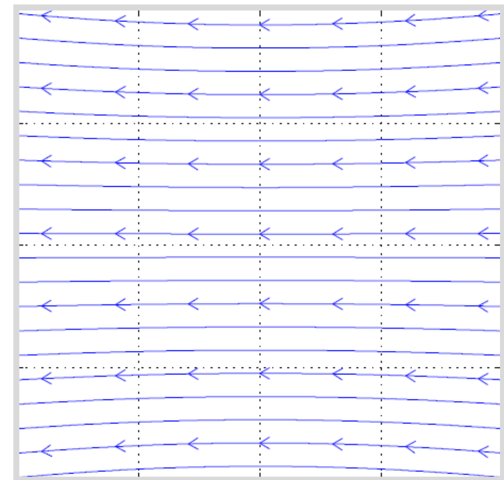
Simpler Surfaces

- Depth becomes a function of the radius

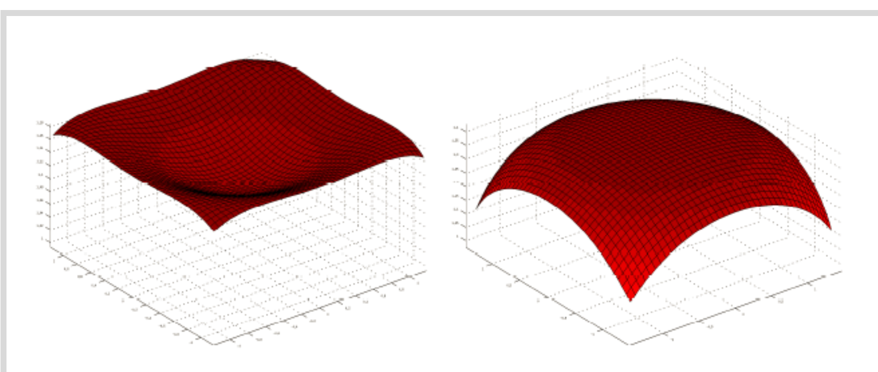
$$Z_1 = \frac{2\,(t_1 \cdot t_2)\,\boxed{f' \neq 0}}{\begin{pmatrix} -(t_2 \cdot (\omega_2 \times \hat{z}))f^2 + (t_2 \cdot (\omega_1 \times \hat{z}))f \\ +2\,(t_2 \cdot (\omega_1 \times \hat{z}))(p^T p)f' \end{pmatrix}}$$

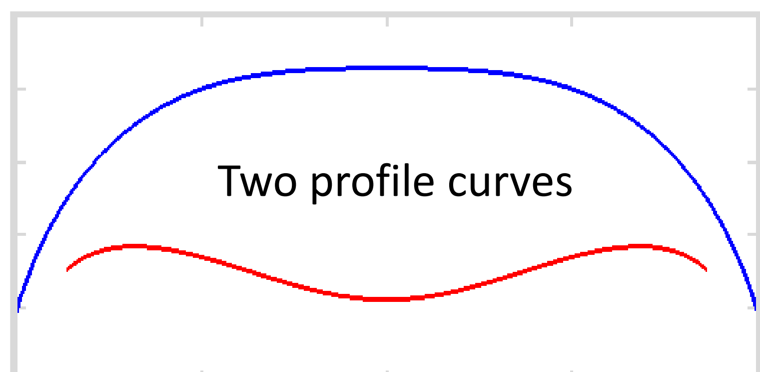
$$Z_2 = \frac{(t_1 \cdot t_2)\; Z_1}{(t_1 \cdot t_1)\; f - t_1 \cdot \; (\hat{z} \times (\omega_2 - f\,\omega_1))\; Z_1}$$

  - Both are rotational symmetric surfaces
  - Different surface curvatures (even signs)
  - Does not exist without radial distortion!



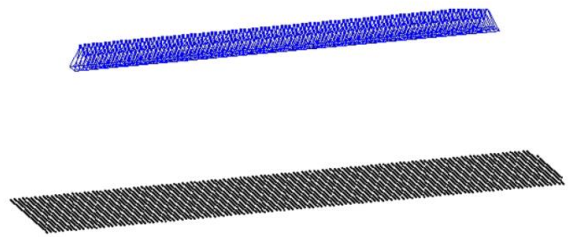
Motion field p'



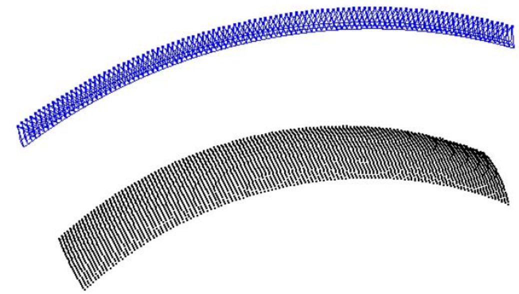


Impact on Multi-view Reconstruction

Synthetic captures with radial distortion

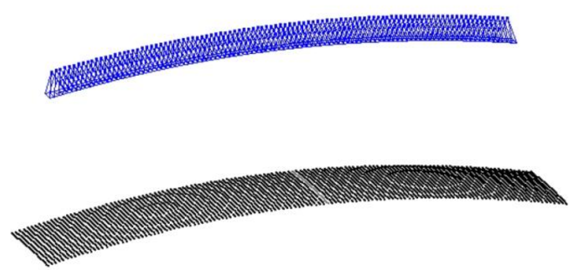


Capture#1 - plane

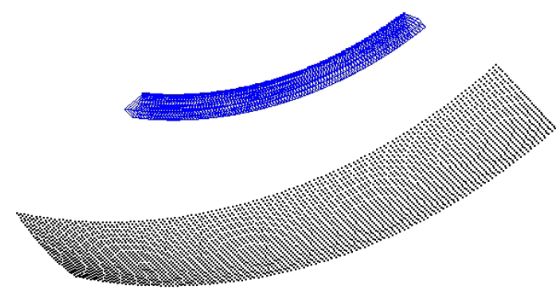


Capture#2 - Sphere

Self-calibration using VisualSFM



Result#1



Result#2

- Persistent local ambiguity leads to accumulated error

In Real Life

To Thomas Gröninger:

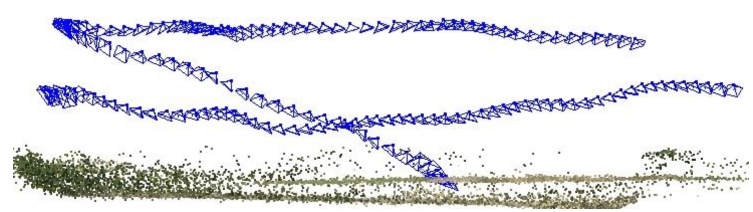
– For your particular capture, the distortion cannot be solved by standard self-calibration

– Using camera calibration should resolve the problem

– (months later..) or, you try can change the motion pattern:
  - not always looking straight-down, or
  - not at constant height

From Thomas Gröninger:

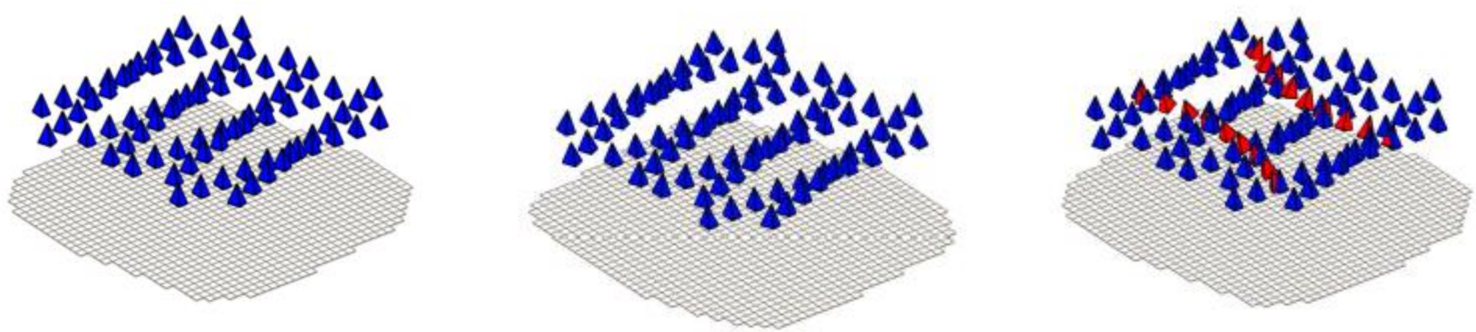
Using approximate calibration



New capture & self-calibration

Recent Experimental Study

- Mike James and Stuart Robson, *Systematic vertical error in UAV-derived topographic models: Origins and solutions*, EGU 2014



✗ Straight-down

✓ Forward-looking 5°

✓ Straight-down + 20° banked views

~~$t \perp \hat{z}$~~

Conclusions

- Summary
  - Critical configurations for radial distortion self-calibration.
  - Radial distortion can be easily ambiguous (e.g. nadir capture).
    - Calibrate the camera, or alter the camera motion
    - Use additional motion priors in the reconstruction
- Future work
  - Extend the study to discrete viewpoints.
- Sincere thanks to Thomas Gröninger!

Questions?