P3.5P: Pose Estimation With Unknown Focal Length

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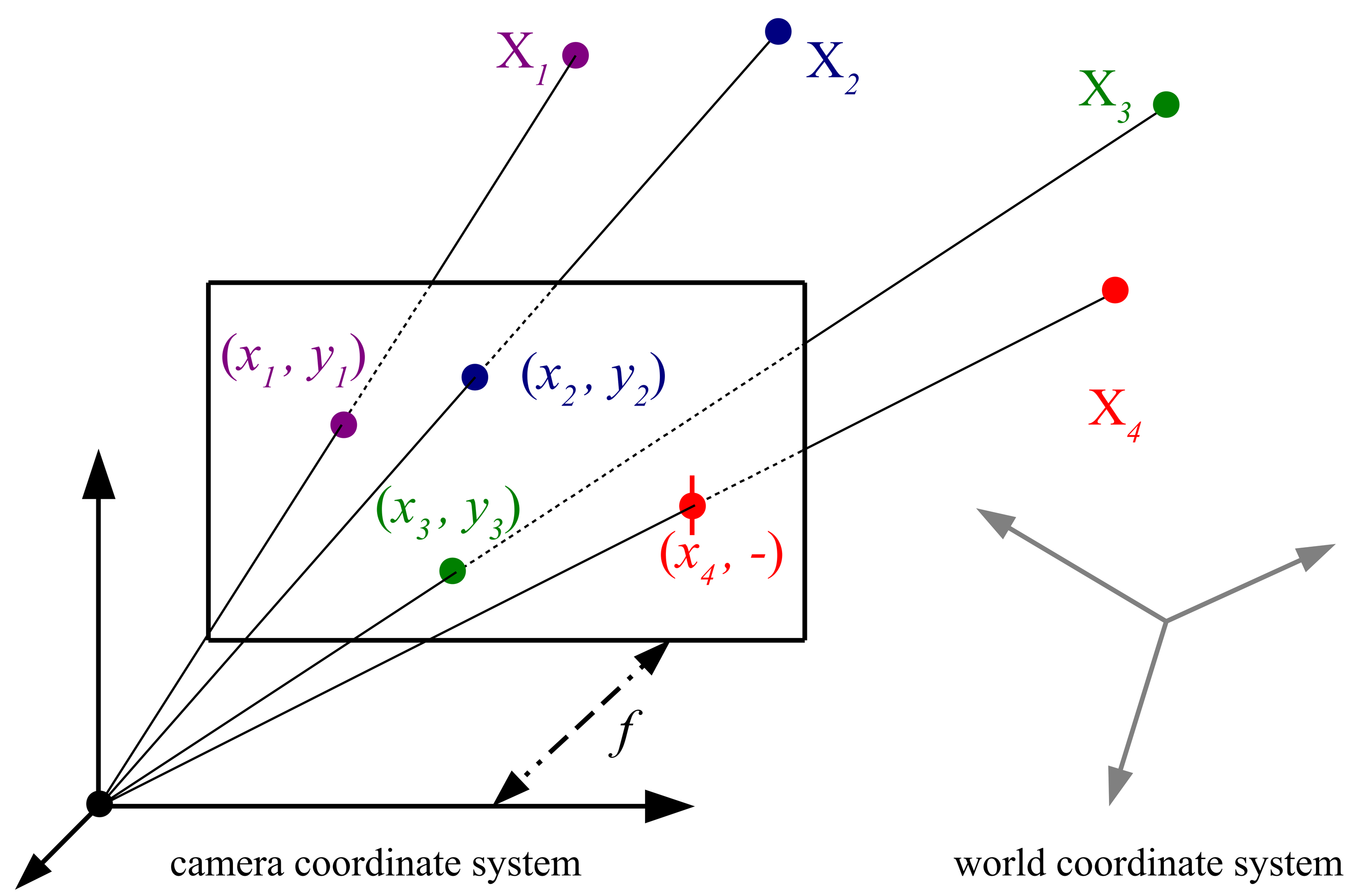
The First Minimal Solution for The 7-DoF Problem



- The problem has 7 DoF (rotation, translation, focal length);
- Common approximation for pose estimation of uncalibrated camera;
- The input are 7 image coordinates (3.5 points) of 4 known 3D points.

Related PnP Problems

Problem	Unknowns	DoF	Input	Minimal?
P3P	R, T	6	3	Y
P3.5P	**f, R, T**	**7**	**3.5**	**Y**
P4P	f, R, T	7	4	N
P5.5P	3x4 P	11	5.5	Y
P6P	3x4 P	11	6	N

Common Constraint and Parametrization

Constraints	Parametrization	Application
(x_i, y_i) : perspective projection	Linear combination of null space	P5.5P / P6P
$P\Omega P^\top \sim \mathrm{diag}(f^2, f^2, 1)$: focal length		P4P [1]
$\lvert X_i - X_j \rvert$: 3D Similarity between the camera and world coordinate systems	Point depth and its derivatives	P3P P4P [2, 3]

A New Camera Parametrization

- The naive parametrization leads to 2x solutions:

$$P = \begin{bmatrix} f & & \\ & f & \\ & & 1 \end{bmatrix} R \begin{bmatrix} I & -C \end{bmatrix} = \begin{bmatrix} -f & & \\ & -f & \\ & & 1 \end{bmatrix} \left(\begin{bmatrix} -1 & & \\ & -1 & \\ & & 1 \end{bmatrix} R \right) \begin{bmatrix} I & -C \end{bmatrix}.$$

  - Also degenerate for planar points (not general).
- Decompose the camera rotation matrix:

$$R = R_\theta \, R_\rho = \underbrace{R(z,\, \theta)}_{\text{around } z} \;\; \underbrace{R(\Phi,\, \rho)}_{\text{around } \Phi \perp z}$$

- A compact parametrization for camera with unknown focal length:

$$P = \begin{bmatrix} K_\theta \, R_\rho & T \end{bmatrix} \Longleftarrow K_\theta = \begin{bmatrix} f_c & -f_s & \\ f_s & f_c & \\ & & 1 \end{bmatrix} = \begin{bmatrix} f\cos\theta & -f\sin\theta & \\ f\sin\theta & f\cos\theta & \\ & & 1 \end{bmatrix}$$

  - No redundancy; Works for planar points.

Solving the P3.5P Problem

$$K_\theta(f_c, f_s), \;\; R_\rho \;\text{ as quaternion} = (1, q_x, q_y, 0)^\top, \;\; T = (t_x, t_y, t_z)^\top$$

Projection constraints — *7 unknowns 7 equations Degree 3*

⇓ Eliminate translation

Quadruple constraints — *4 unknowns 4 equations Degree 3*

⇓ Eliminate f_c, f_s

Determinant constraints — *2 unknowns 4 equations Degree 6*

⇓ Gröbner basis

≤10 real solutions for q_x, q_y

⇓ SVD

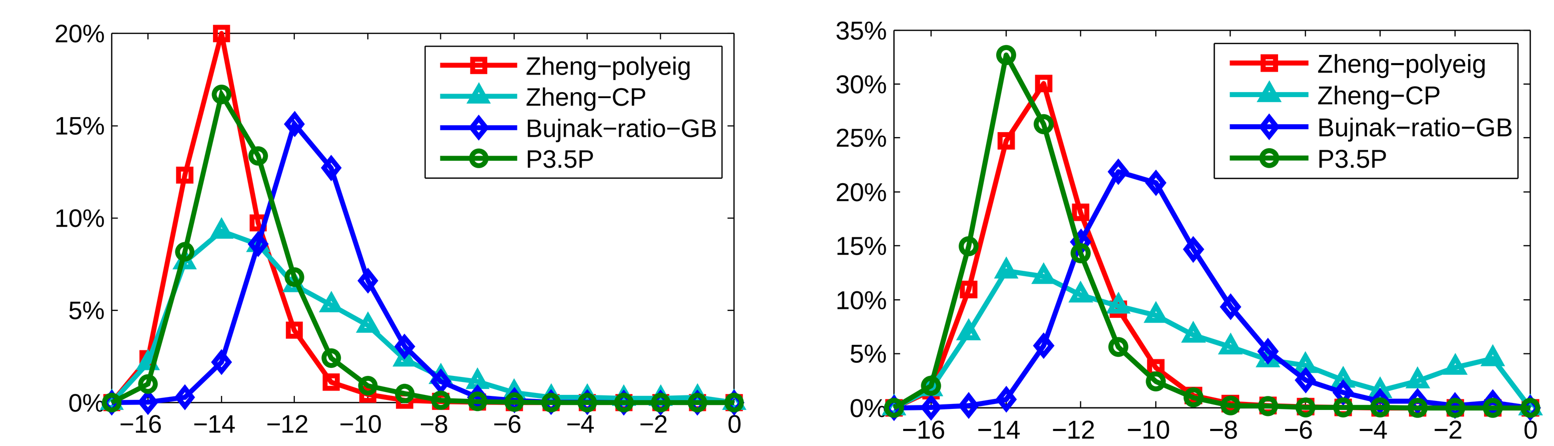
Solving for other unknowns ⇒ Filtering by the 8th coordinate

- Exact 7 image coordinates
- Minimal number of solutions
- Lowest polynomial degrees
- Smallest elimination template

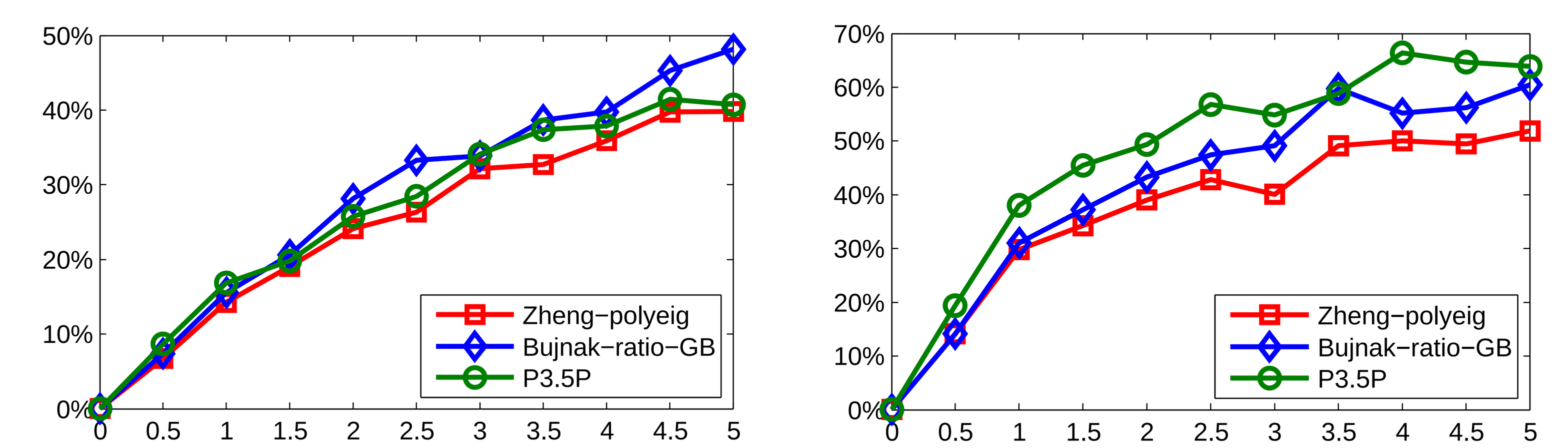
Experimental Results

Solver	Polynomial degree	Solving method	Speed
P3.5P	6	GB (20x30)	0.108ms
Zheng [3]	7	GB (36x53)	0.257ms
		Polyeig	1.648ms
		Characteristic Polynomial	0.067ms
Bujnak [2]	8	GB (53x63)	0.336ms
		GB (139x153)	3.320ms

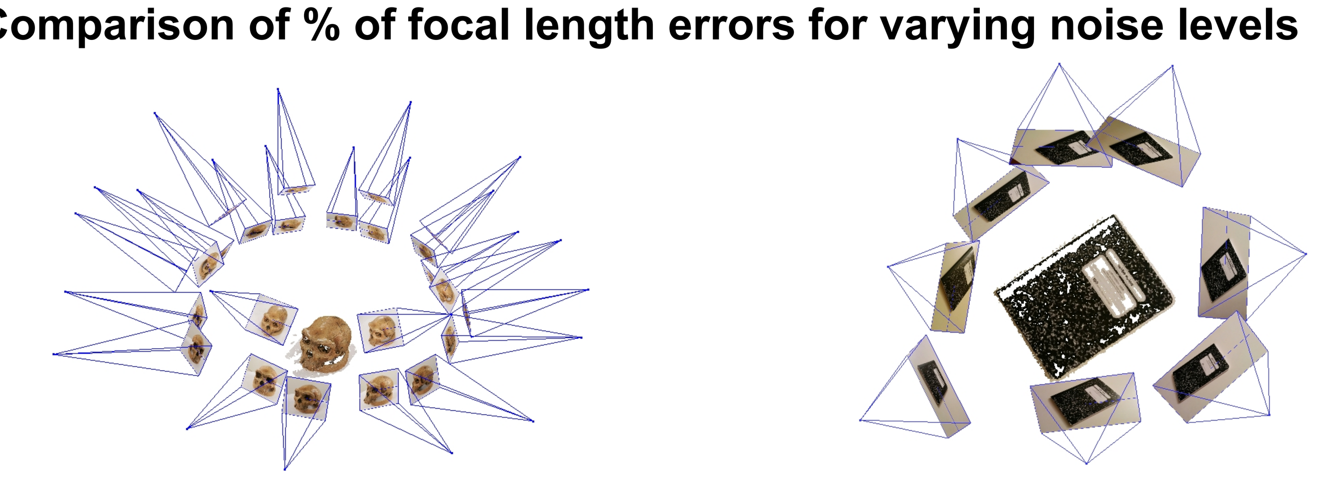
- **Comparison of polynomial system and speed**



- **Comparison of log of focal length errors for noise-free data**



- **Comparison of % of focal length errors for varying noise levels**



- **Reconstruction from real images (general and planar scene)**

References

[1] B. Triggs. Camera pose and calibration from 4 or 5 known 3D points. In CVPR, 1999.

[2] M. Bujnak, Z. Kukelova, and T. Pajdla. A general solution to the P4P problem for camera with unknown focal length. In CVPR, 2008

[3] Y. Zheng, S. Sugimoto, I. Sato, and M. Okutomi. A general and simple method for camera pose and focal length determination. In CVPR, 2014