Rewrite Rule Inference Using Equality Saturation

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OOPSLA 2021









PLSE

PAUL G. ALLEN SCHOOL
OF COMPUTER SCIENCE & ENGINEERING

Rewrite Rules Are Ubiquitous!

CVC4

HERBIE

Z3

tvm

Halide

LLVM COMPILER INFRASTRUCTURE

GCC

- Compilers
- Program Synthesizers
- Simplifiers / Optimizers
- SMT Solvers
- ML Frameworks

Rewrite Engines must be Efficient and Reliable!

- Compilers
- Program Synthesizers
- Simplifiers / Optimizers
- SMT Solvers
- ML Frameworks

Performance and reliability are key for a TRS [Newcomb et al. OOPSLA'20]

But…Designing Rewrite Rules is still Hard!

Who *writes* the *rewrite* rules?

Typically hand written by experts

Time consuming, often takes years

Too few / too many rules

Unsound rules

ility are key for a
TRS [Newcomb et al. OOPSLA'20]

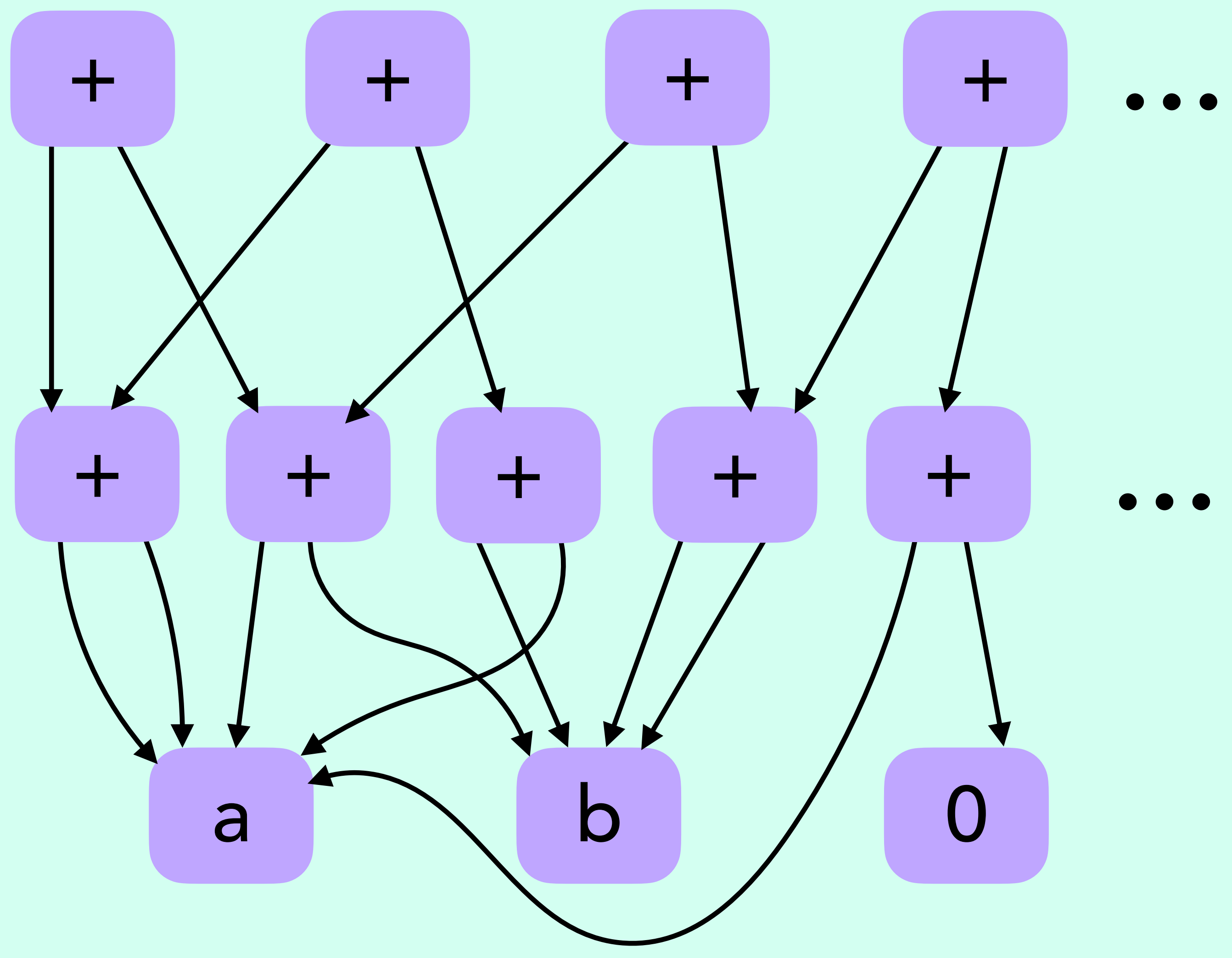
A 3-Step Approach for Inferring Rewrite Rules

Joshi et al. 2002, Bansal et al. 2006, Singh et al. 2016, Menendez et al. 2017, …

A 3-Step Approach for Inferring Rewrite Rules

Enumerate terms
from a grammar

a, b, 0, +, …



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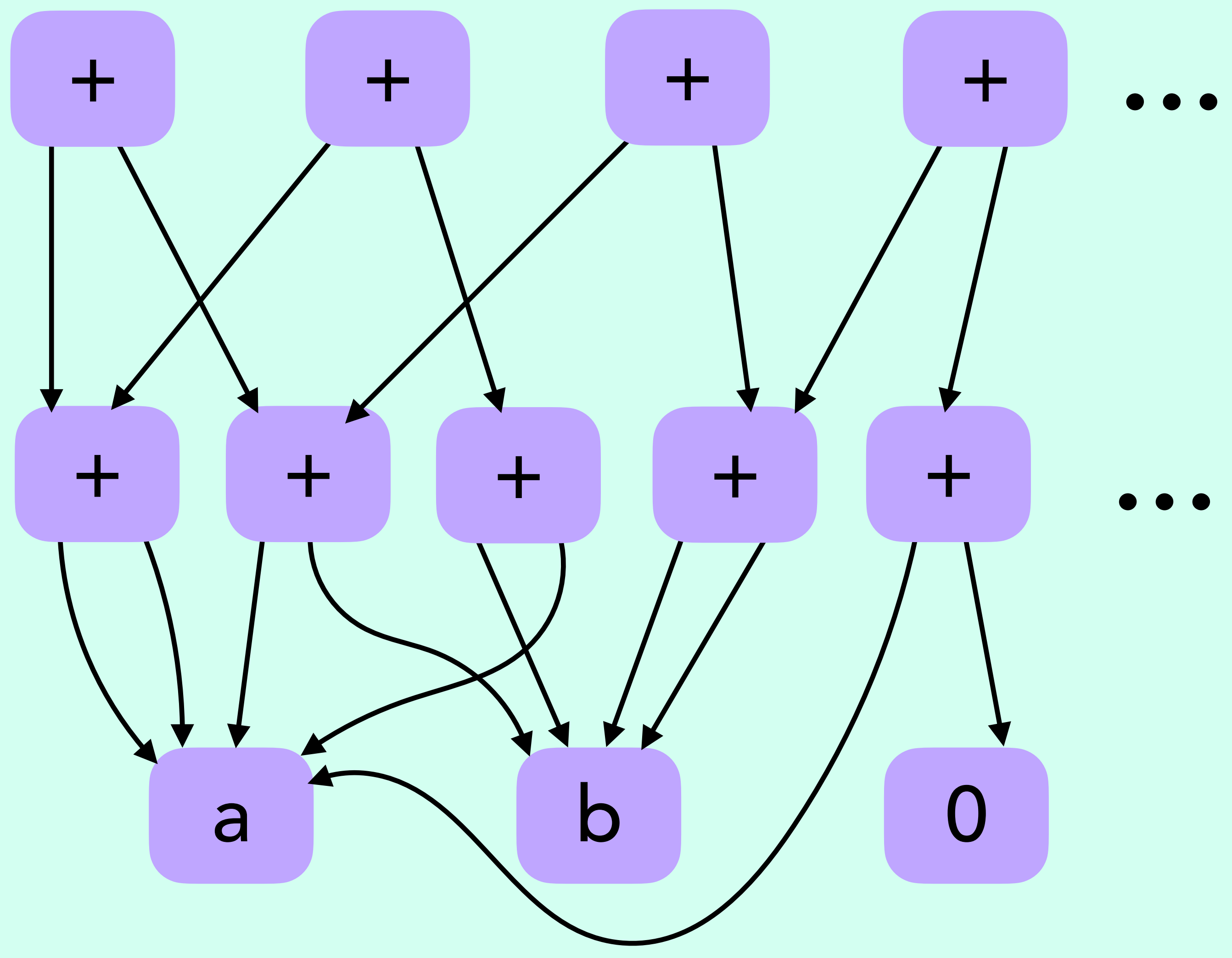
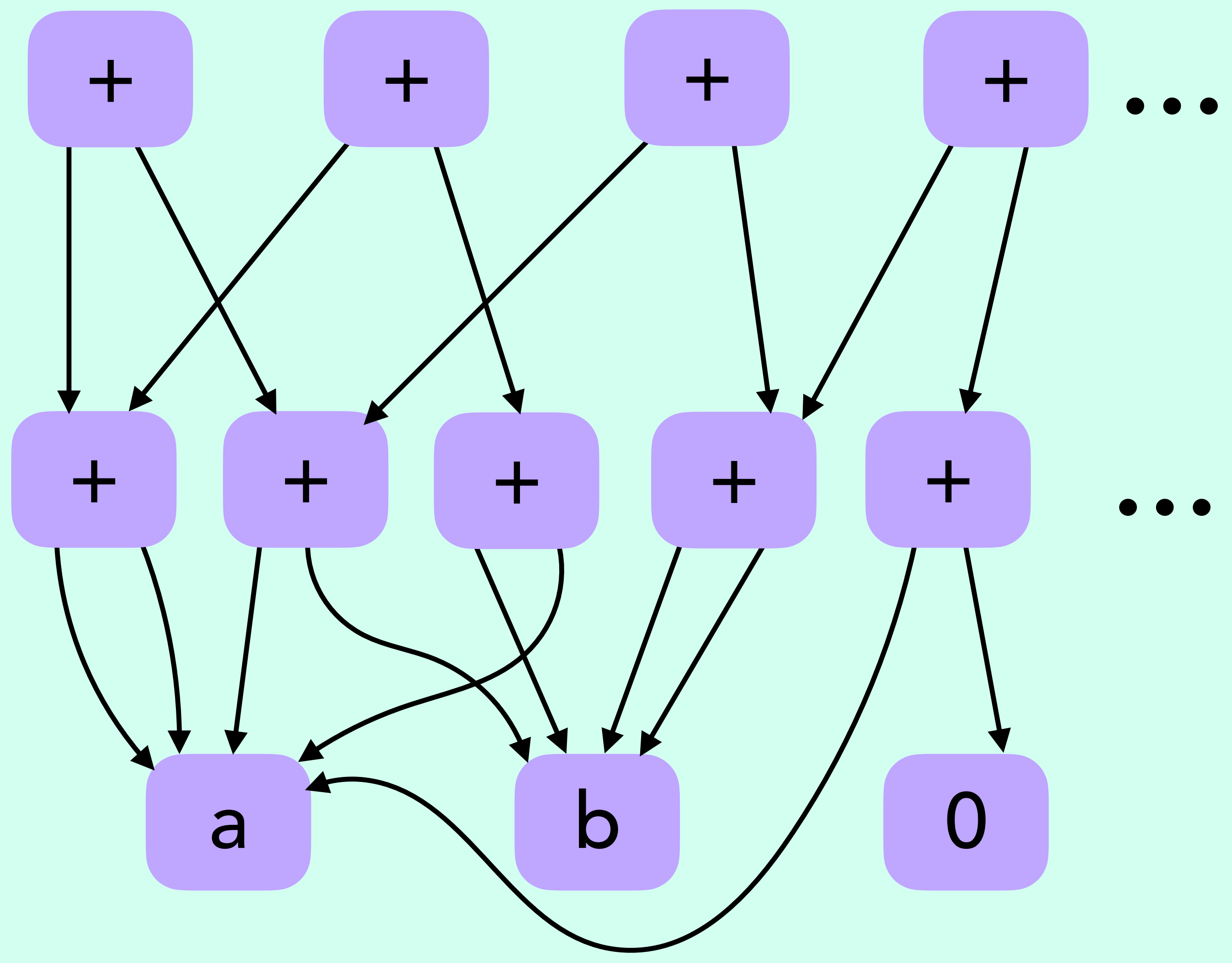


Find candidates: interpret over concrete inputs

 "Fingerprints"



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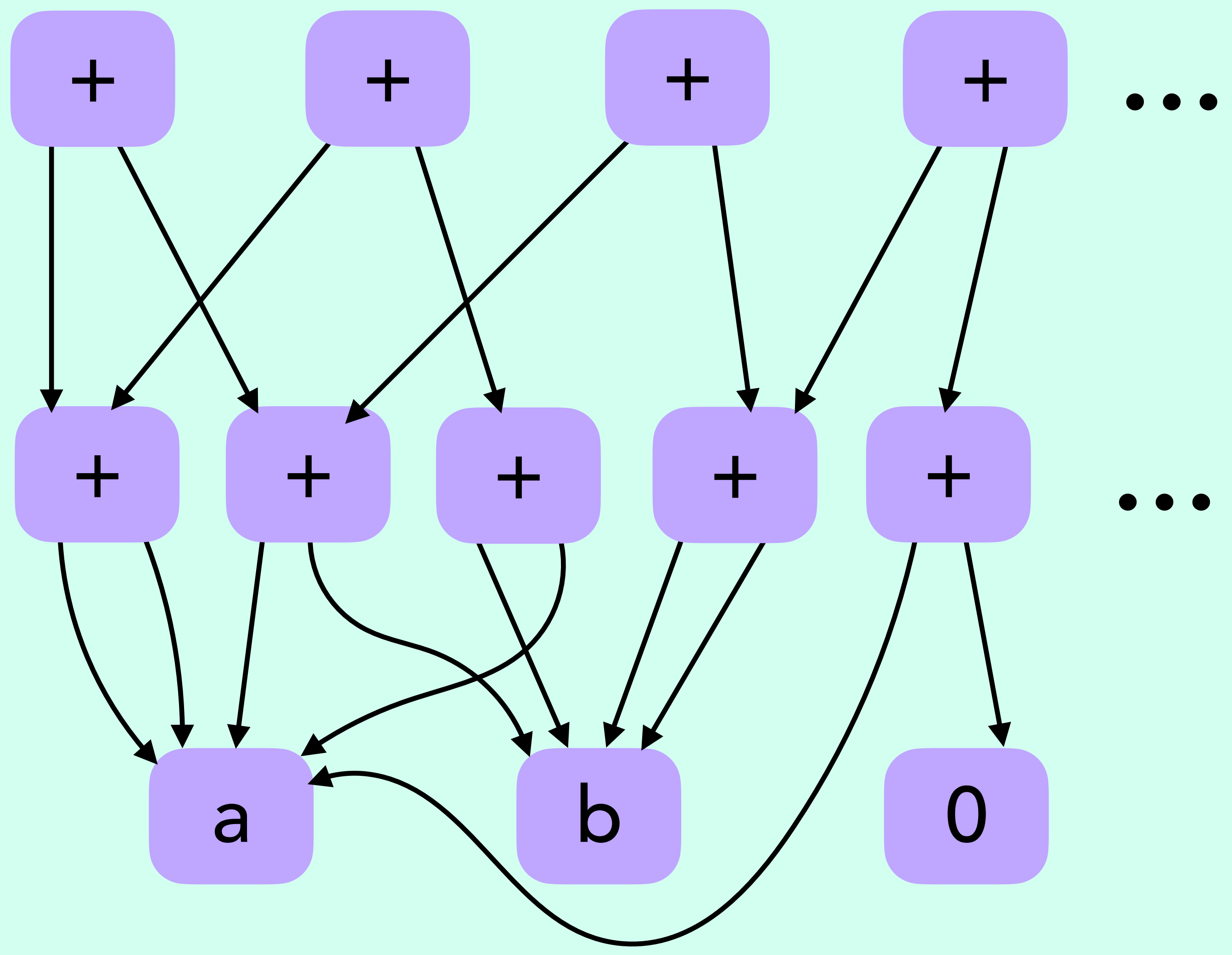

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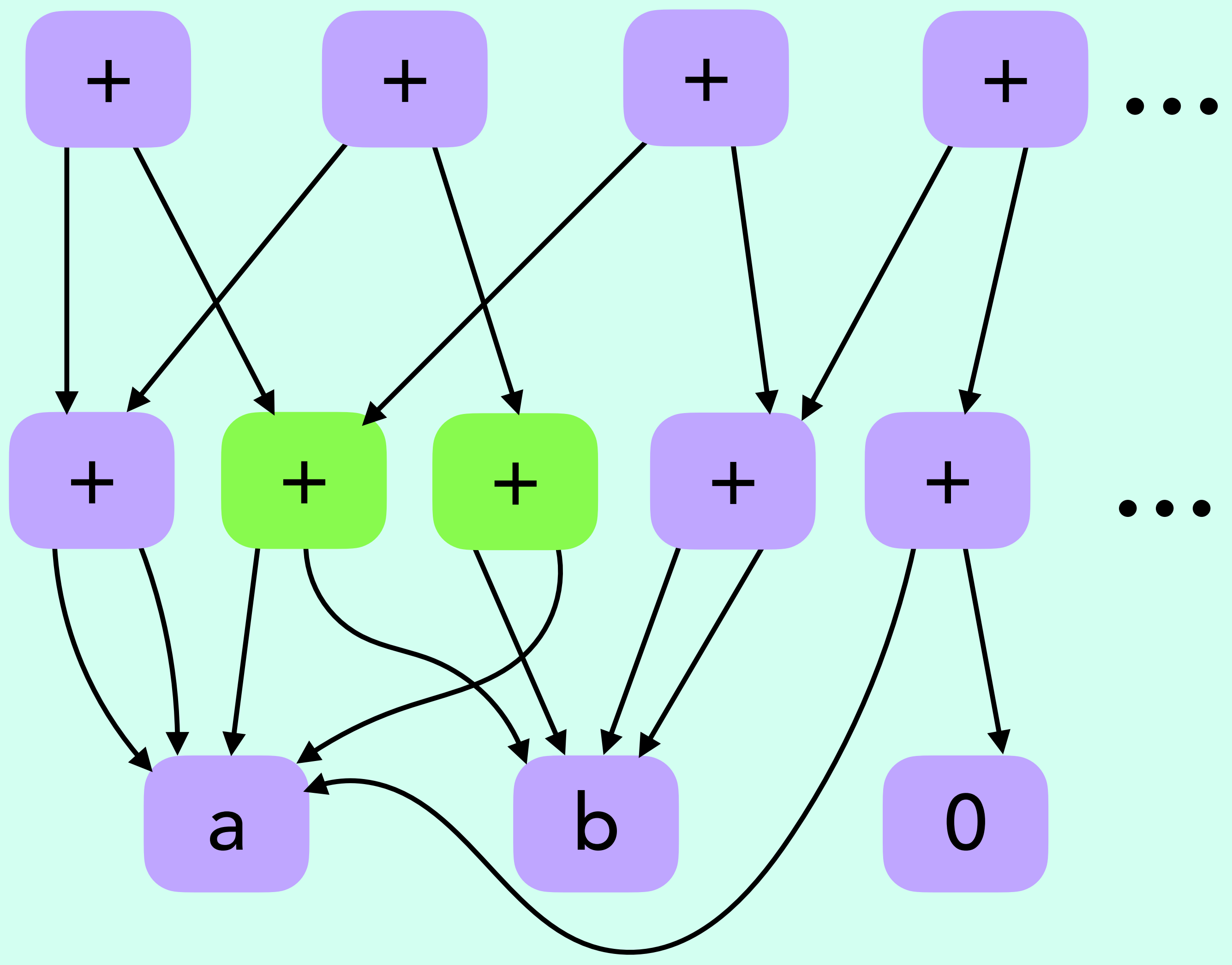


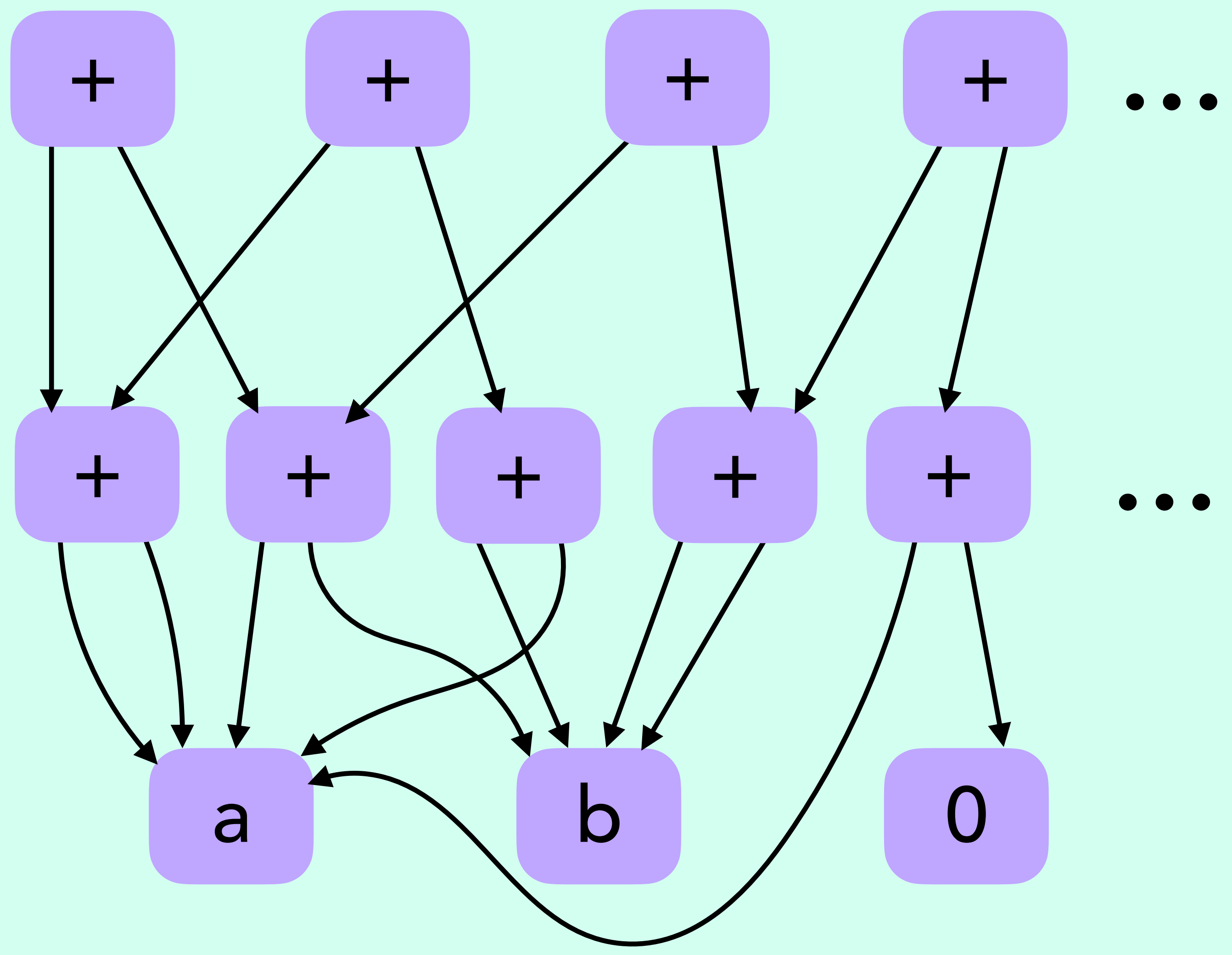
$(x + y) \longleftrightarrow (y + x)$

Joshi et al. 2002, Bansal et al. 2006, Singh et al. 2016, Menendez et al. 2017, …

A 3-Step Approach for Inferring Rewrite Rules

Enumerate terms from a grammar

a, b, 0, +, …



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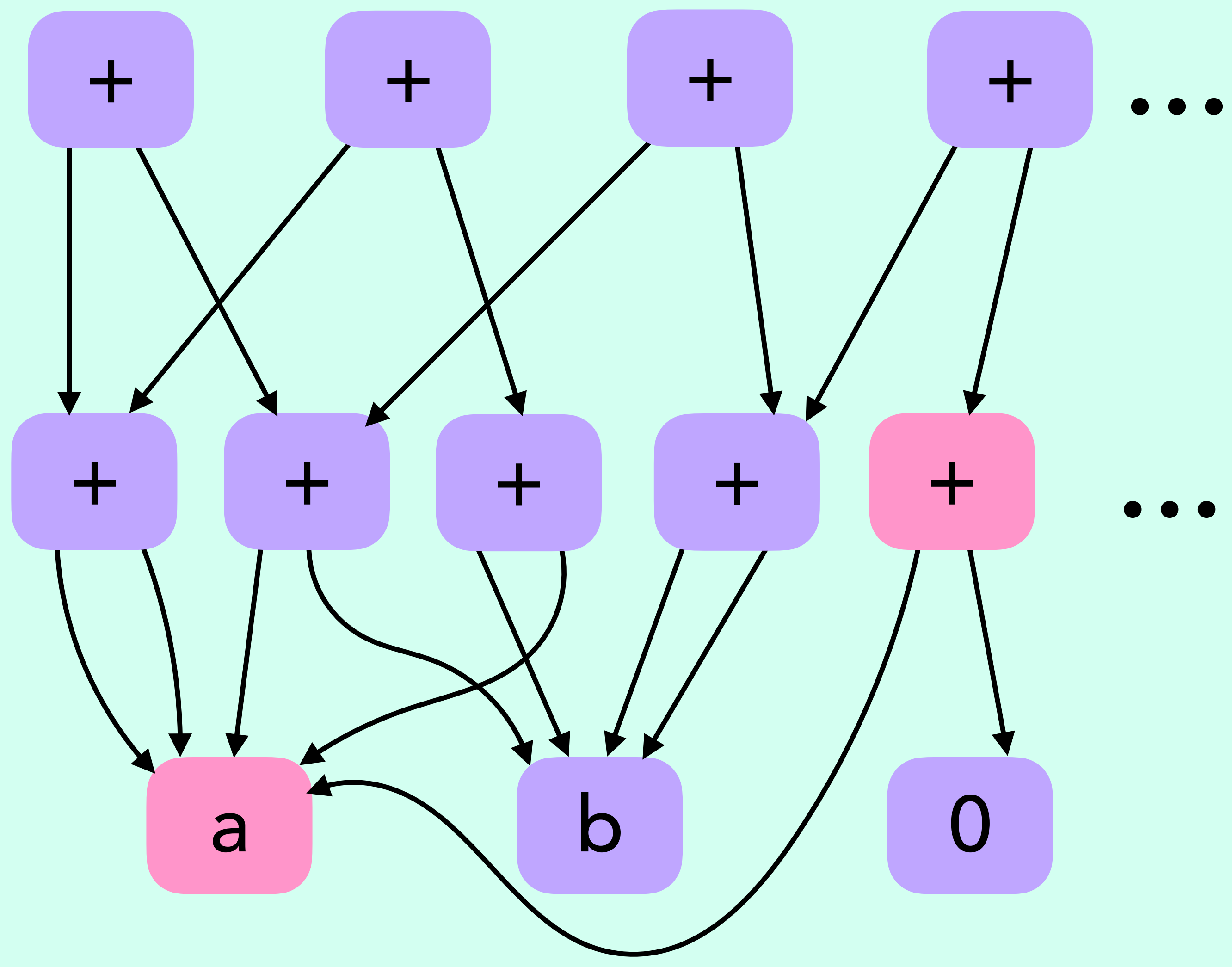

$(x + 0) \longleftrightarrow x$

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A 3-Step Approach for Inferring Rewrite Rules

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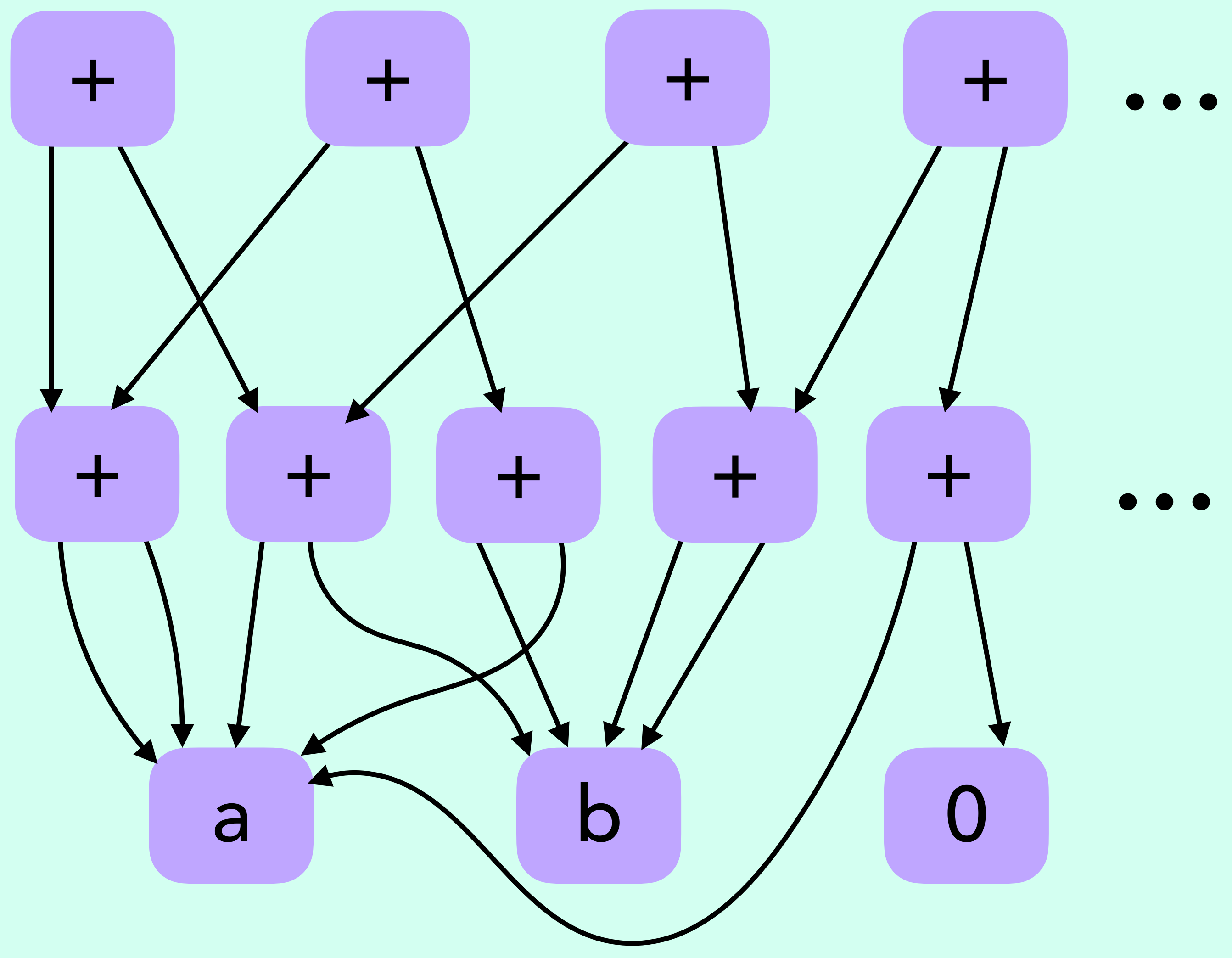

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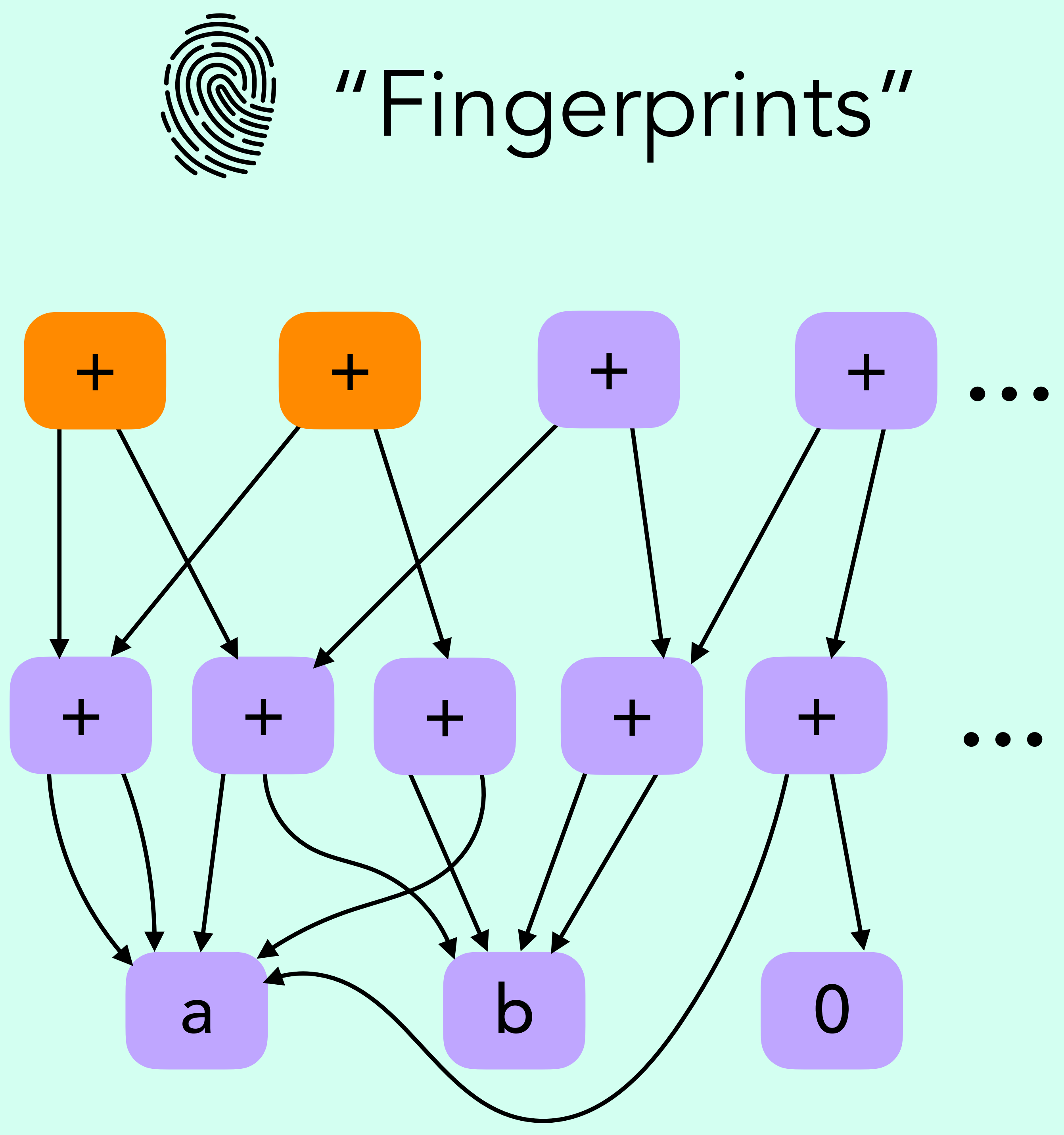


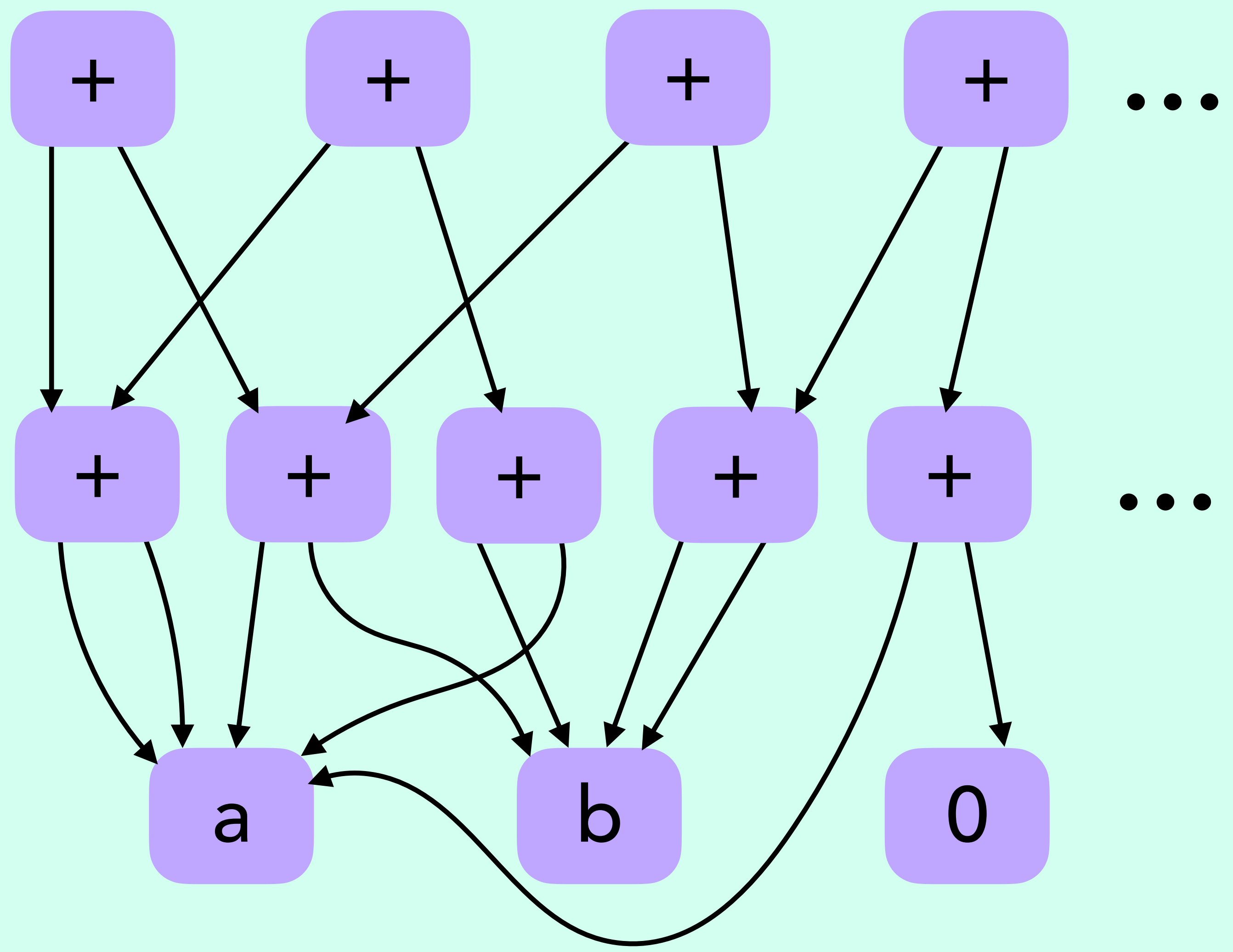
$(x + x) + (x + y) \leftrightarrow (x + x) + (y + x)$

Joshi et al. 2002, Bansal et al. 2006, Singh et al. 2016, Menendez et al. 2017, …

A 3-Step Approach for Inferring Rewrite Rules

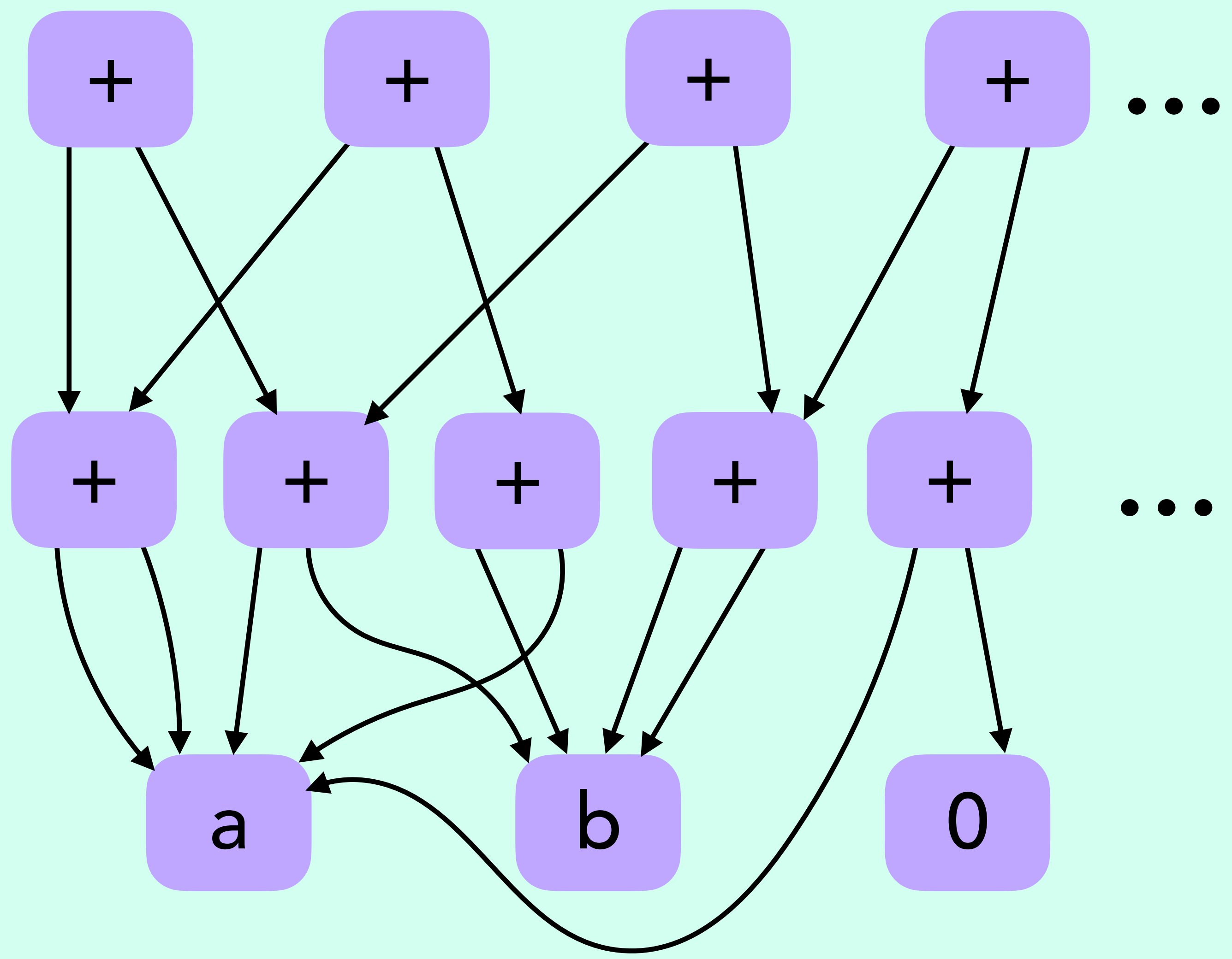
Enumerate terms from a grammar

a, b, 0, +, …



Find candidates: interpret over concrete inputs

"Fingerprints"



Filter candidates to get final ruleset

Remove redundant rules

x + 0 ⟷ 0 + x

y + 0 ⟷ 0 + y

x + y ⟷ y + x

Joshi et al. 2002, Bansal et al. 2006, Singh et al. 2016, Menendez et al. 2017, …

A 3-Step Approach for Inferring Rewrite Rules

Enumerate terms from a grammar

Exponentially many terms!

Find candidates: interpret over concrete inputs

Too many candidates, some potentially unsound!

Filter candidates to get final ruleset

Hard to find a small, useful ruleset

Joshi et al. 2002, Bansal et al. 2006, Singh et al. 2016, Menendez et al. 2017, …

Equality Saturation for Inferring Rewrite Rules

This Talk:

Inferring ***Small***, ***Useful*** Rulesets ***Faster*** using **Equality Saturation**!

Joshi et al. 2002, Bansal et al. 2006, Singh et al. 2016, Menendez et al. 2017, …

What is *Equality Saturation?*

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$(a * 2) / 2$

What is *Equality Saturation*?

$(a * 2) / 2$ $\qquad\qquad$ a

What is *Equality Saturation*?

$$(a * 2) / 2 \quad \Rightarrow \quad a$$

???

What is *Equality Saturation?*

$$(a * 2) / 2 \quad \Rightarrow \quad a$$

Rewrite rules!

$$(x * y) / z \longleftrightarrow x * (y / z)$$

$$y / y \longrightarrow 1$$

$$x * 1 \longleftrightarrow x$$

How to Apply Rewrite Rules?

$$(a * 2) / 2 \Rightarrow a * (2 / 2)$$

$$(x * y) / z \longleftrightarrow x * (y / z)$$

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$$(a * 2) / 2 \Rightarrow a * (2 / 2)$$

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$$a * (2 / 2) \Rightarrow a * 1$$

$$y / y \longrightarrow 1$$

How to Apply Rewrite Rules?

$$(a * 2) / 2 \Rightarrow a * (2 / 2)$$

$(x * y) / z \longleftrightarrow x * (y / z)$

$$a * (2 / 2) \Rightarrow a * 1$$

$y / y \longrightarrow 1$

$$a * 1 \Rightarrow a$$

$x * 1 \longleftrightarrow x$

Destructively, In a Specific Order

(a * 2 … 2 / 2)

a * (2 … * 1

Order of rule application affects result

Missed opportunities for optimizations

Same order may not work for all inputs

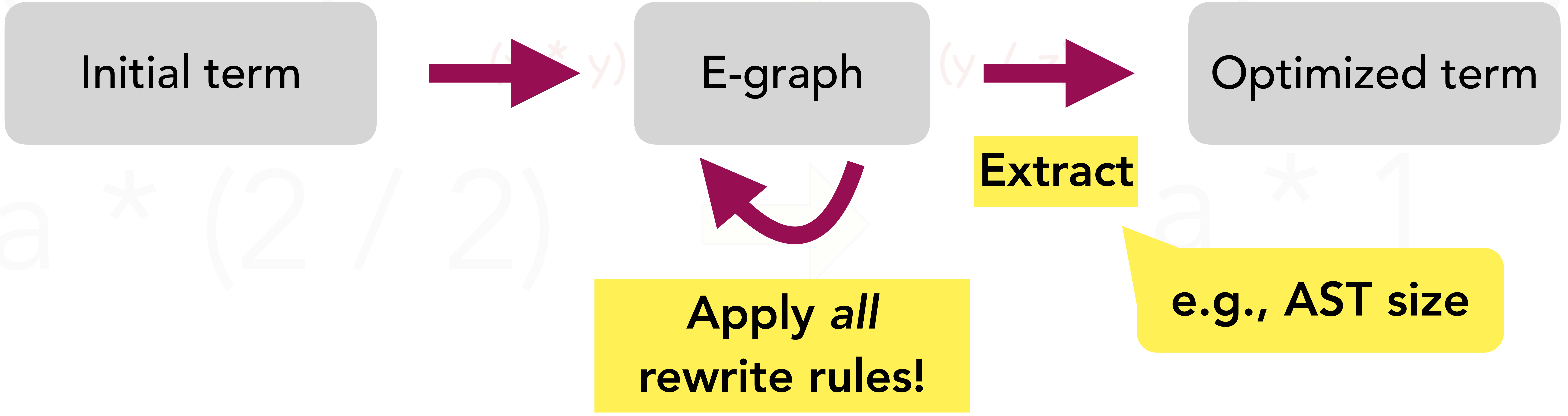
Old expression is lost

a

e.g., supporting commutativity is hard without additional tricks to ensure termination!

$x * 1 \leftrightarrow x$

Equality Saturation Mitigates Phase Ordering!



How Does Equality Saturation Work?

(a * 2) / 2

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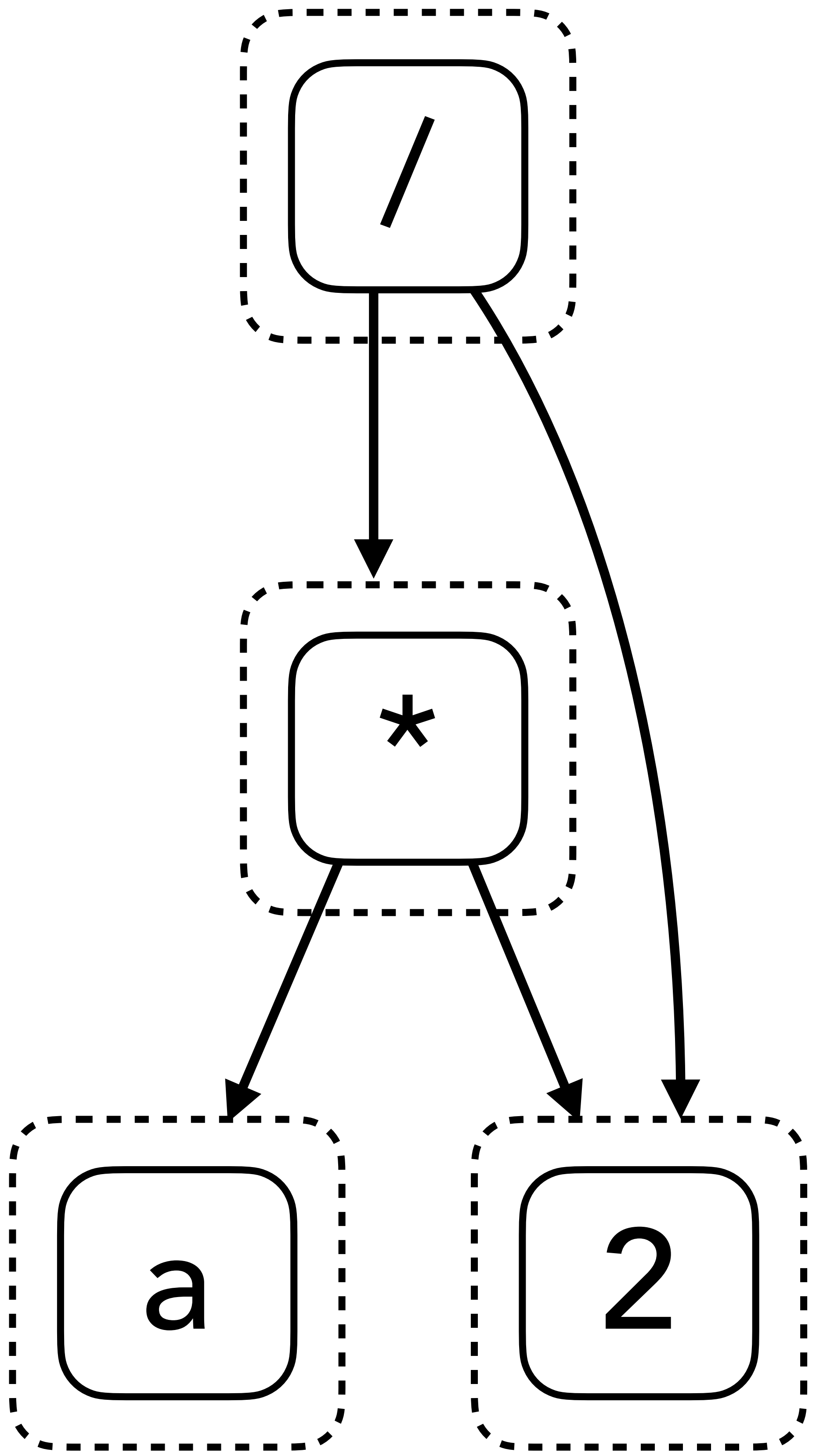
(a * 2) / 2



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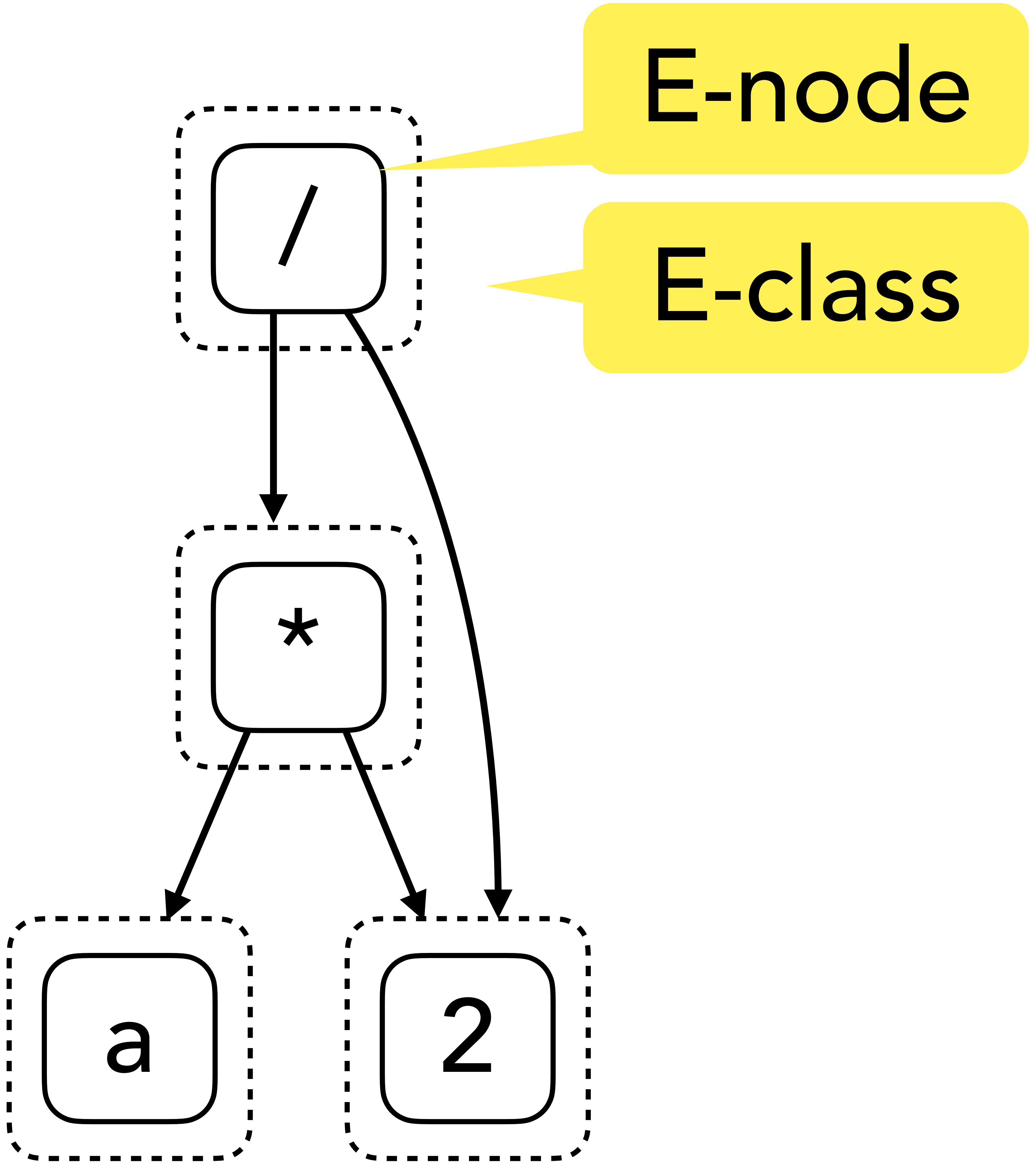

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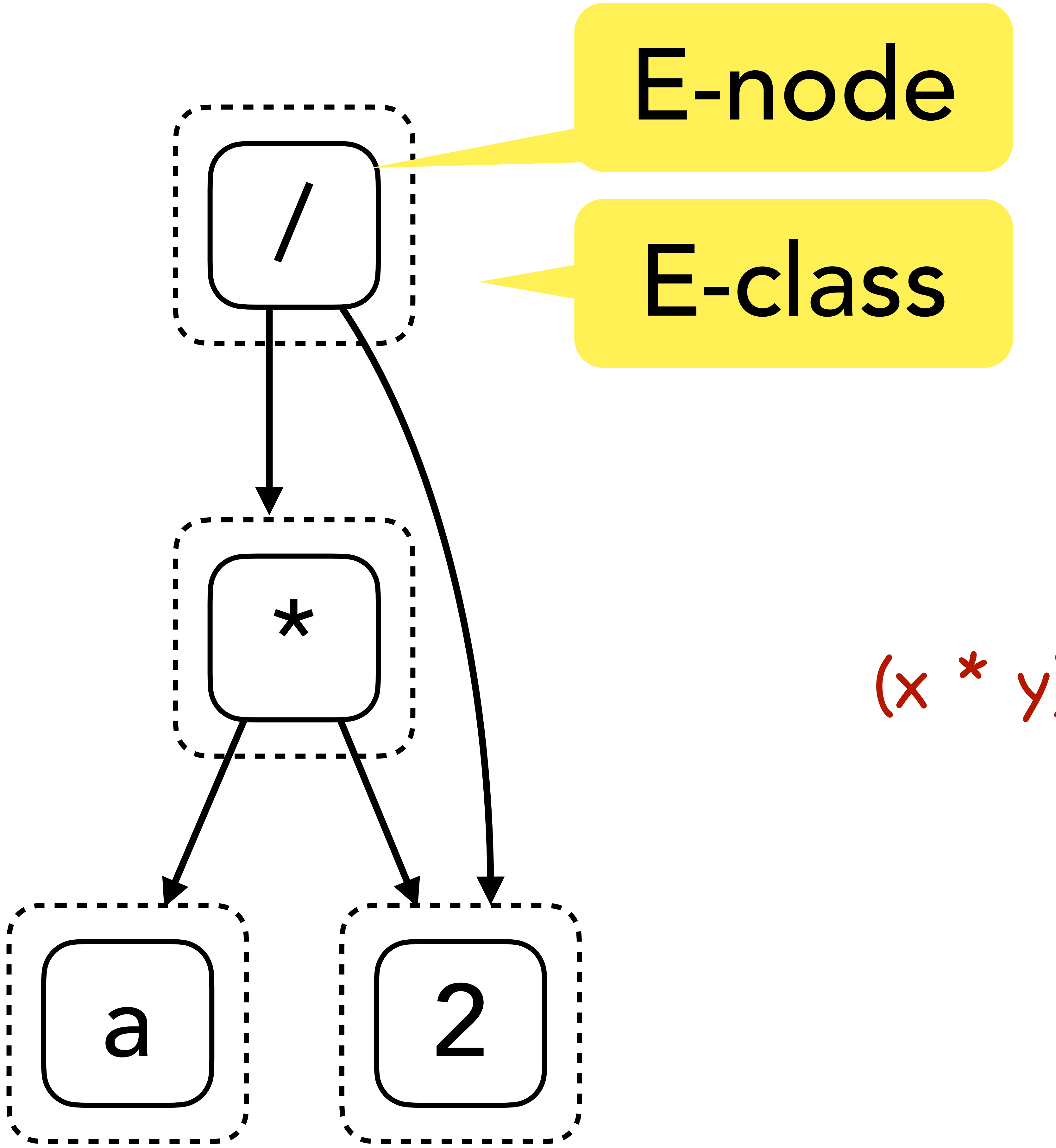
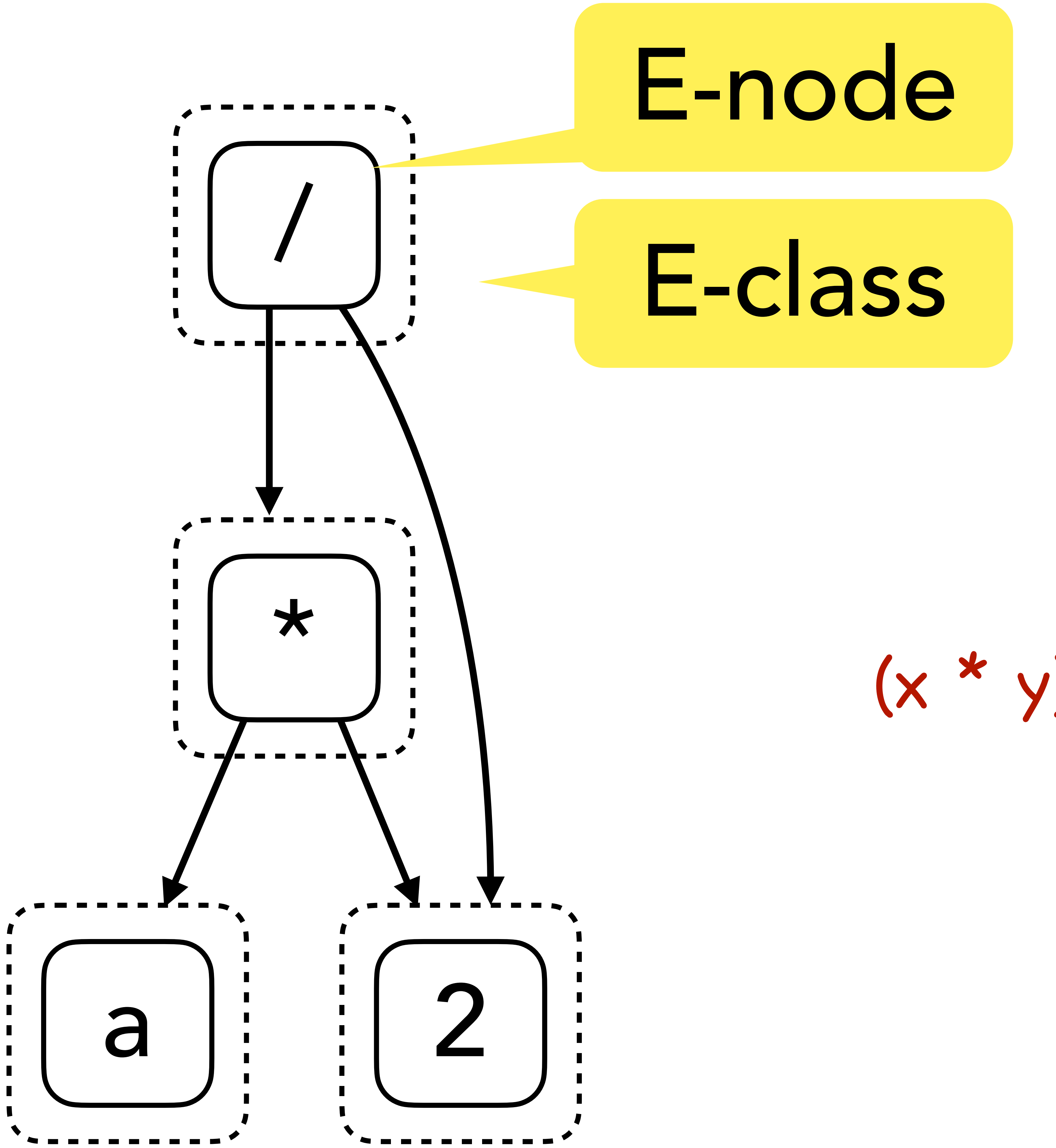


(x * y) / z ⟷ x * (y / z)

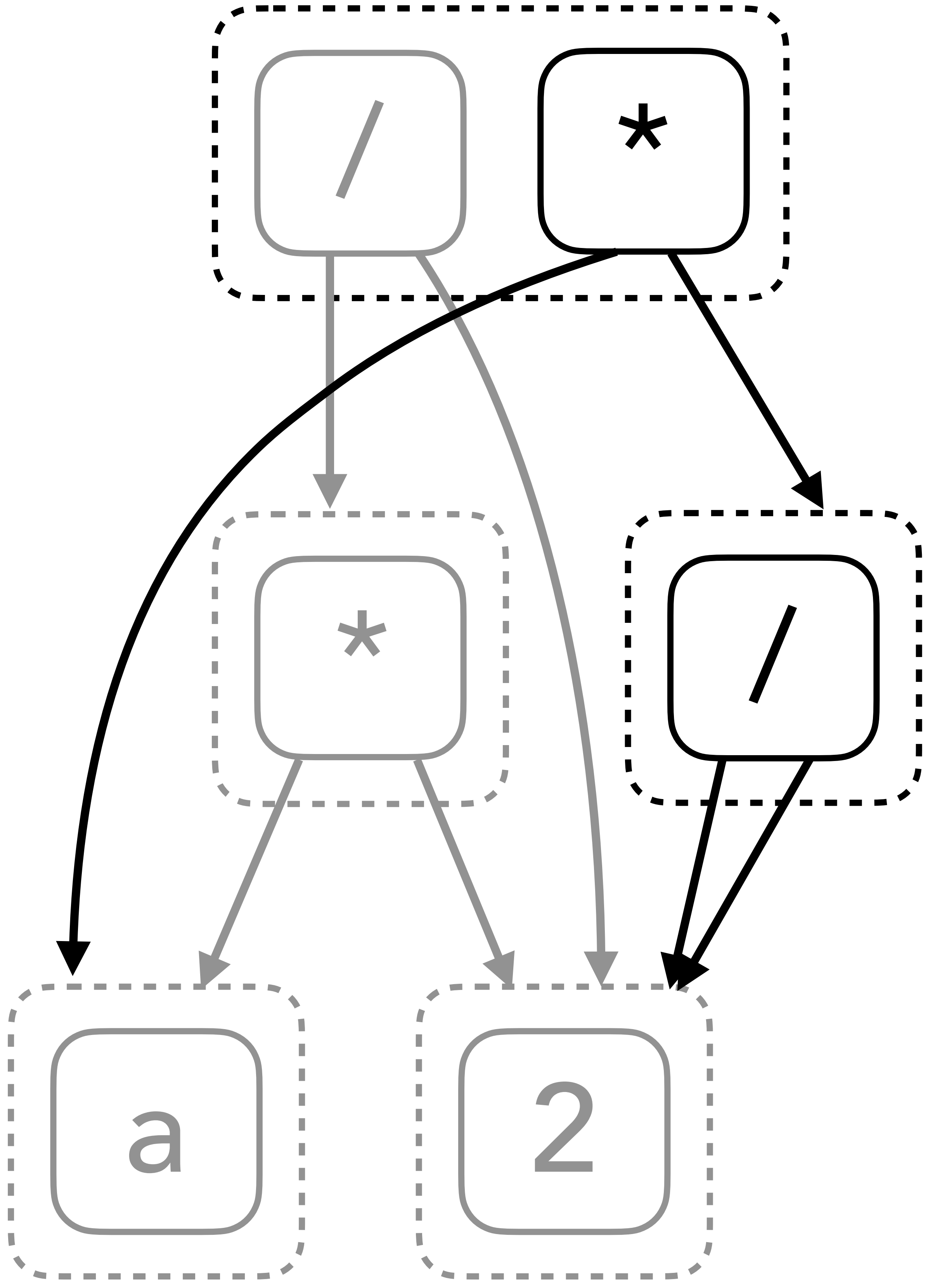
How Does Equality Saturation Work?

(a * 2) / 2



(x * y) / z ⟷ x * (y / z)

(a * 2) / 2, a * (2 / 2)



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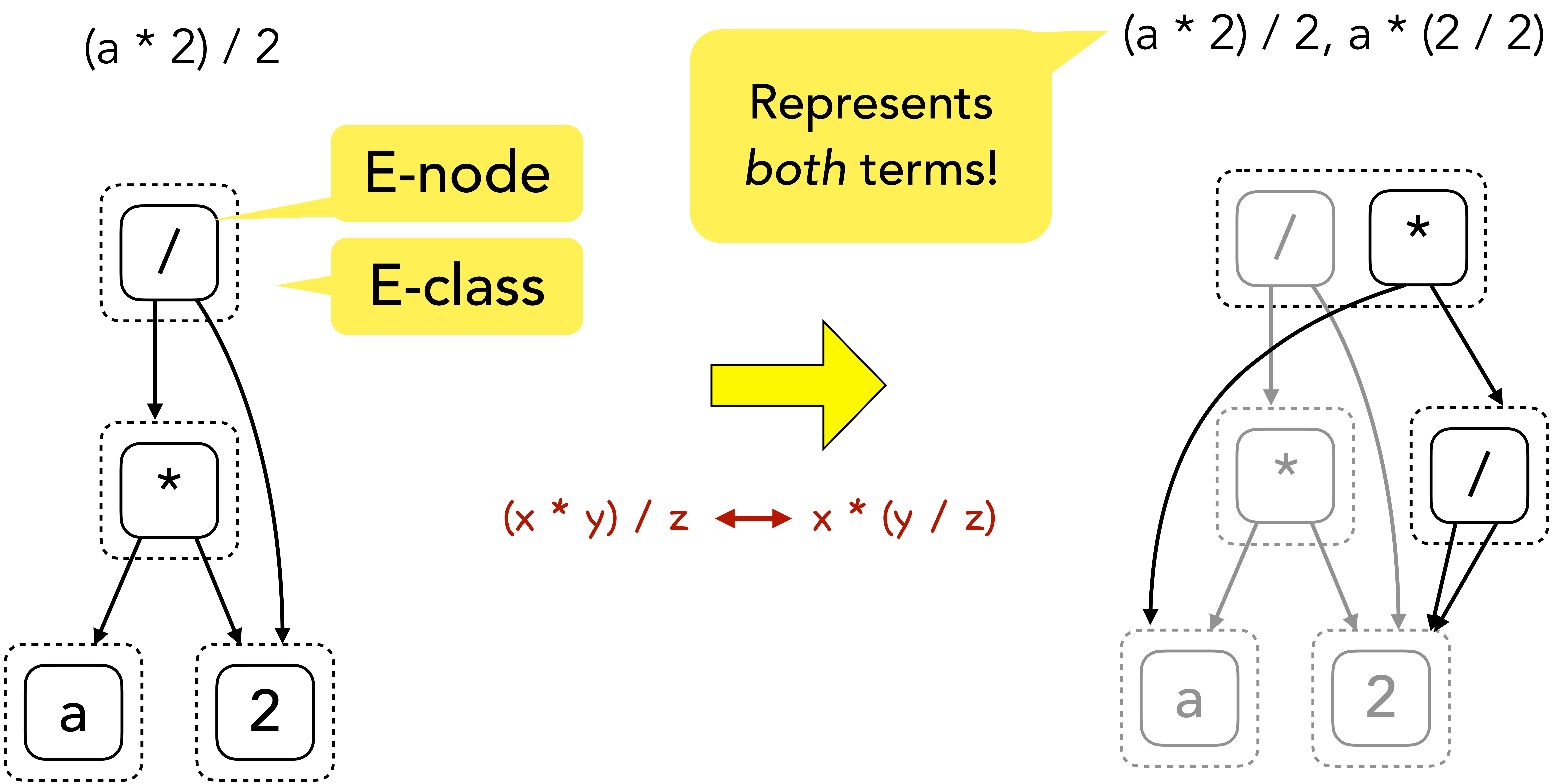
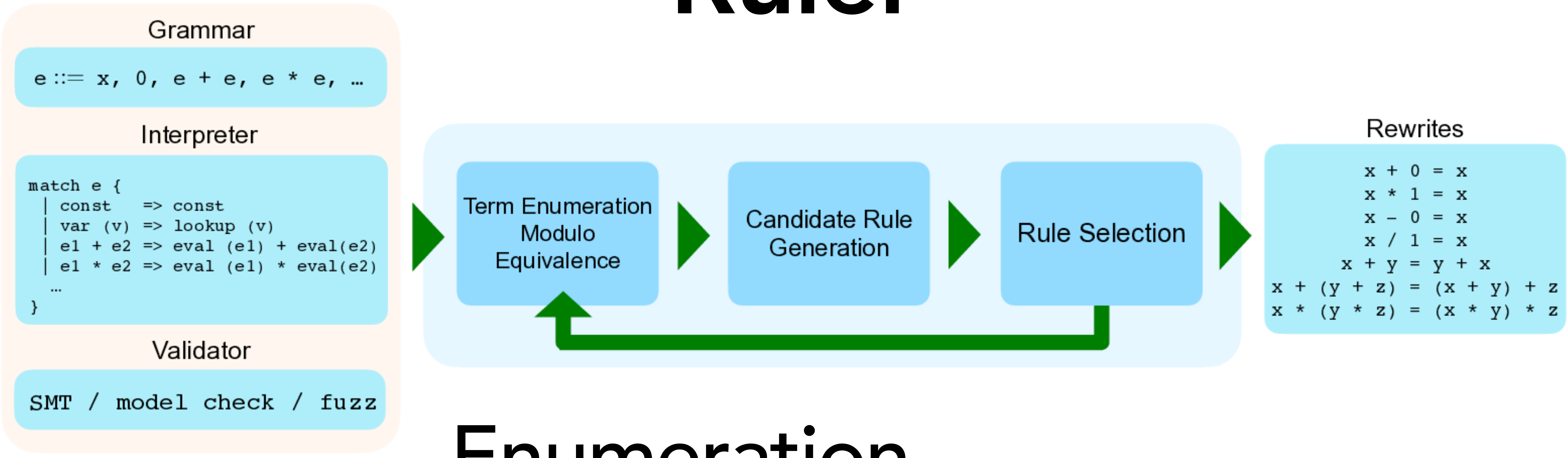


Equality Saturation for Inferring Rewrite Rules

Equality Saturation for not just applying rewrites, but to also *infer* them!

Joshi et al. 2002, Bansal et al. 2006, Singh et al. 2016, Menendez et al. 2017, ...

Ruler

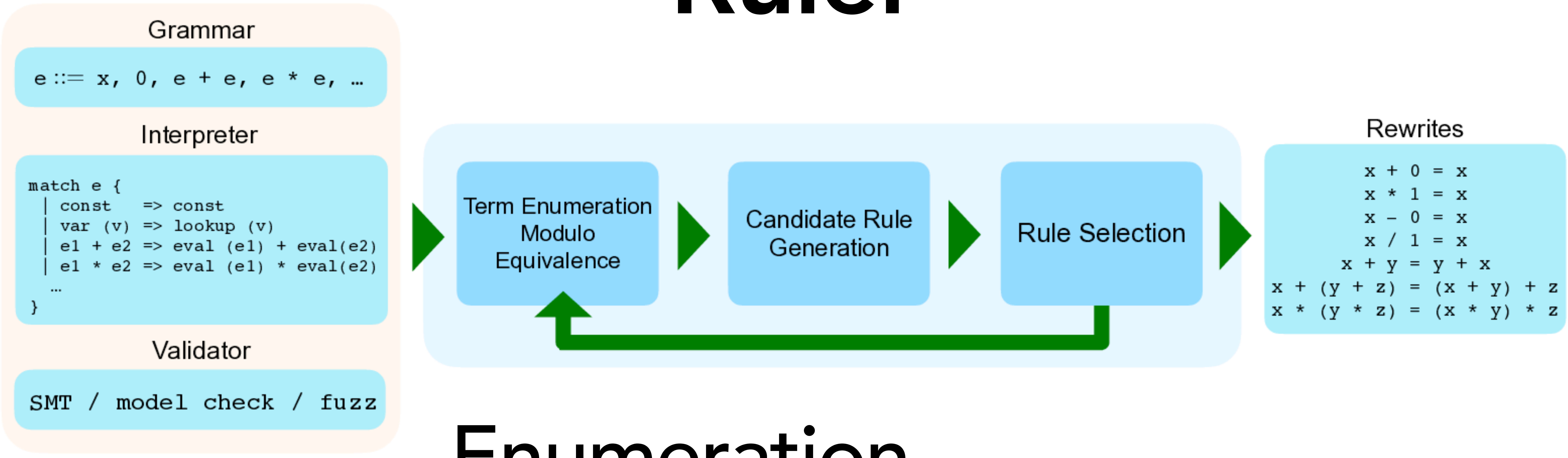


Enumeration

Candidate Generation

Rule Selection

Ruler



Enumeration

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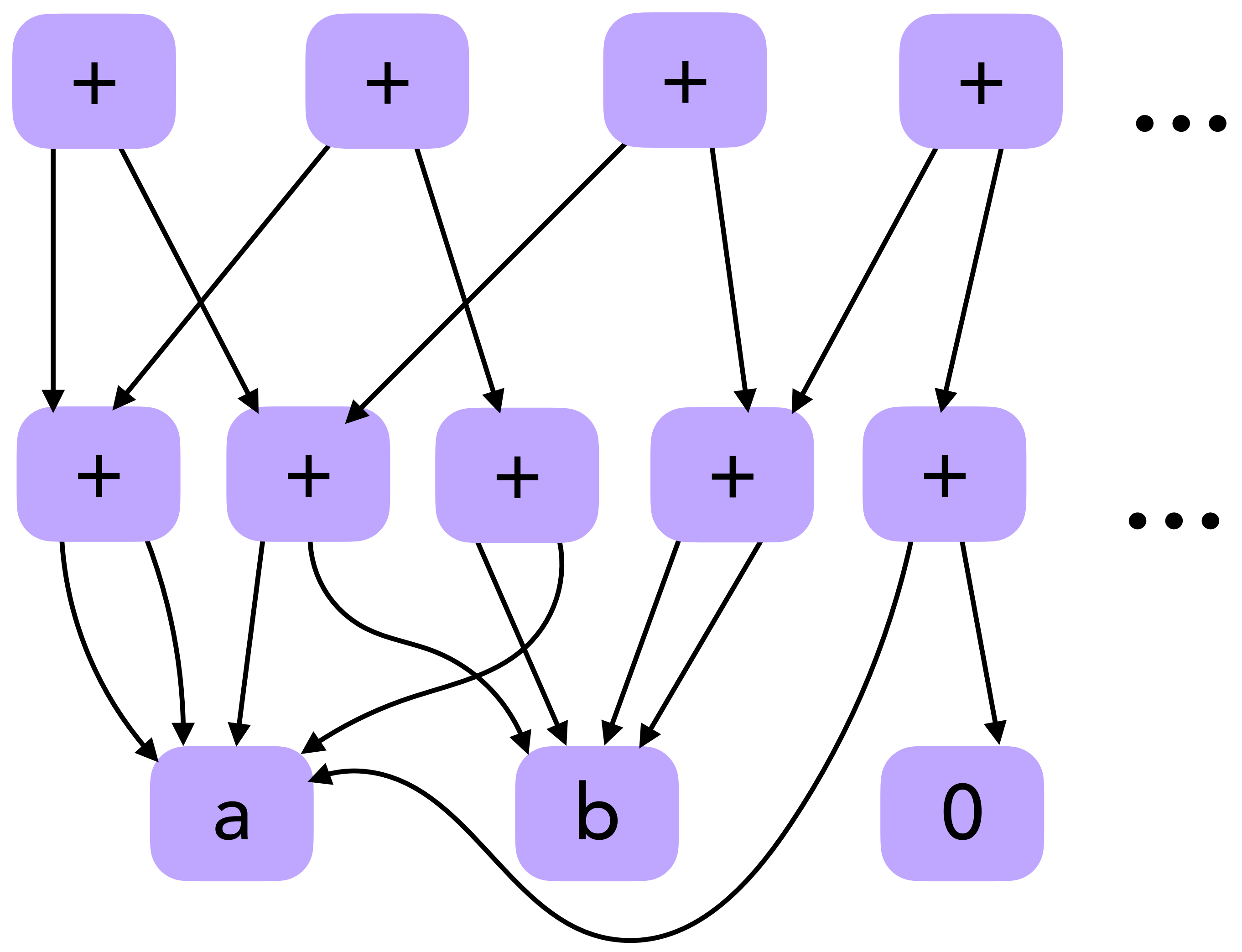
Enumeration Modulo Equality Saturation

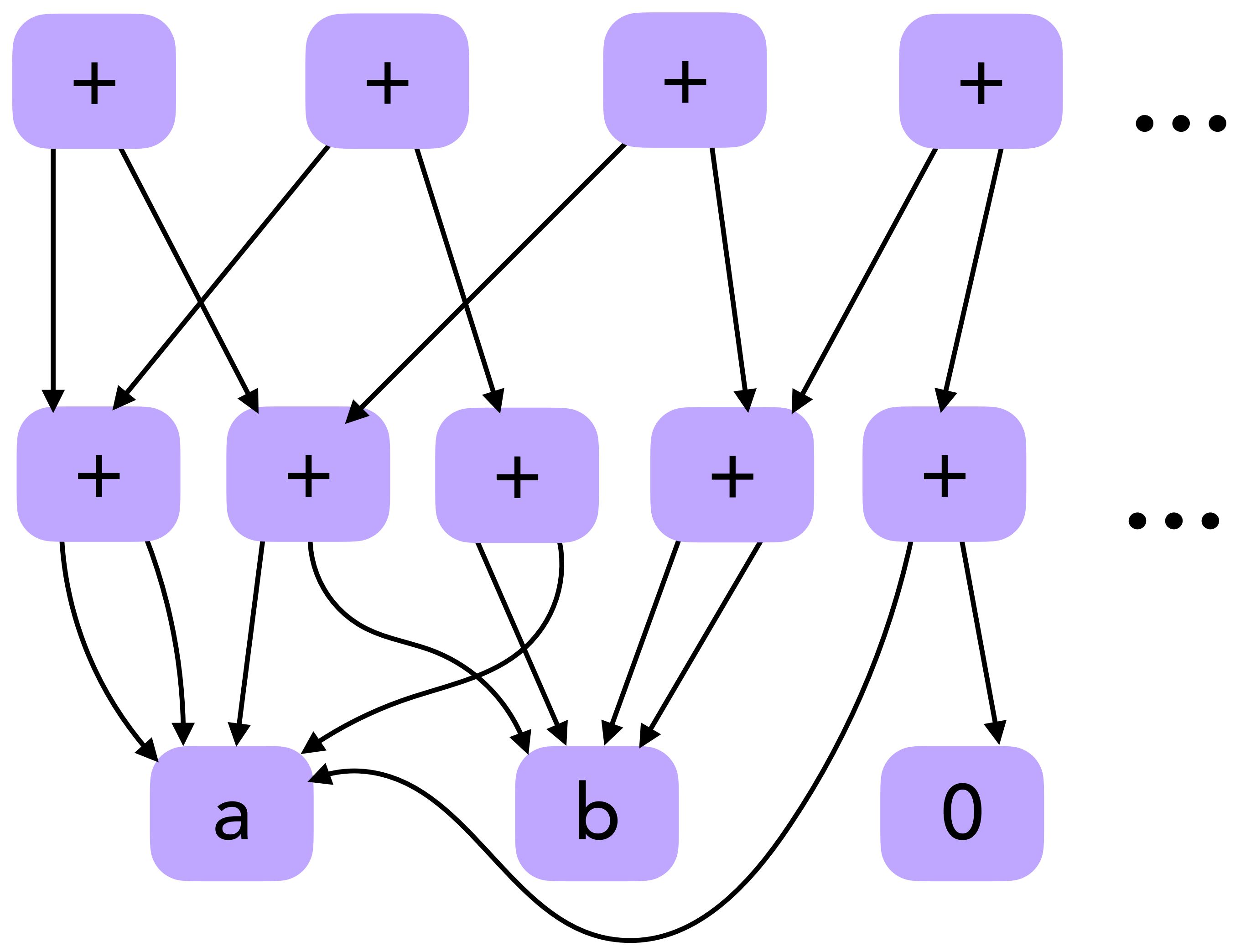
a, b, 0, +, …

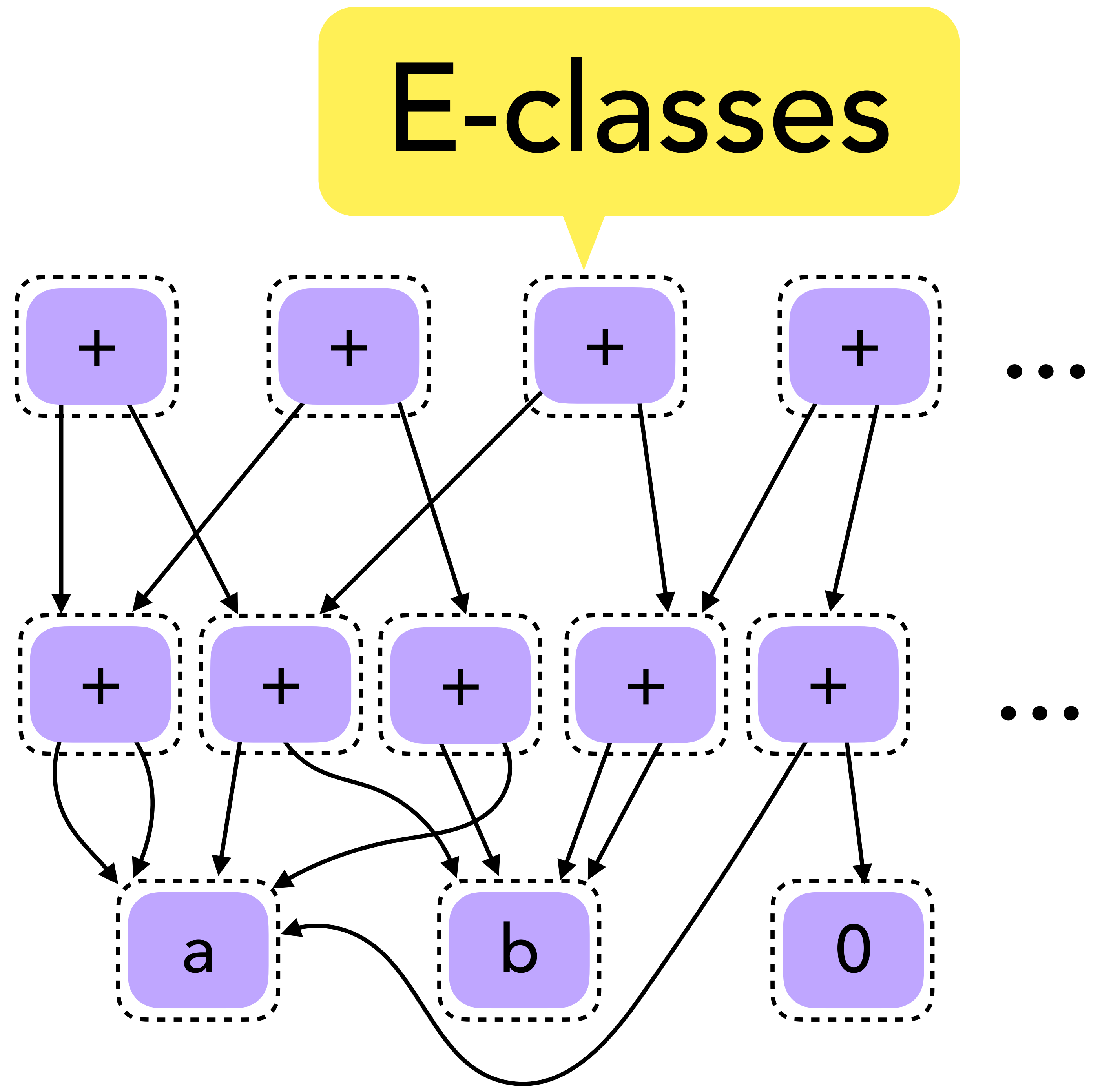


Exponentially many terms!

Enumeration Modulo Equality Saturation

a, b, 0, +, …



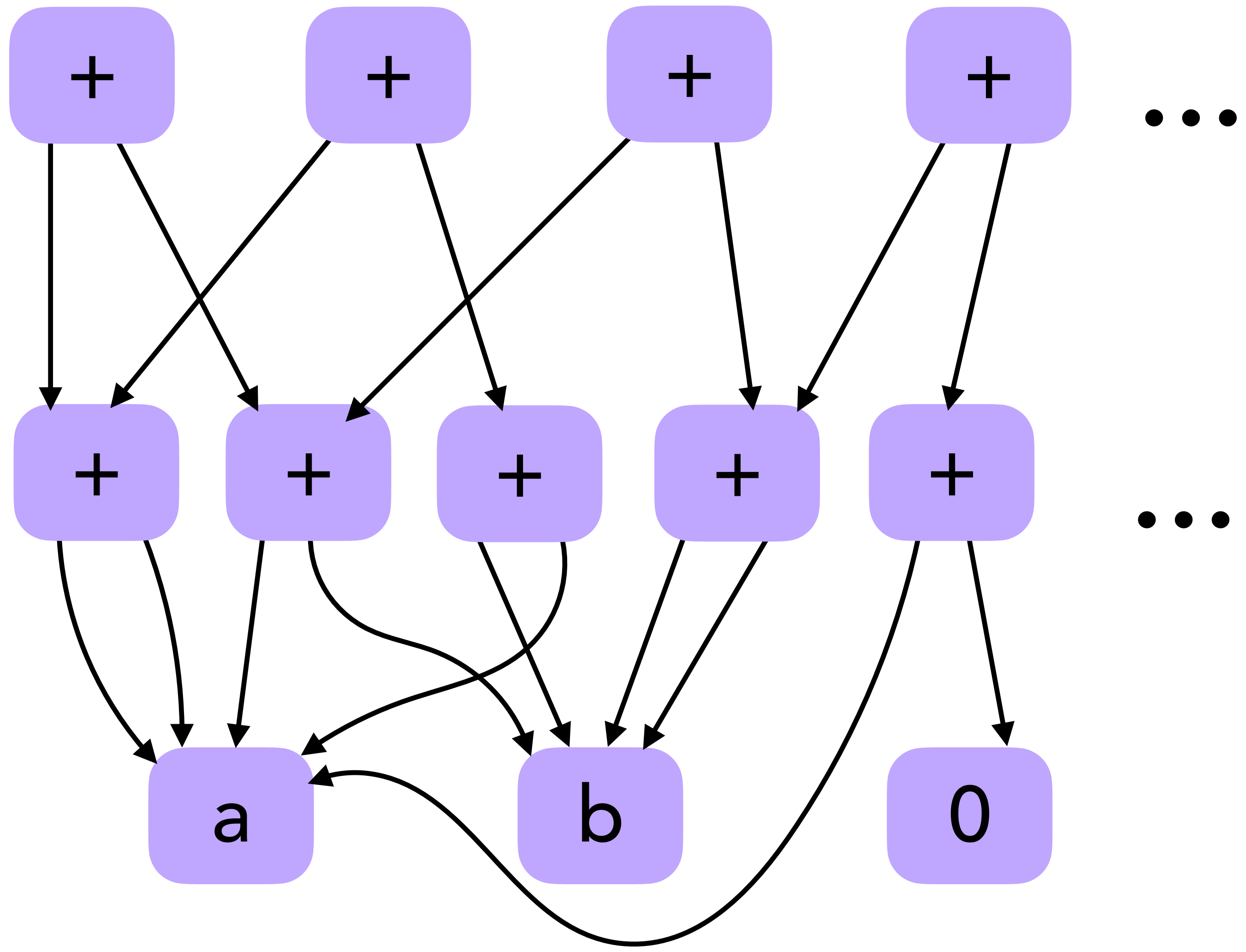


Exponentially many terms!

Enumerate *over* an E-graph

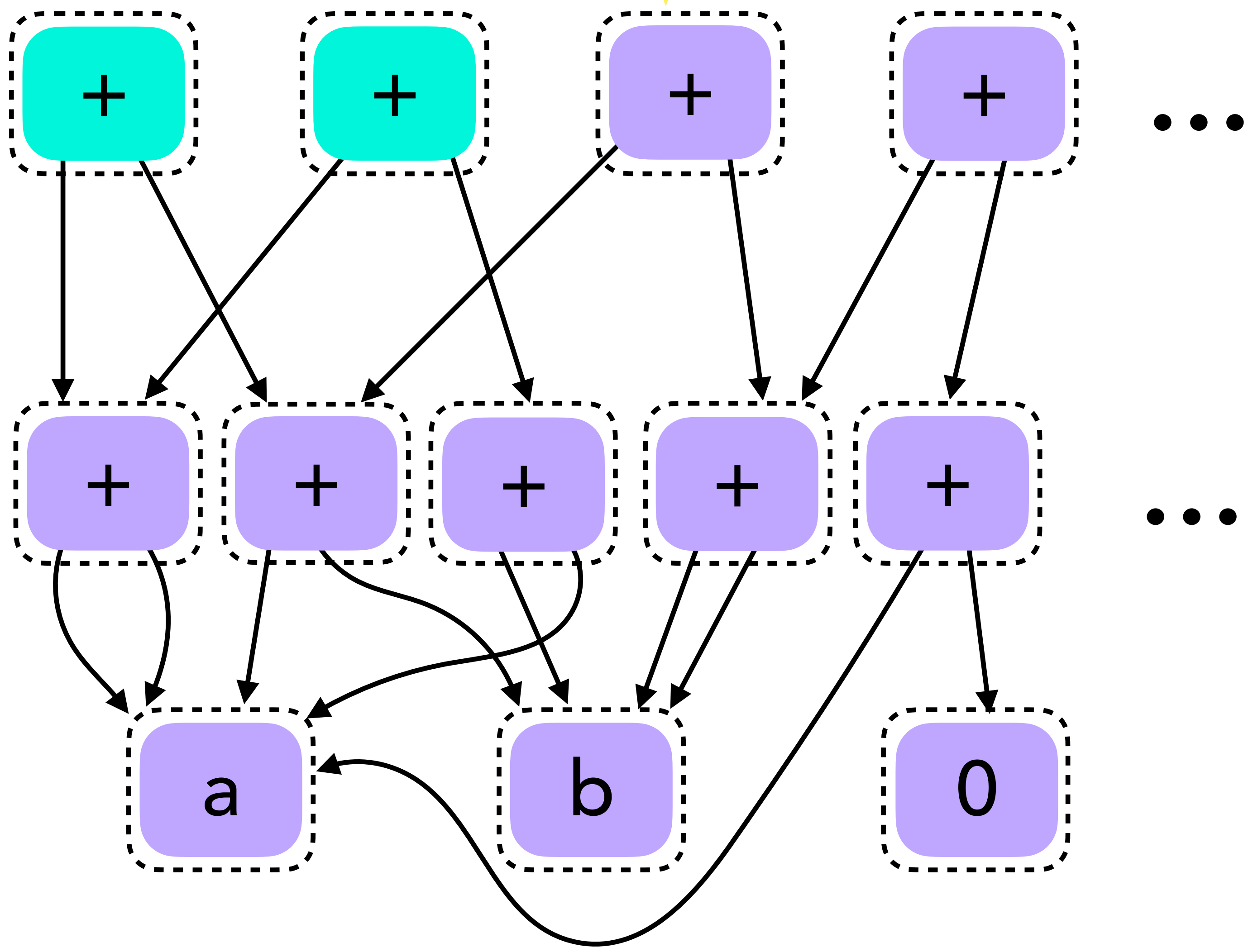
Enumeration Modulo Equality Saturation

a, b, 0, +, …



Exponentially many terms!

E-classes



Enumerate *over* an E-graph

(x + x) + (x + y)

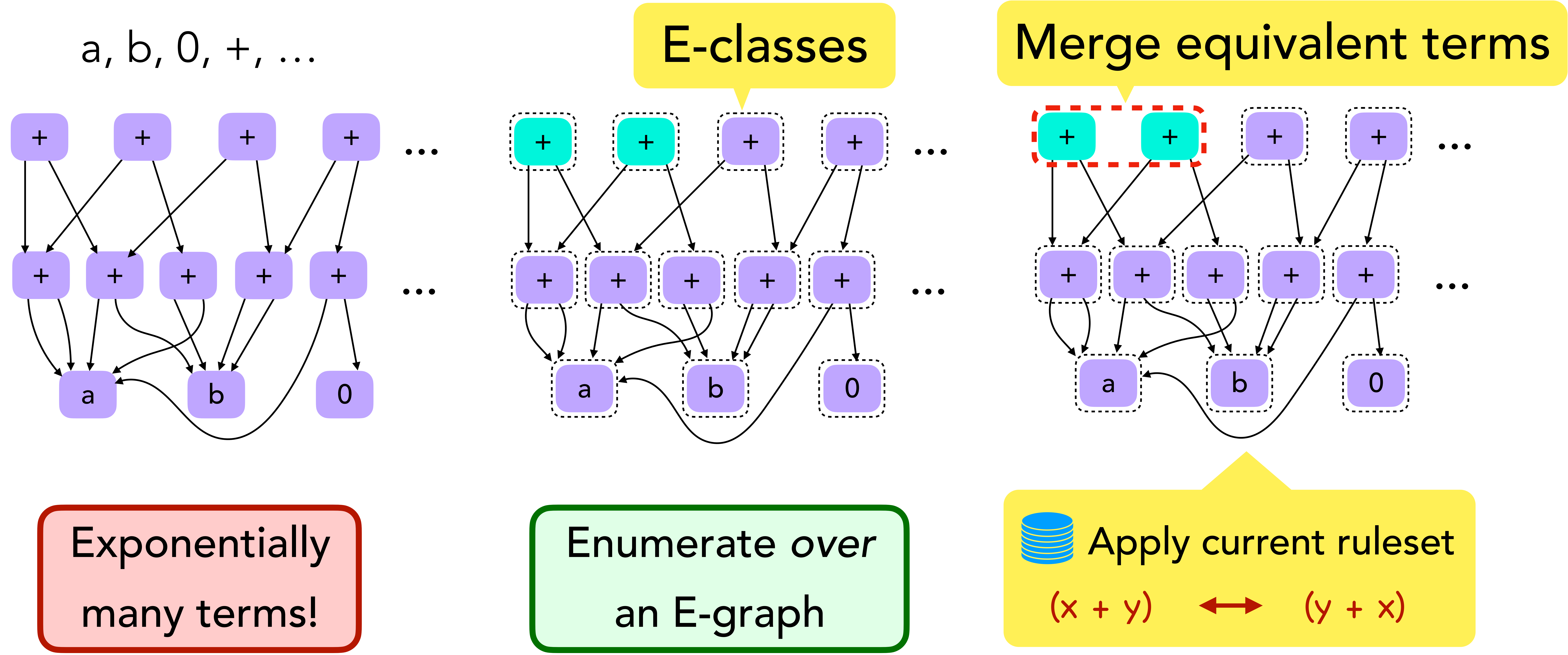
=

(x + x) + (y + x)

Apply current ruleset

(x + y) ⟷ (y + x)

Enumeration Modulo Equality Saturation



Enumeration Modulo Equality Saturation

E-classes

Merge equivalent terms

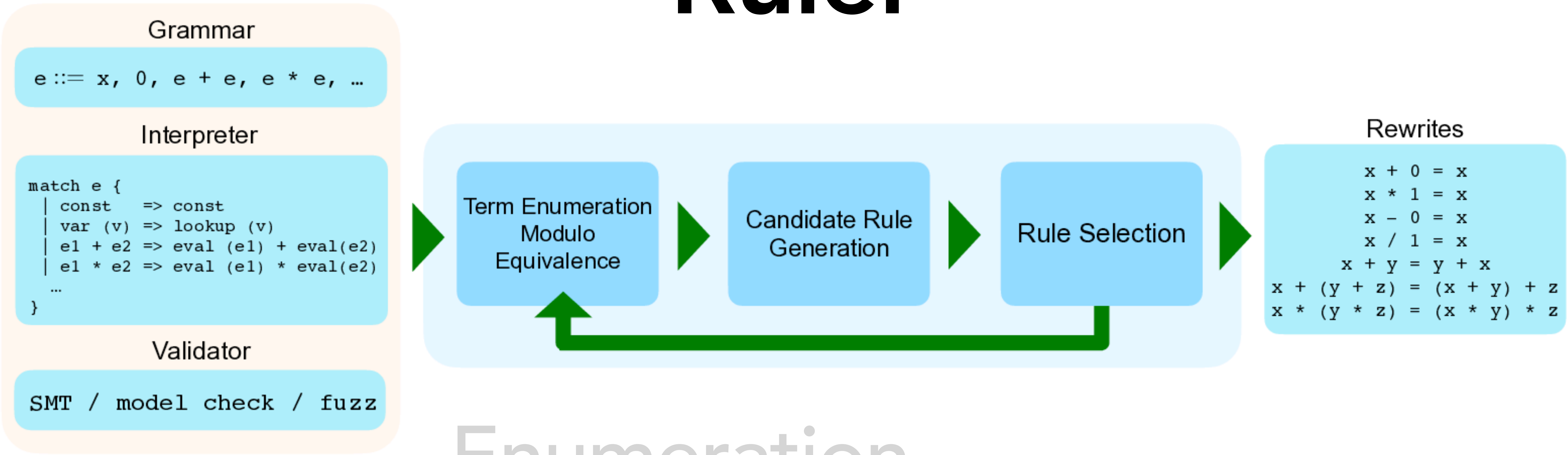
Shrinks the term space by applying rewrites as they are learned!

many terms!

an E-graph

(x + y) ⟷ (y + x)

Ruler

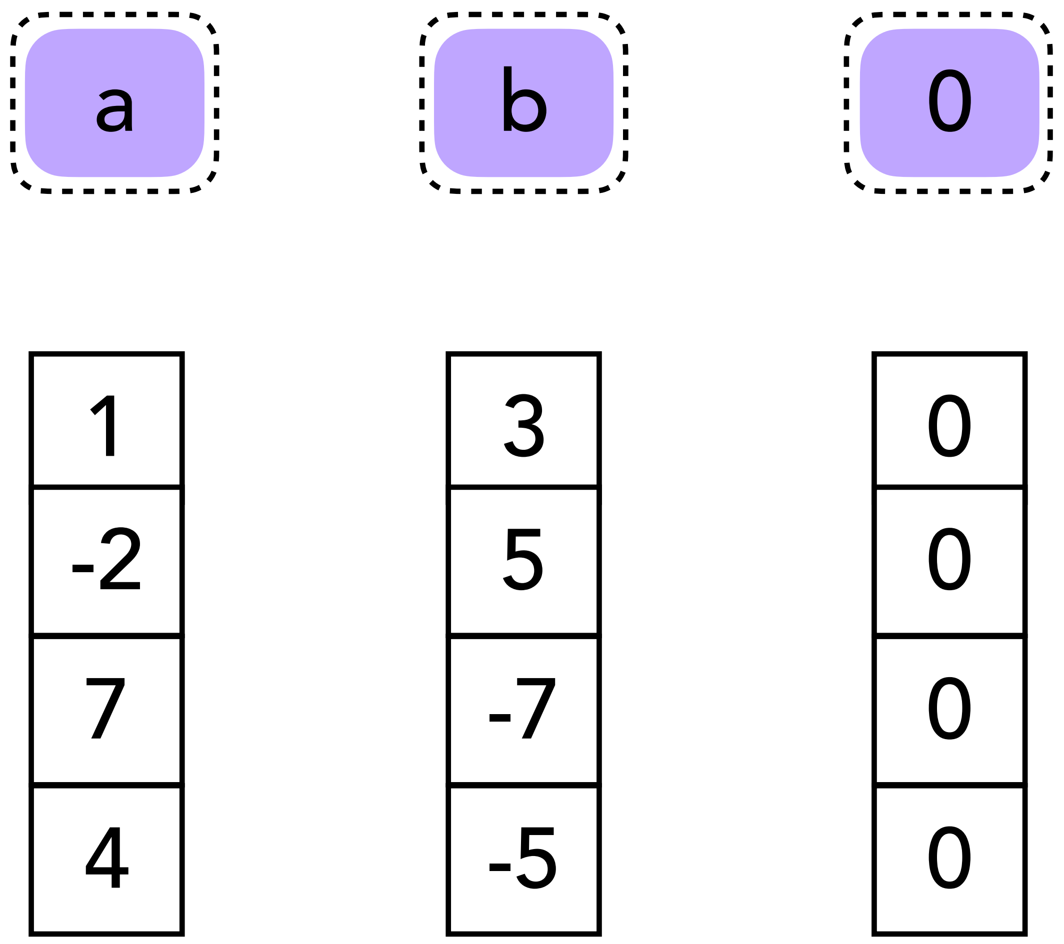


Enumeration

Candidate Generation

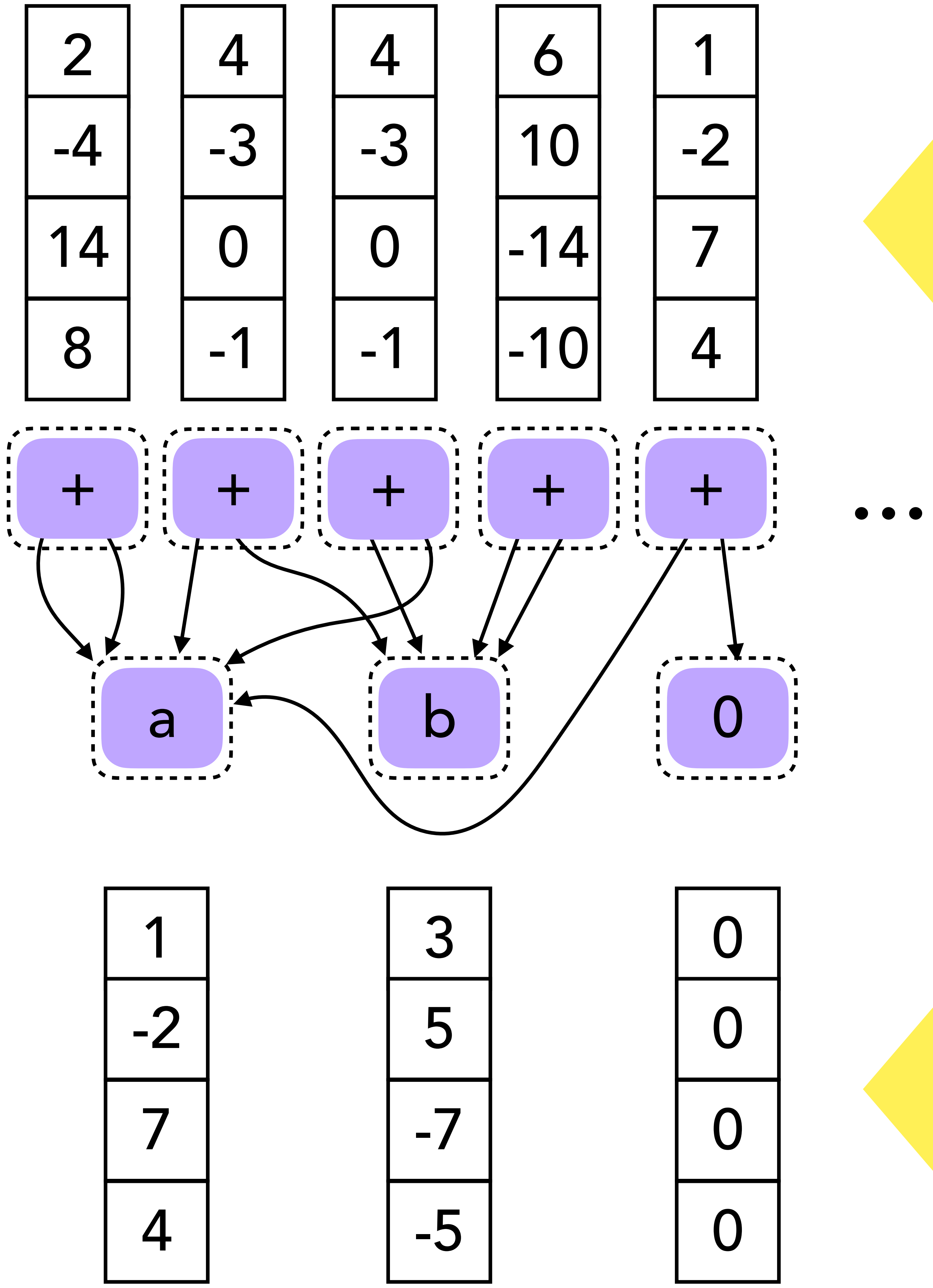
Rule Selection

Candidate Generation by Characteristic Vector Matching



Seed initial E-classes with concrete values (cvecs) from the domain

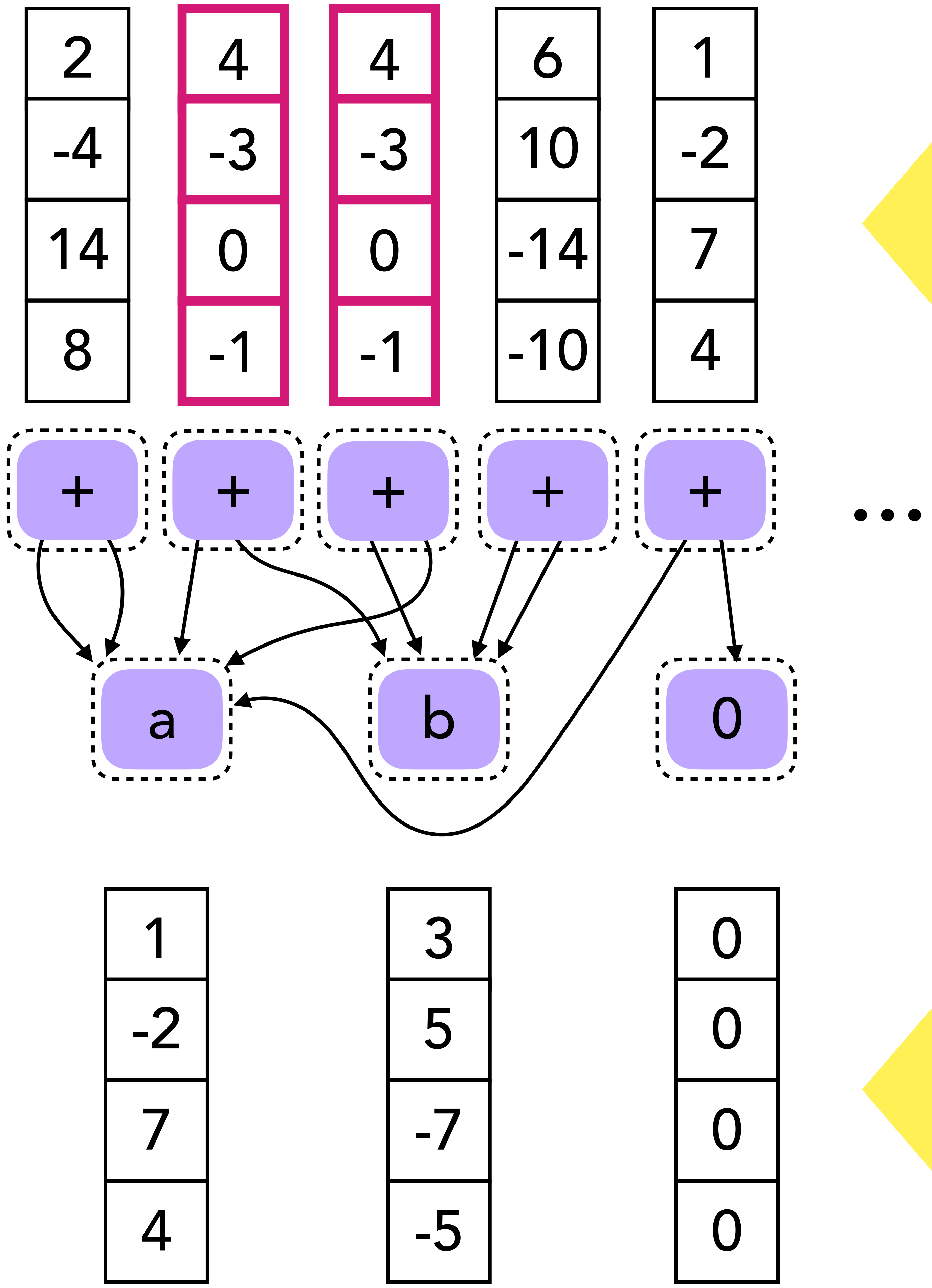
Candidate Generation by Characteristic Vector Matching



Compute the cvecs for newly enumerated E-classes

Seed initial E-classes with concrete values (cvecs) from the domain

Candidate Generation by Characteristic Vector Matching

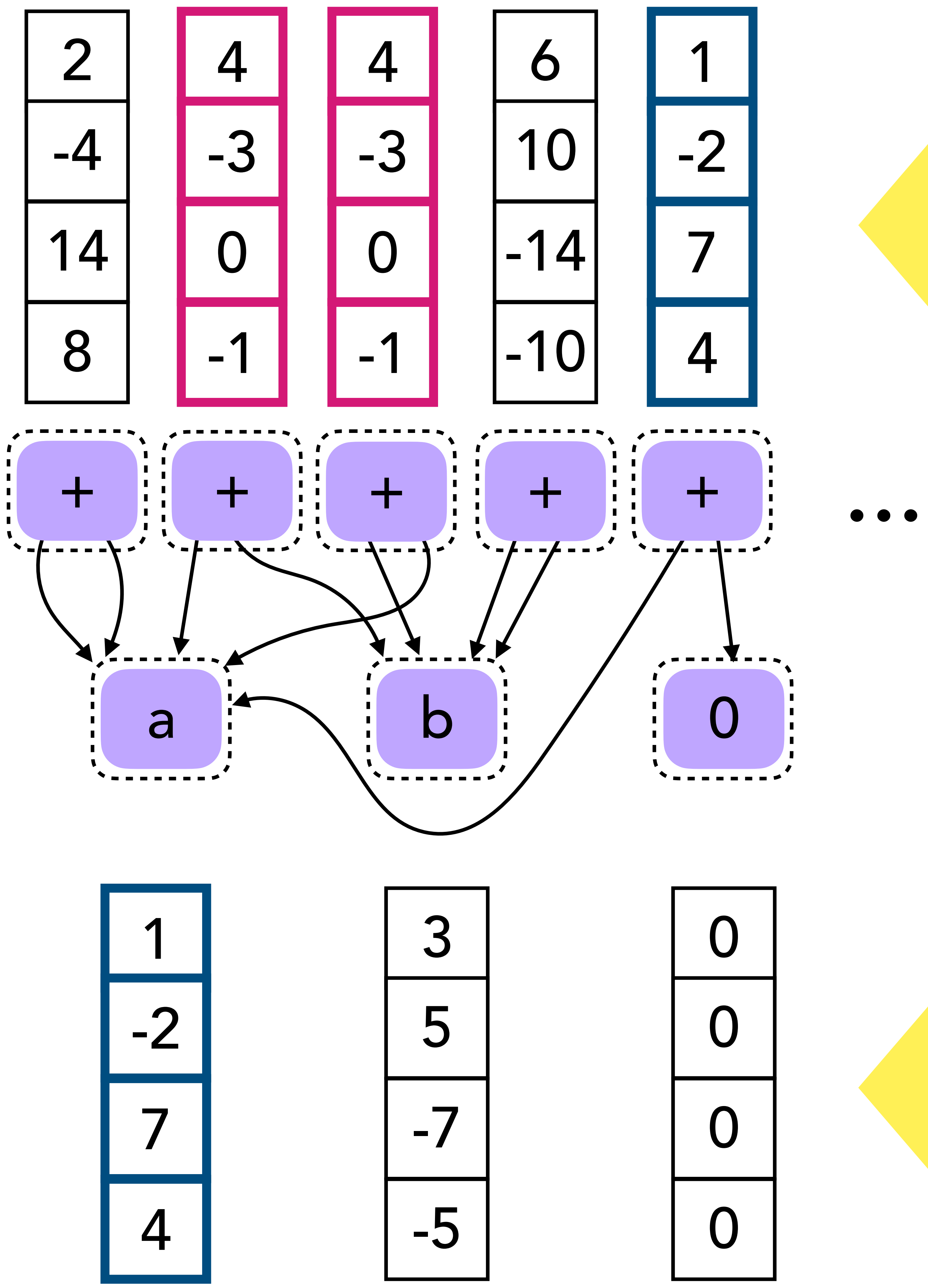


Compute the cvecs for newly enumerated E-classes

Seed initial E-classes with concrete values (cvecs) from the domain

$(x + y) \longleftrightarrow (y + x)$

Candidate Generation by Characteristic Vector Matching



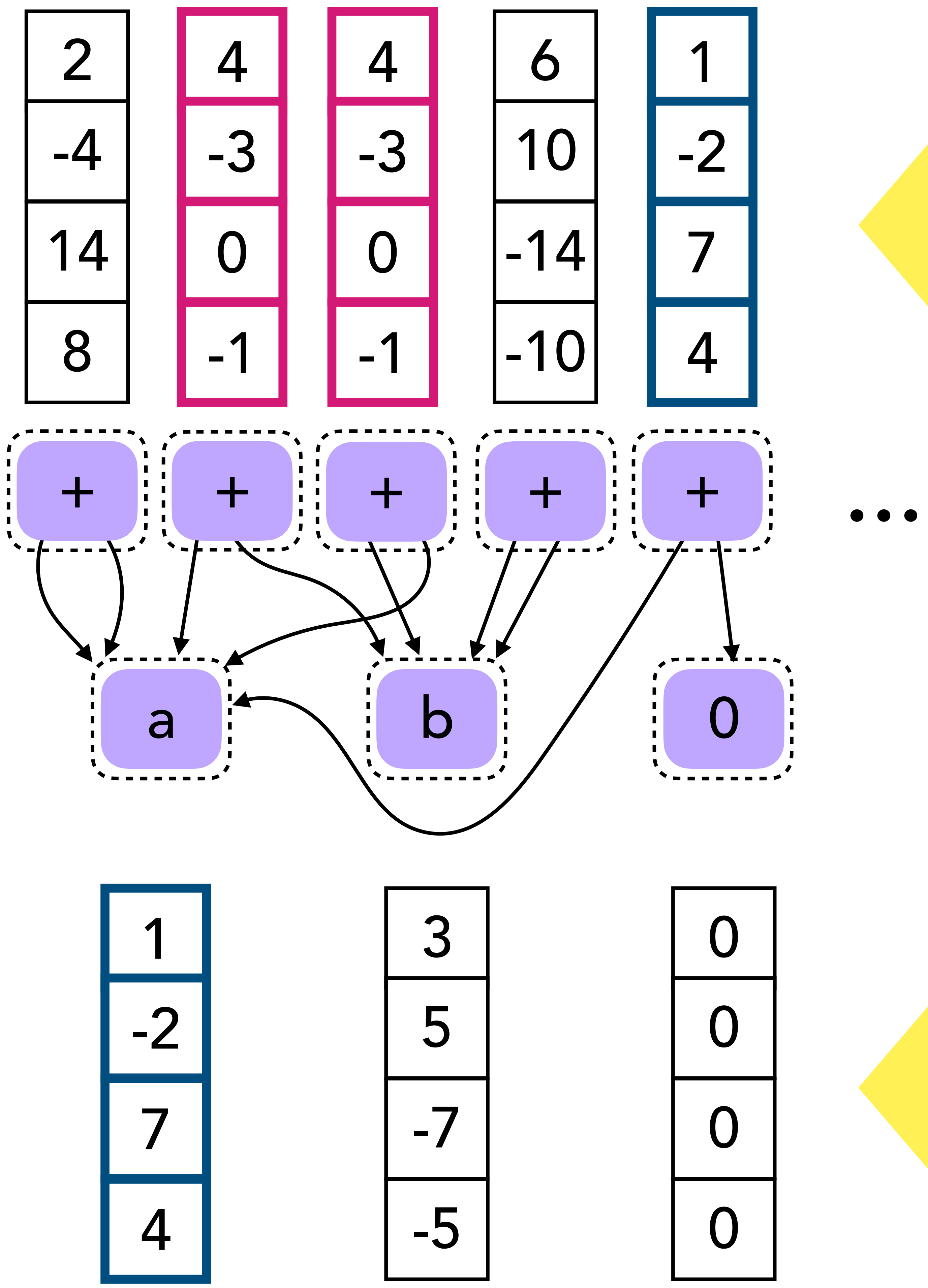
Compute the cvecs for newly enumerated E-classes

Seed initial E-classes with concrete values (cvecs) from the domain

$(x + y) \longleftrightarrow (y + x)$

$(x + 0) \longleftrightarrow x$

Candidate Generation by Characteristic Vector Matching



Compute the cvecs for newly enumerated E-classes

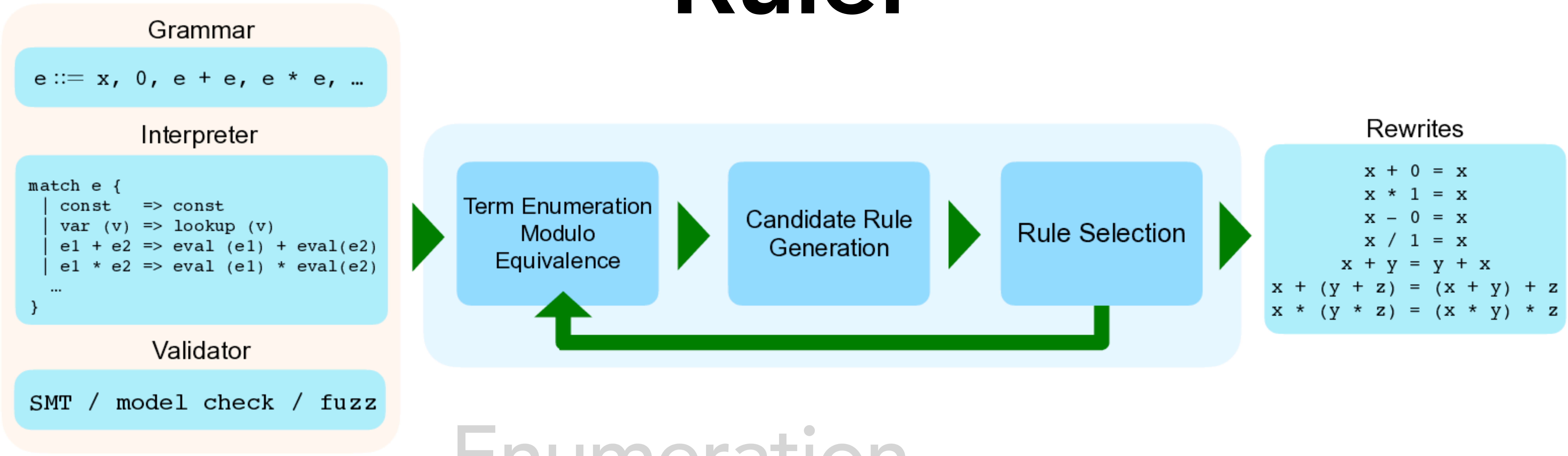
Seed initial E-classes with concrete values (cvecs) from the domain

(x + y) ⟷ (y + x)

(x + 0) ⟷ x

Validate candidates using SMT, fuzzing, model checking

Ruler



Enumeration

Candidate Generation

Rule Selection

Rule Selection with Equality Saturation

$$C = \begin{array}{ccc} (x + y) & \longleftrightarrow & (y + x) \\ (x + 0) & \longleftrightarrow & (0 + x) \\ (y + 0) & \longleftrightarrow & (0 + y) \\ (x * y) & \longleftrightarrow & (y * x) \\ (x * 1) & \longleftrightarrow & (1 * x) \\ (y * 1) & \longleftrightarrow & (1 * y) \end{array}$$

Rule Selection with Equality Saturation

Rank sound candidates based on generality and pick top-k (2)

C =

(x + y) ⟷ (y + x)

(x * y) ⟷ (y * x)

(x + 0) ⟷ (0 + x)

(y + 0) ⟷ (0 + y)

(x * 1) ⟷ (1 * x)

(y * 1) ⟷ (1 * y)

→ R

Rule Selection with Equality Saturation

Rank sound candidates based on generality and pick top-k (2)

(x + y) ⟷ (y + x)

(x * y) ⟷ (y * x)

(x + 0) ⟷ (0 + x)

(y + 0) ⟷ (0 + y)

(x * 1) ⟷ (1 * x)

(y * 1) ⟷ (1 * y)

R

Instantiate and add to rule E-graph

Rule Selection with Equality Saturation

Rank sound candidates based on generality and pick top-k (2)

(x + y) ⟷ (y + x)

(x * y) ⟷ (y * x)

(x + 0) ⟷ (0 + x)

(y + 0) ⟷ (0 + y)

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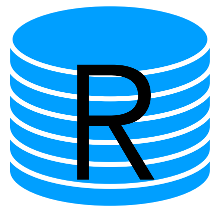
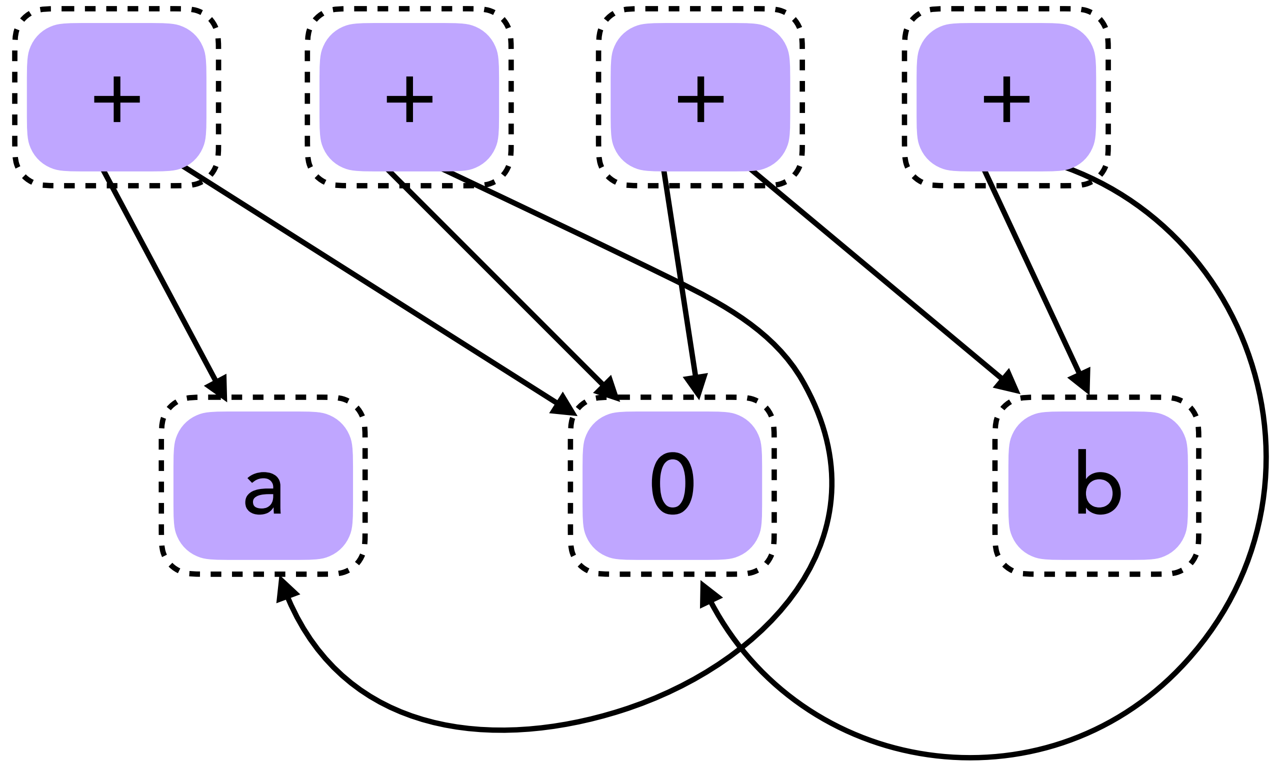
(y * 1) ⟷ (1 * y)



Instantiate and add to rule E-graph



Rule Selection with Equality Saturation

Rank sound candidates based on generality and pick top-k (2)

(x + y) ⟷ (y + x)

(x * y) ⟷ (y * x)

R

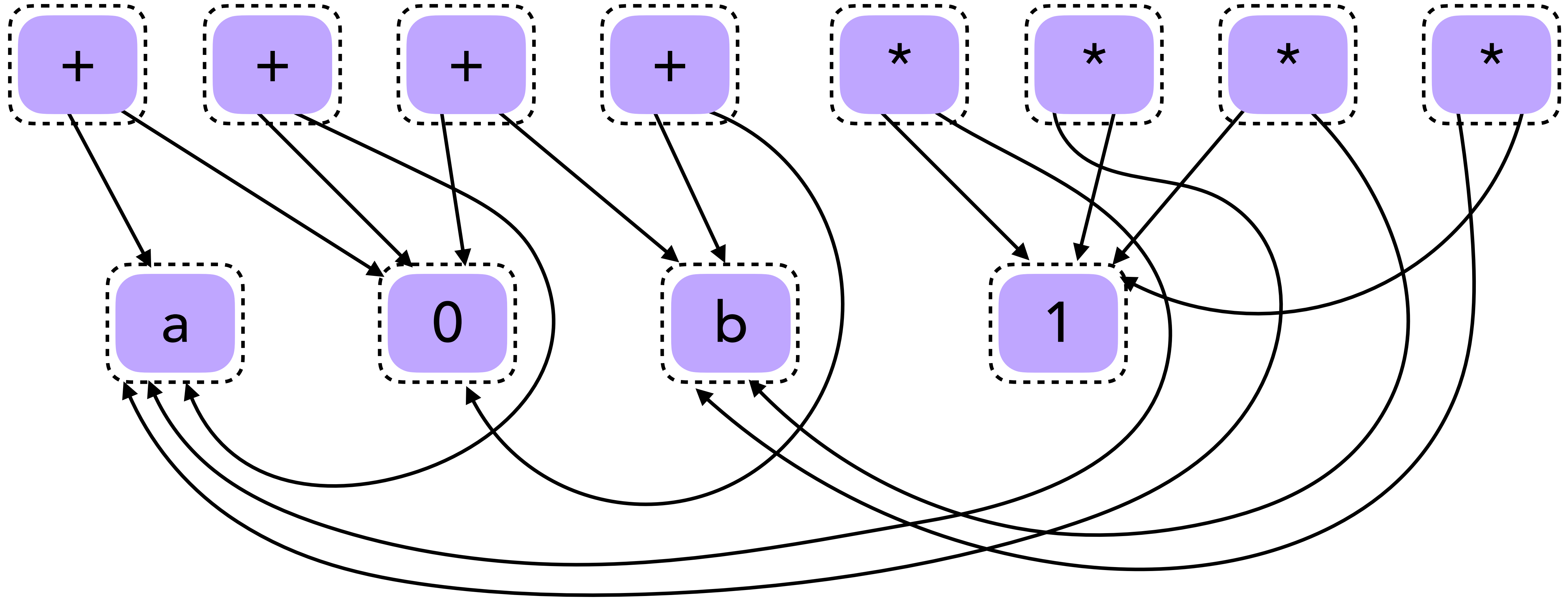
(x + 0) ⟷ (0 + x)

(y + 0) ⟷ (0 + y)

(x * 1) ⟷ (1 * x)

(y * 1) ⟷ (1 * y)

Instantiate and add to rule E-graph



Rule Selection with Equality Saturation

R

$(x + y) \longleftrightarrow (y + x)$

$(x * y) \longleftrightarrow (y * x)$

Run equality saturation

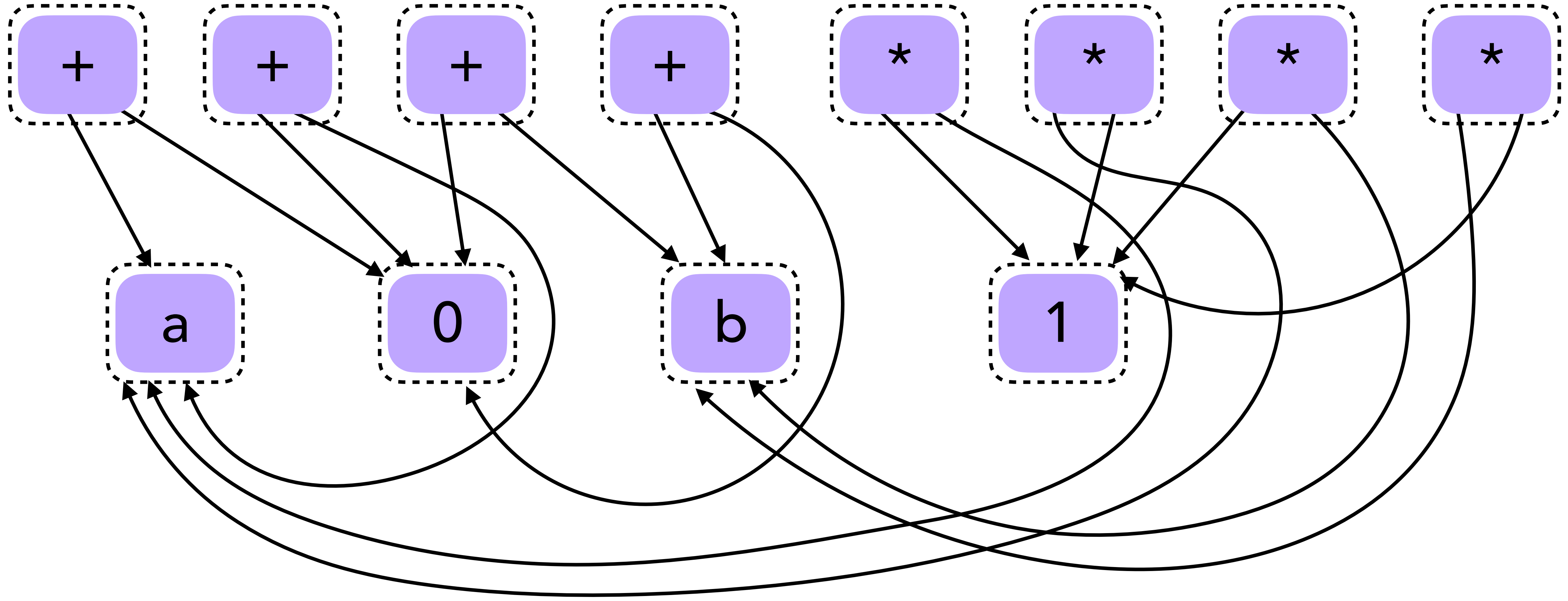
$(x + 0) \longleftrightarrow (0 + x)$

$(y + 0) \longleftrightarrow (0 + y)$

$(x * 1) \longleftrightarrow (1 * x)$

$(y * 1) \longleftrightarrow (1 * y)$

Instantiate and add to rule E-graph



Rule Selection with Equality Saturation

R

(x + y) ⟷ (y + x)

(x * y) ⟷ (y * x)

All four rules are redundant and therefore discarded!

Run equality saturation

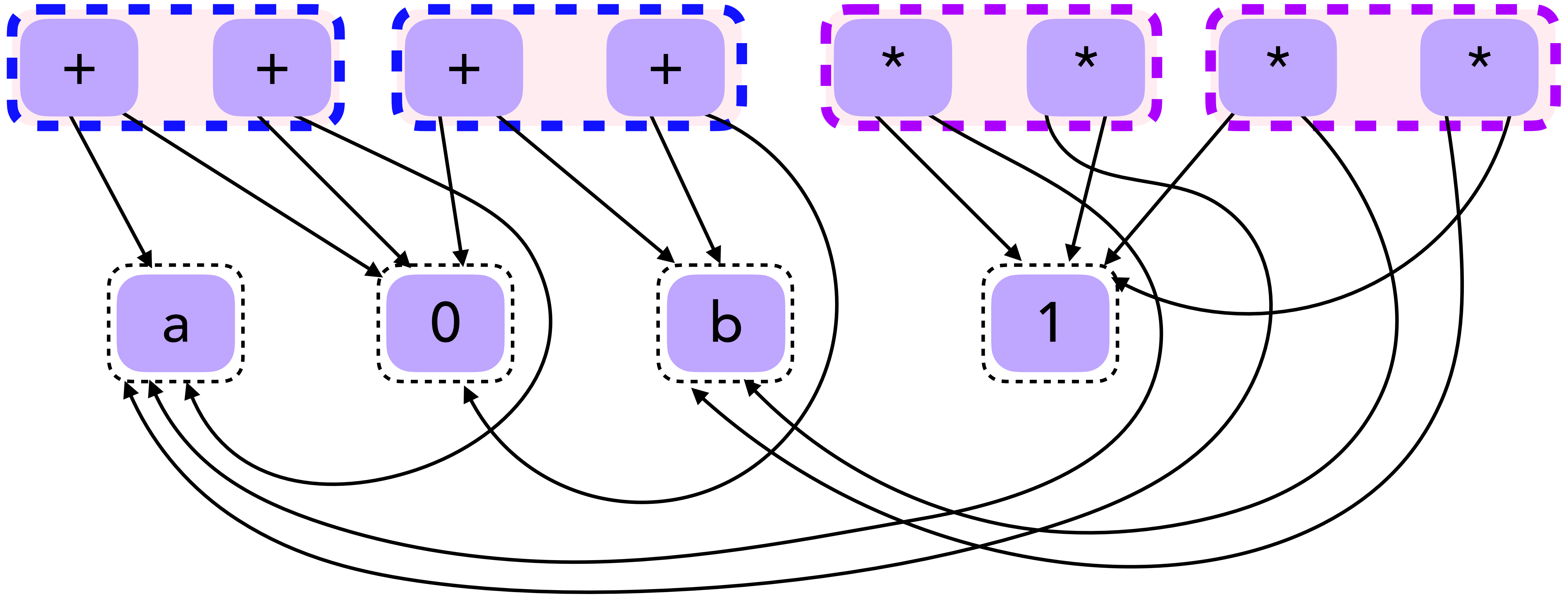
(x + 0) ⟷ (0 + x)

(y + 0) ⟷ (0 + y)

(x * 1) ⟷ (1 * x)

(y * 1) ⟷ (1 * y)

Instantiate and add to rule E-graph



Rule Selection with Equality Saturation

Continue processing until candidate set is empty or has only unsound ones left!

R

(x + y) ⟷ (y + x)

(x * y) ⟷ (y * x)

All four rules are redundant and therefore discarded!

Run equality saturation

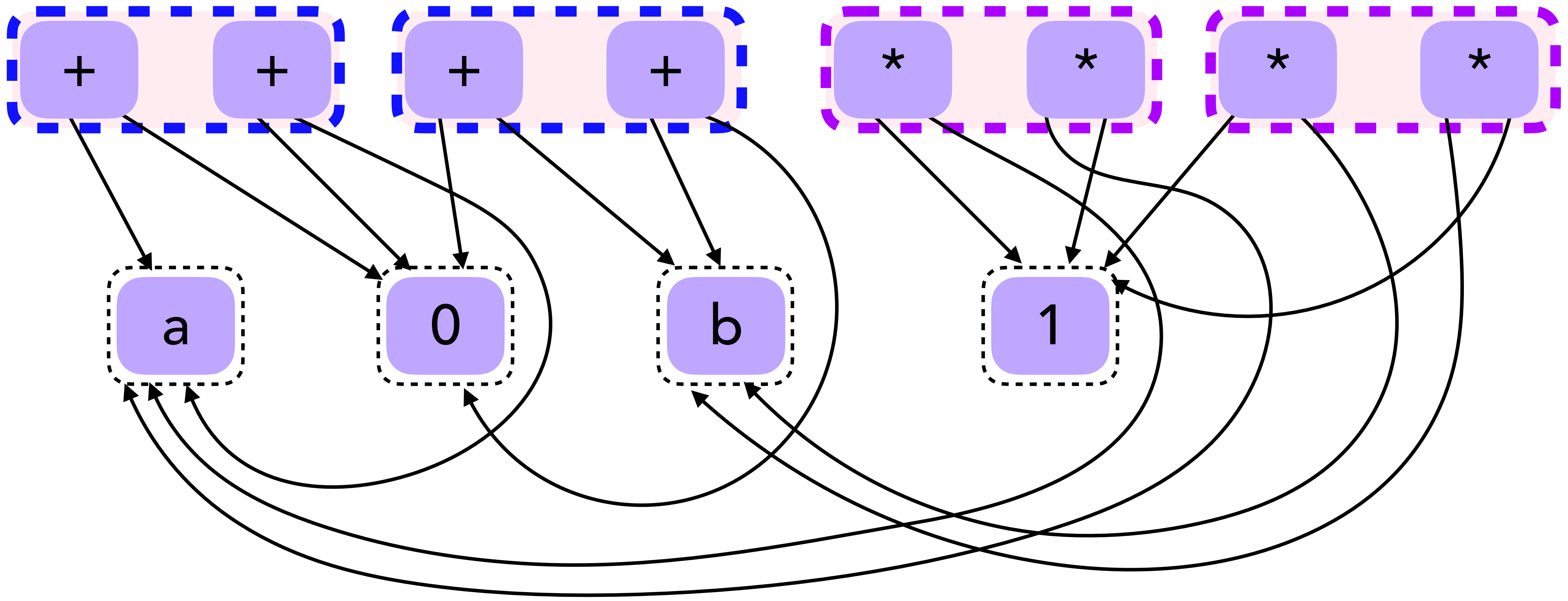
(x + 0) ⟷ (0 + x)

(y + 0) ⟷ (0 + y)

(x * 1) ⟷ (1 * x)

(y * 1) ⟷ (1 * y)

Instantiate and add to rule E-graph



Rule Selection with Equality Saturation

Larger top-k makes Ruler faster

Smaller top-k gives smaller rulesets

See paper for detailed comparison!

R

(x + y) ⟷ (y + x)

(x * y) ⟷ (y * x)

Run equality saturation

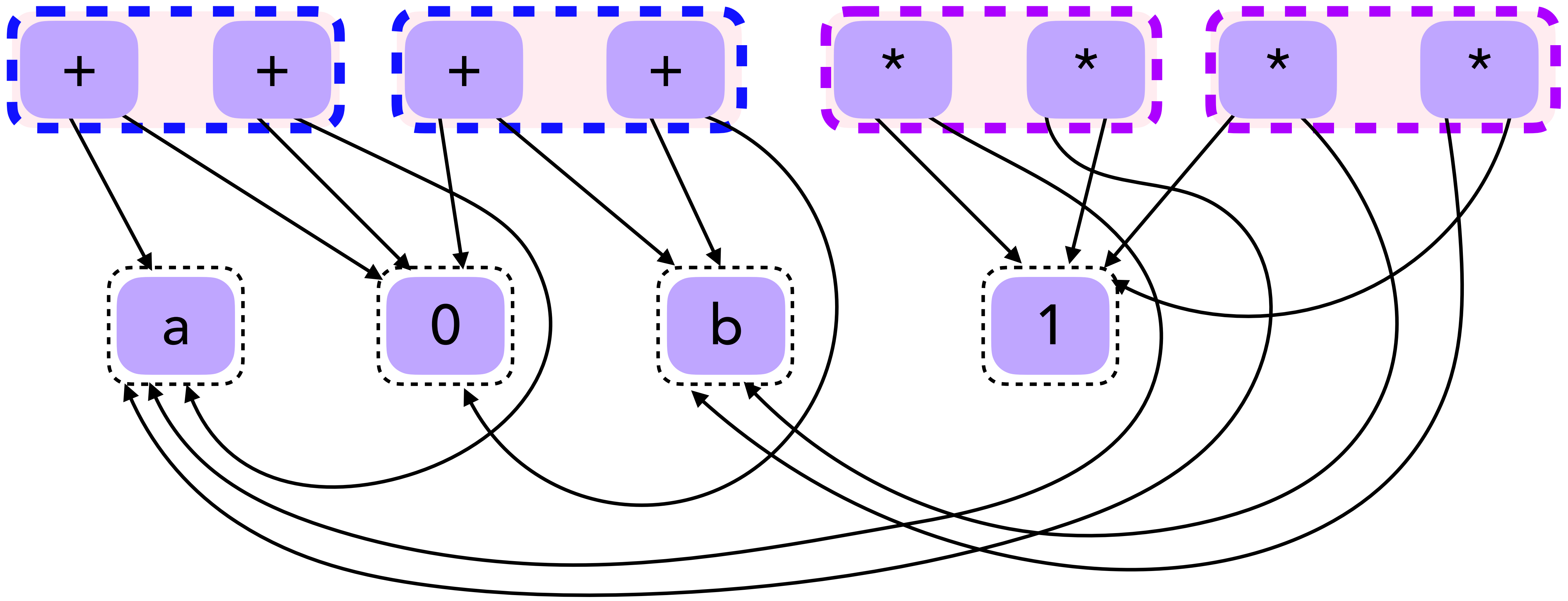
(x + 0) ⟷ (0 + x)

(y + 0) ⟷ (0 + y)

(x * 1) ⟷ (1 * x)

(y * 1) ⟷ (1 * y)

Instantiate and add to rule E-graph



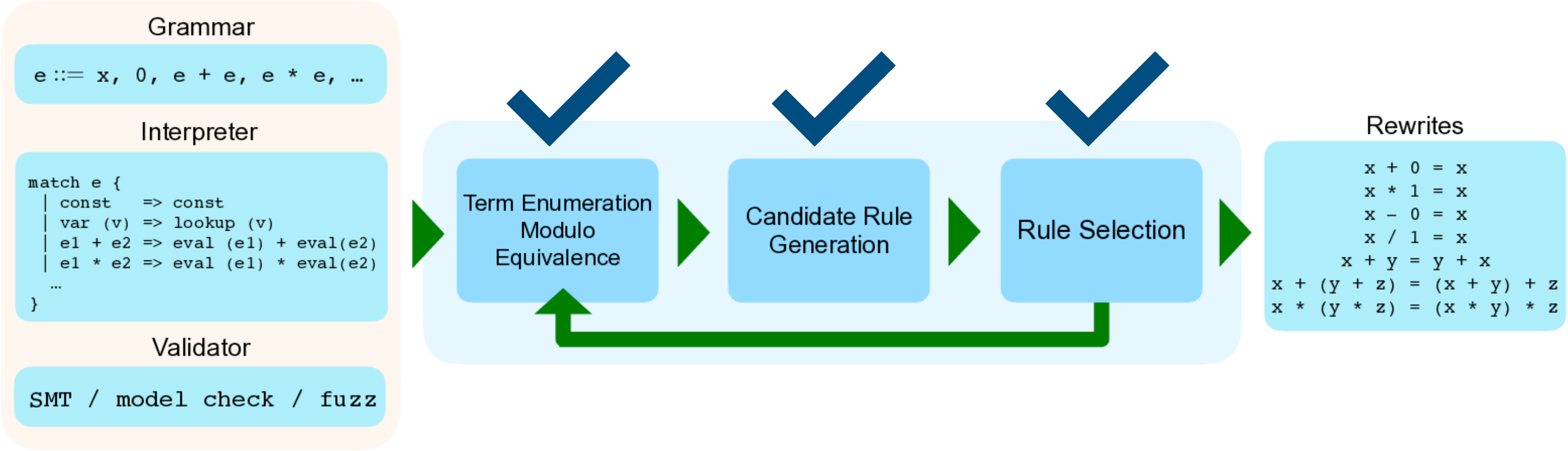
Rule Selection with Equality Saturation

Larger top-k makes Ruler faster

$(y * 1) \longleftrightarrow (1 * y)$

Shrinks the *candidate space* by applying rewrites as they are learned!

Ruler

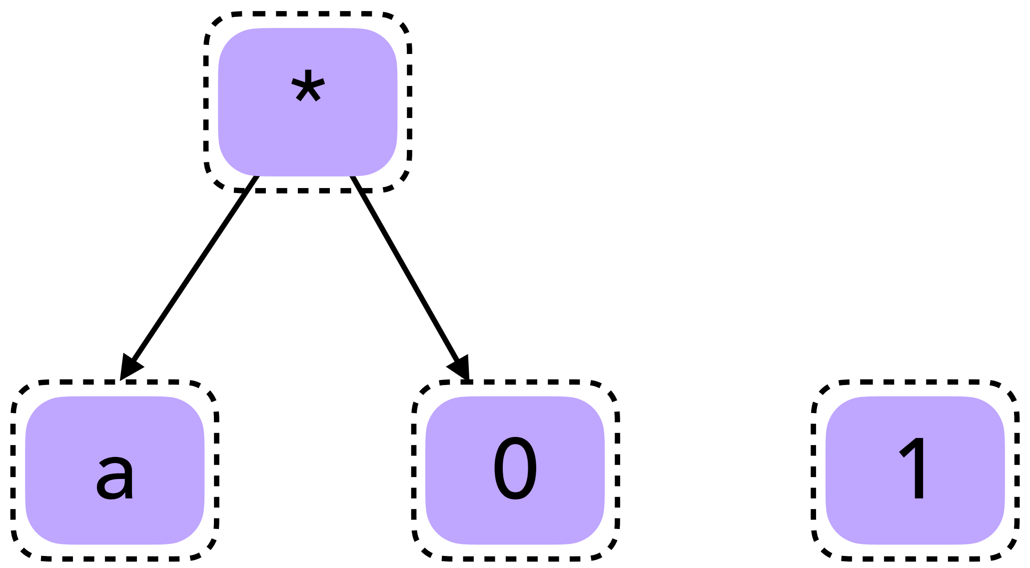

Grammar
e ::= x, 0, e + e, e * e, …
Interpreter
match e {
| const => const
| var (v) => lookup (v)
| e1 + e2 => eval (e1) + eval(e2)
| e1 * e2 => eval (e1) * eval(e2)
…
}
Validator
SMT / model check / fuzz
Term Enumeration Modulo Equivalence
Candidate Rule Generation
Rule Selection
Rewrites
x + 0 = x
x * 1 = x
x – 0 = x
x / 1 = x
x + y = y + x
x + (y + z) = (x + y) + z
x * (y * z) = (x * y) * z

Equality Saturation *"Soundiness"*

Equality Saturation *amplifies* unsoundness!

Equality Saturation “*Soundiness*”

Equality Saturation *amplifies* unsoundness!



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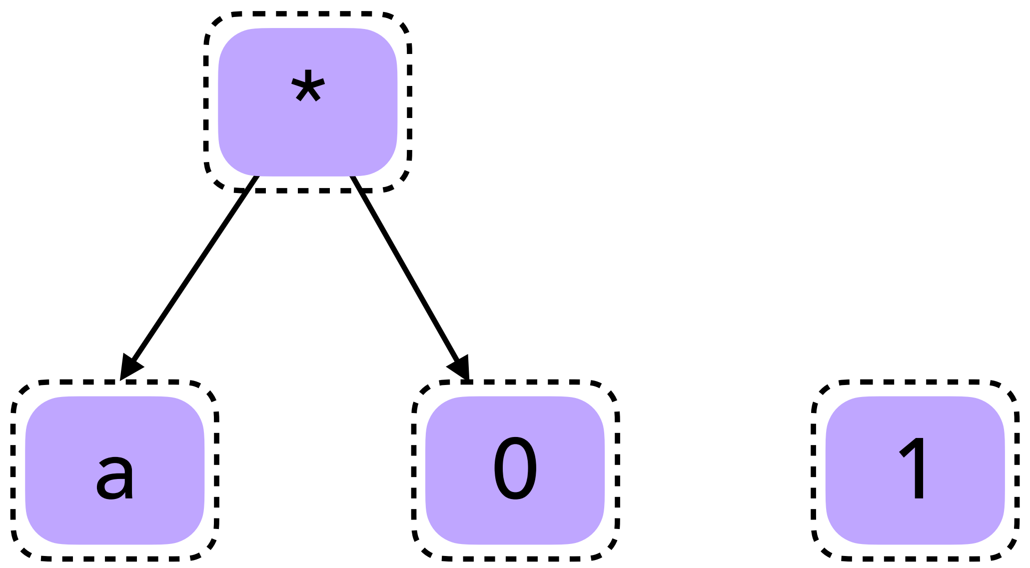
current ruleset

(y * 0) ⟷ 0

(y * 0) ⟷ 1



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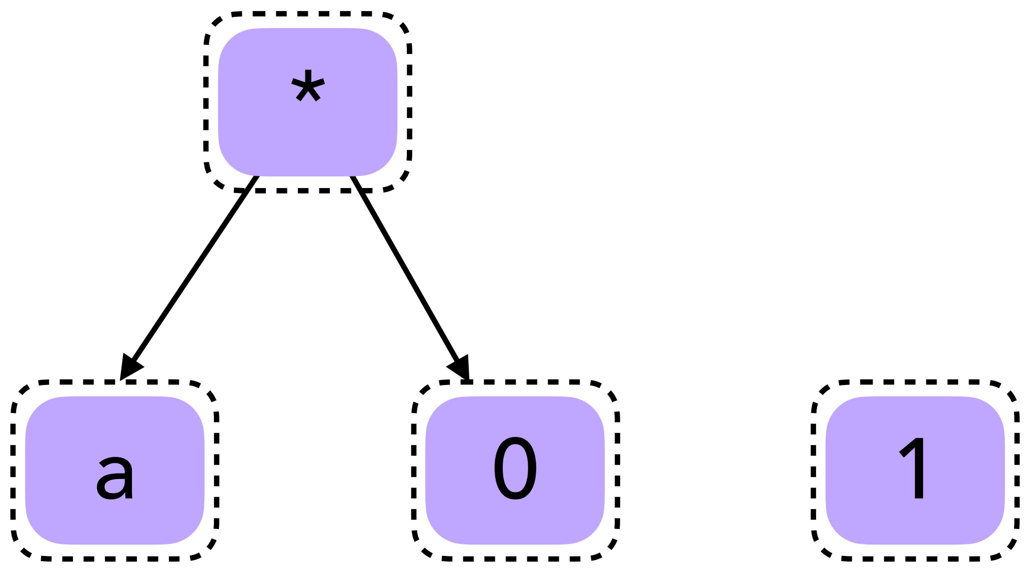

Run equality saturation on term E-graph

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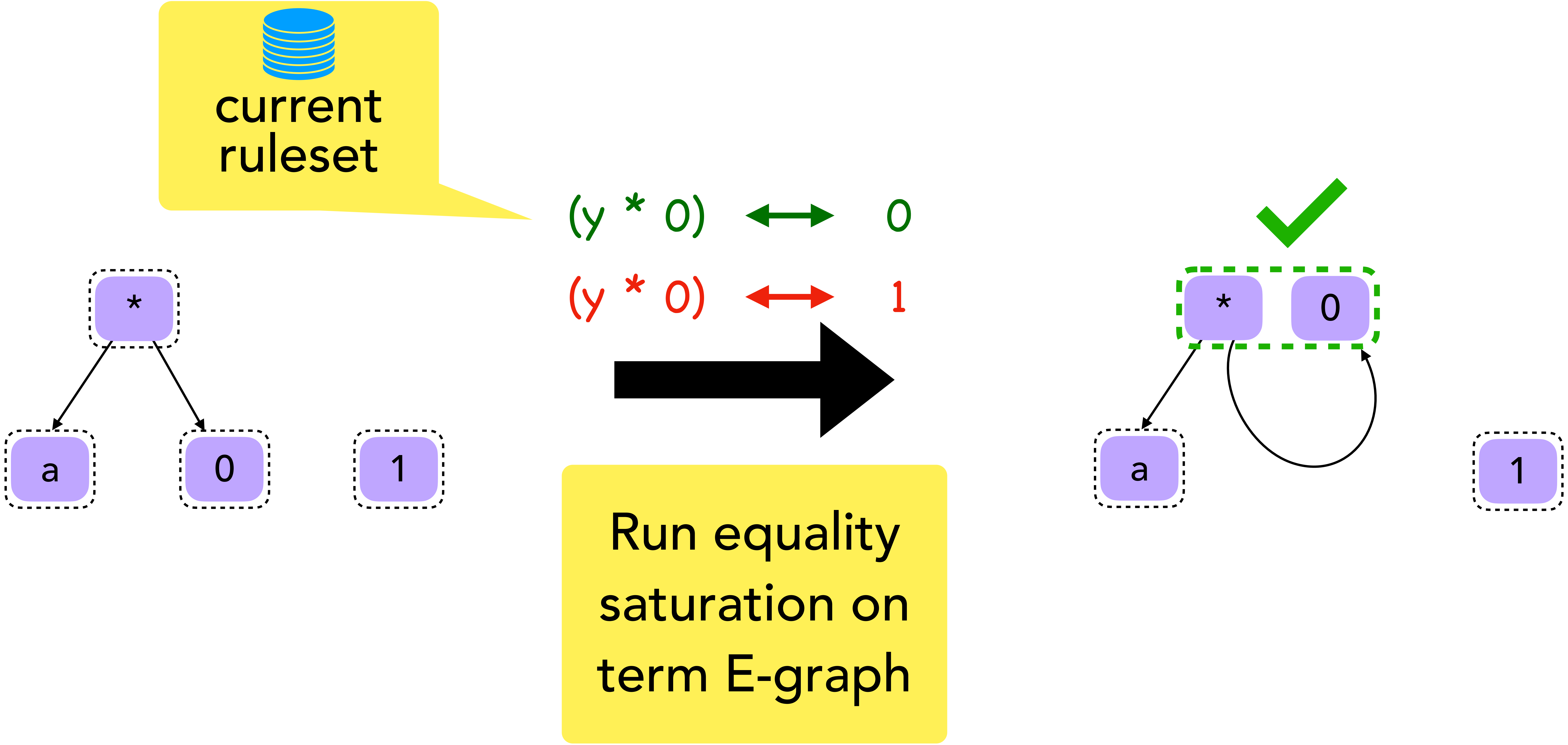

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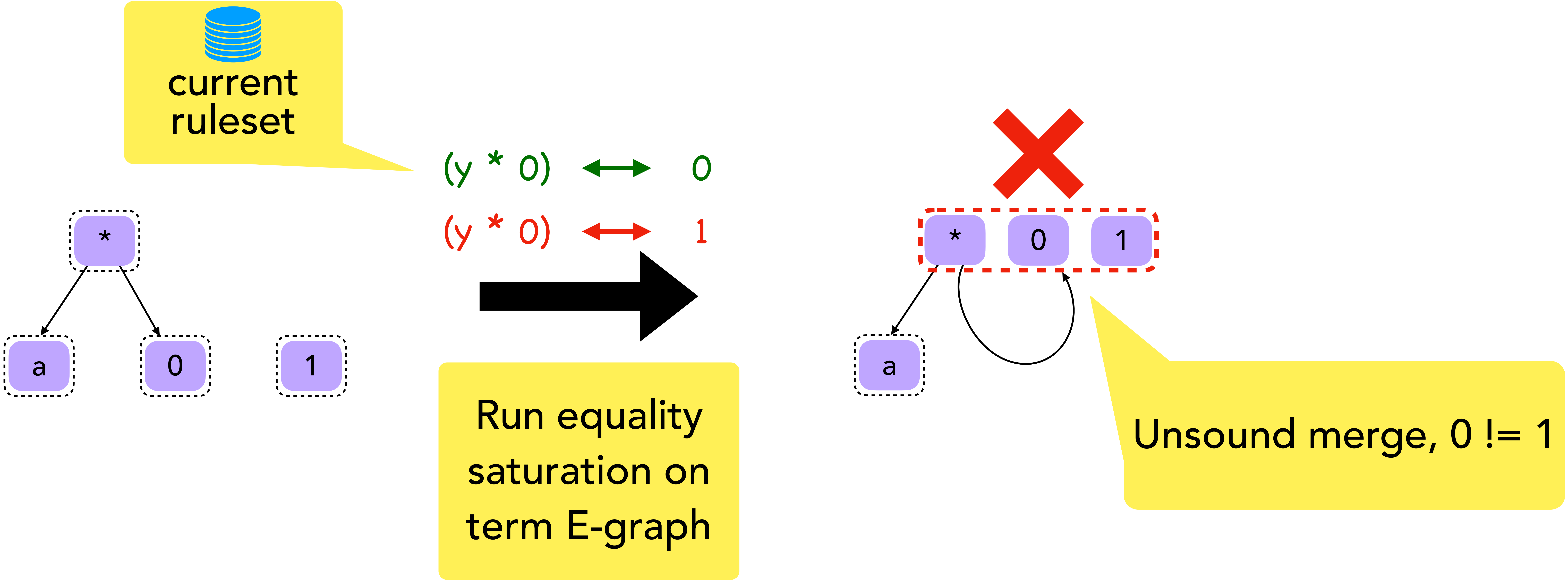

Implementation

https://github.com/uwplse/ruler

Implemented in Rust

Uses **egg** for equality saturation

Evaluation

Ruler vs Other tools (CVC4)

How do the rulesets compare?

Comparison with CVC4

Parameters		Ruler			CVC4			Ruler / CVC4	
Domain	# Conn	Time (s)	# Rules	Drv	Time (s)	# Rules	Drv	Time	Rules
bool	2	0.01	20	1	0.13	53	1	0.06	0.38
bool	3	0.06	28	1	0.82	293	1	0.07	0.10
bv4	2	0.14	49	1	4.47	135	0.98	0.03	0.36
bv4	3	4.30	272	1	372.26	1978	1	0.01	0.14
bv32	2	13.00	46	0.97	18.53	126	0.93	0.70	0.37
bv32	3	630.09	188	0.98	1199.53	1782	0.91	0.53	0.11

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Fraction of the 1782 rules from CVC4 that the 188 rules from Ruler can derive via equality saturation

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Ruler infers a **smaller**, **useful** ruleset **faster**

Evaluation

Ruler vs Other tools (CVC4)

How do the rulesets compare?

Ruler vs Humans (Herbie)

Can Ruler compete with experts?

Comparison with Human-written Rules

```
sqrt(x+1) - sqrt(x)  →  1/(sqrt(x+1) + sqrt(x))
```

Herbie detects inaccurate expressions and finds more accurate replacements. The red expression is inaccurate when $x > 1$; Herbie's replacement, in blue, is accurate for all x.

Comparison with Human-written Rules

HERBIE

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52 ***rational*** rules, designed by the developers over 6 years

55 / 155 benchmarks are purely over rational arithmetic

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Herbie can generate more-complex expressions that aren't more precise #261 Edit New issue

Closed **nbraud** opened this issue on Aug 31, 2019 · 4 comments

Comparison with Human-written Rules

HERBIE

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Closed **nbraud** opened this issue on Aug 31, 2019 · 4 comments

$$|x * y| \leftrightarrow |x| * |y|$$

$$|x * x| \leftrightarrow x * x$$

Discovered by Ruler, resolved the GitHub issue!

End-to-End: Rational Herbie

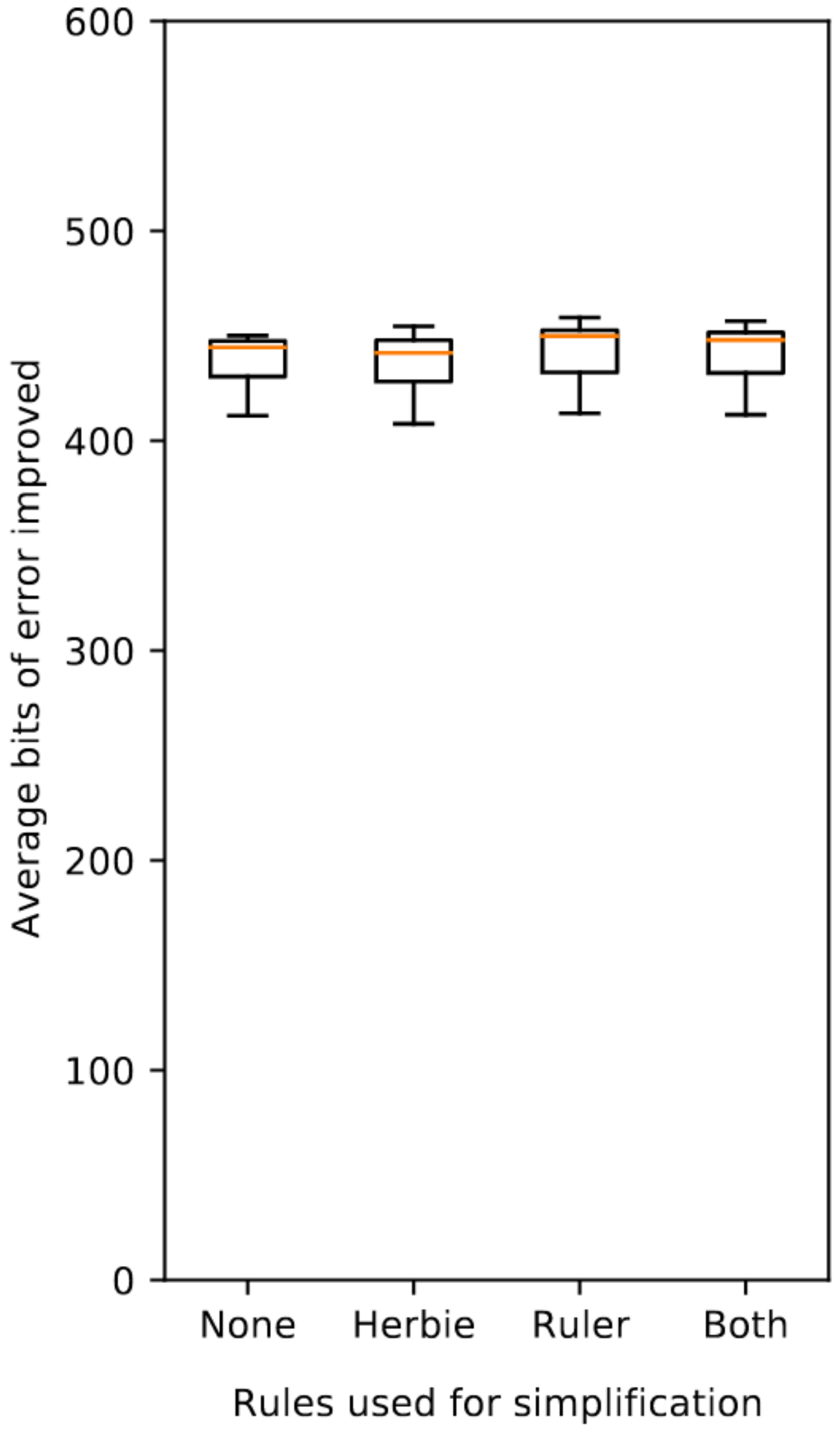
None: Remove all rules

Herbie: Herbie without any changes

Ruler: Herbie with Ruler's rules

Both: Herbie with both original and Ruler's rules

Rational Herbie: Comparing Accuracy



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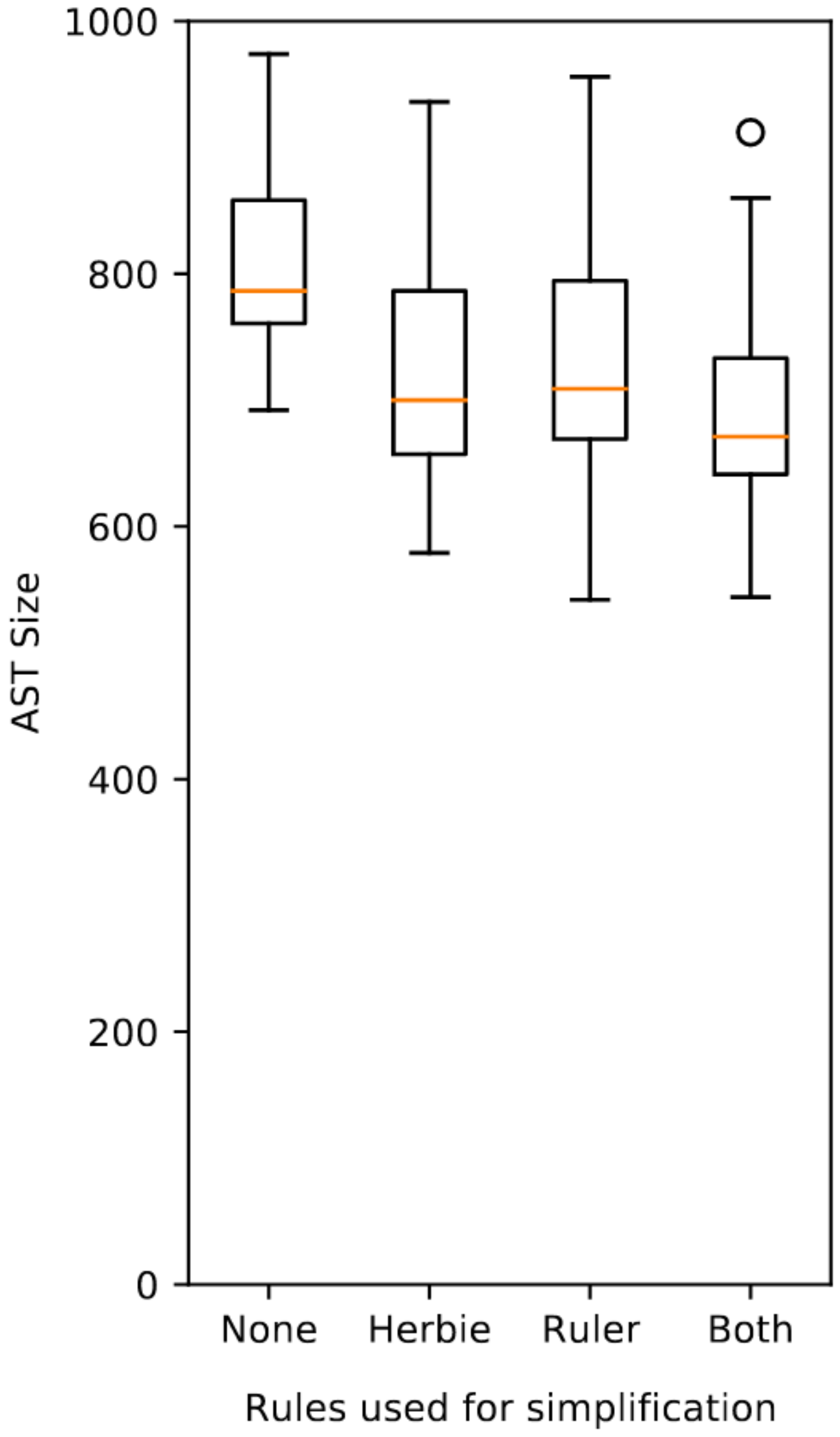
Ruler's rules are at least as good as the original Herbie rules

Rational Herbie: Comparing AST Size



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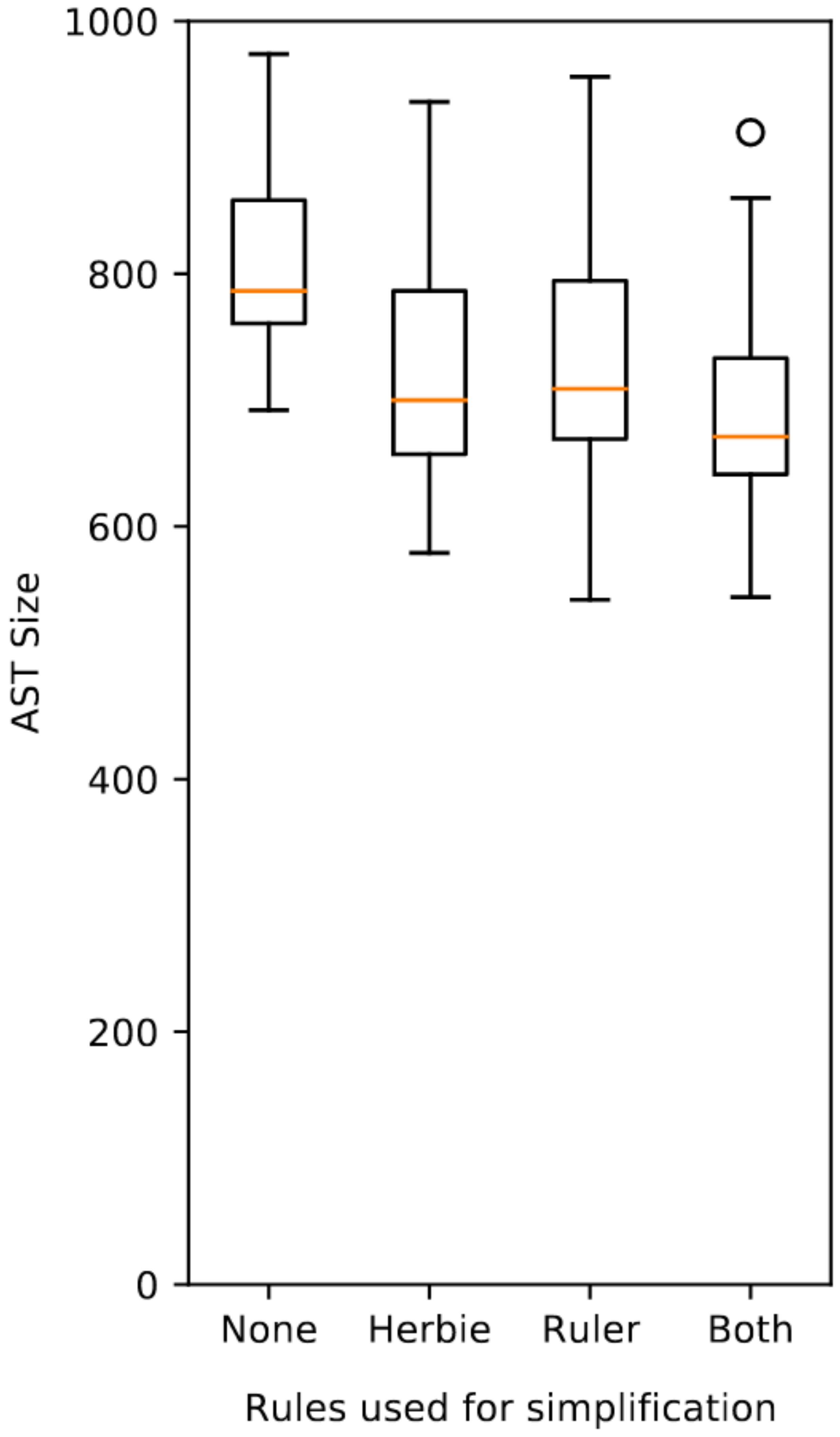

See paper for more results!

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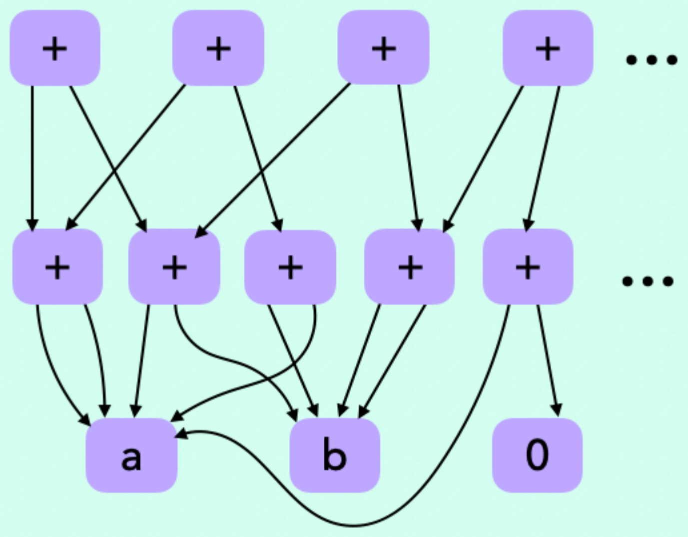
Rewrite Rule Inference Using Equality Saturation

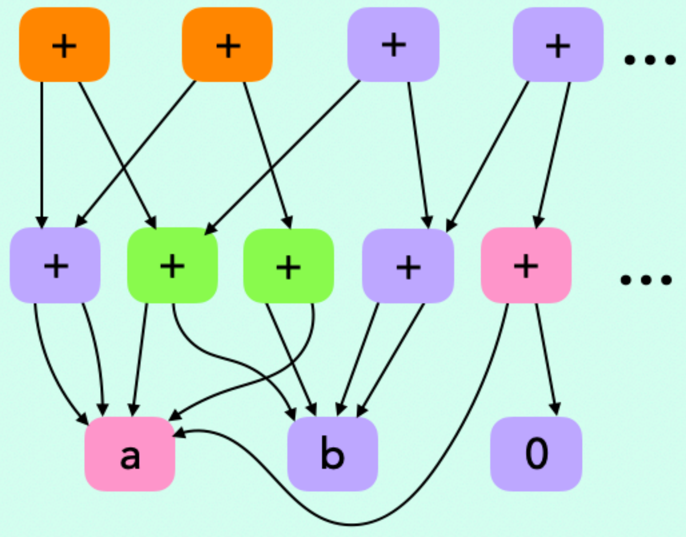
Enumerate terms from a grammar

a, b, 0, +, …



Find candidates: interpret over concrete inputs

"Fingerprints"



Filter candidates to get final ruleset

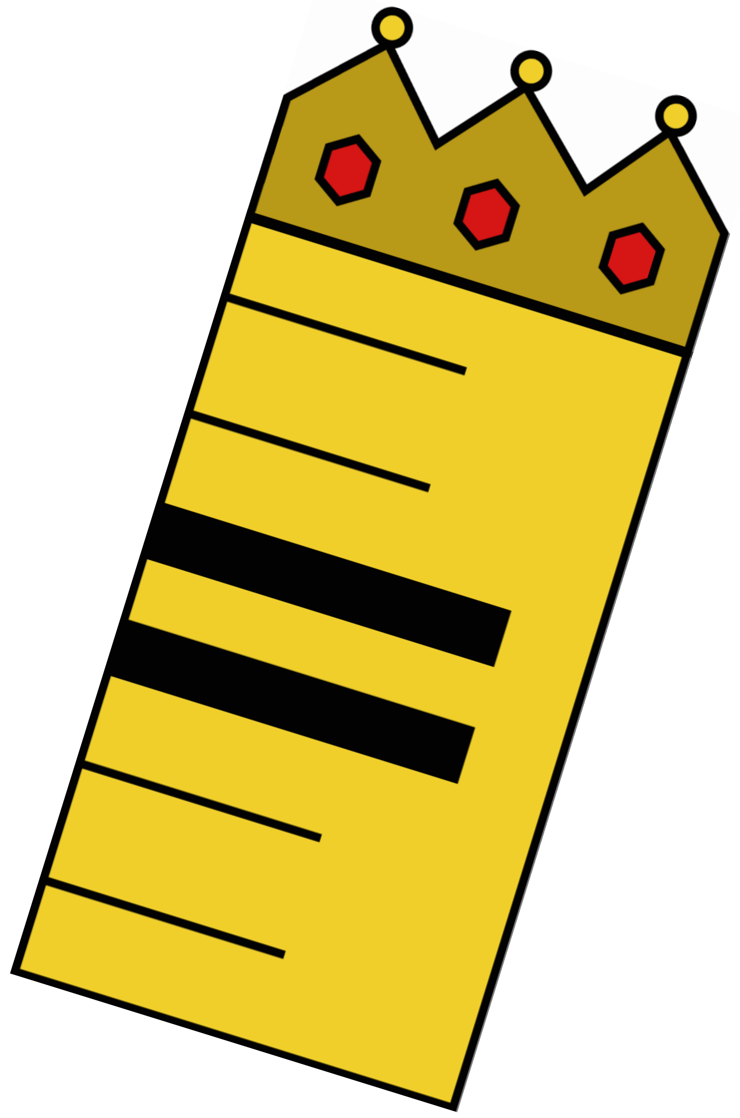
Remove redundant rules

x + 0 ⟷ 0 + x

y + 0 ⟷ 0 + y

x + y ⟷ y + x

Equality Saturation improves all three steps!



Ruler: https://github.com/uwplse/ruler

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