



## Lecture 16

# Unification Static Types

type inference as constraint solving with Prolog

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# Type inference

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In OO static types like Java's, programmers annotate variables/parameters with types.

```
Foo myFunction(Bar b)
```

Why ask for these annotations when the type can be (often) inferred automatically?

today we will look at one such type inference

# Example

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Consider this factorial program.

```
def fact(n):  
  if (n==0) { 1 } else { n * fact(n-1) }
```

Let's *type* this function. *Typing a function* includes type inference and type checking. Three questions:

- what is the type of the parameter  $n$ ?
- what is the return type of `fact`?
- is the function type safe, ie will it perform only operations sanctioned by their type?

# Let's write type rules of our arithmetic

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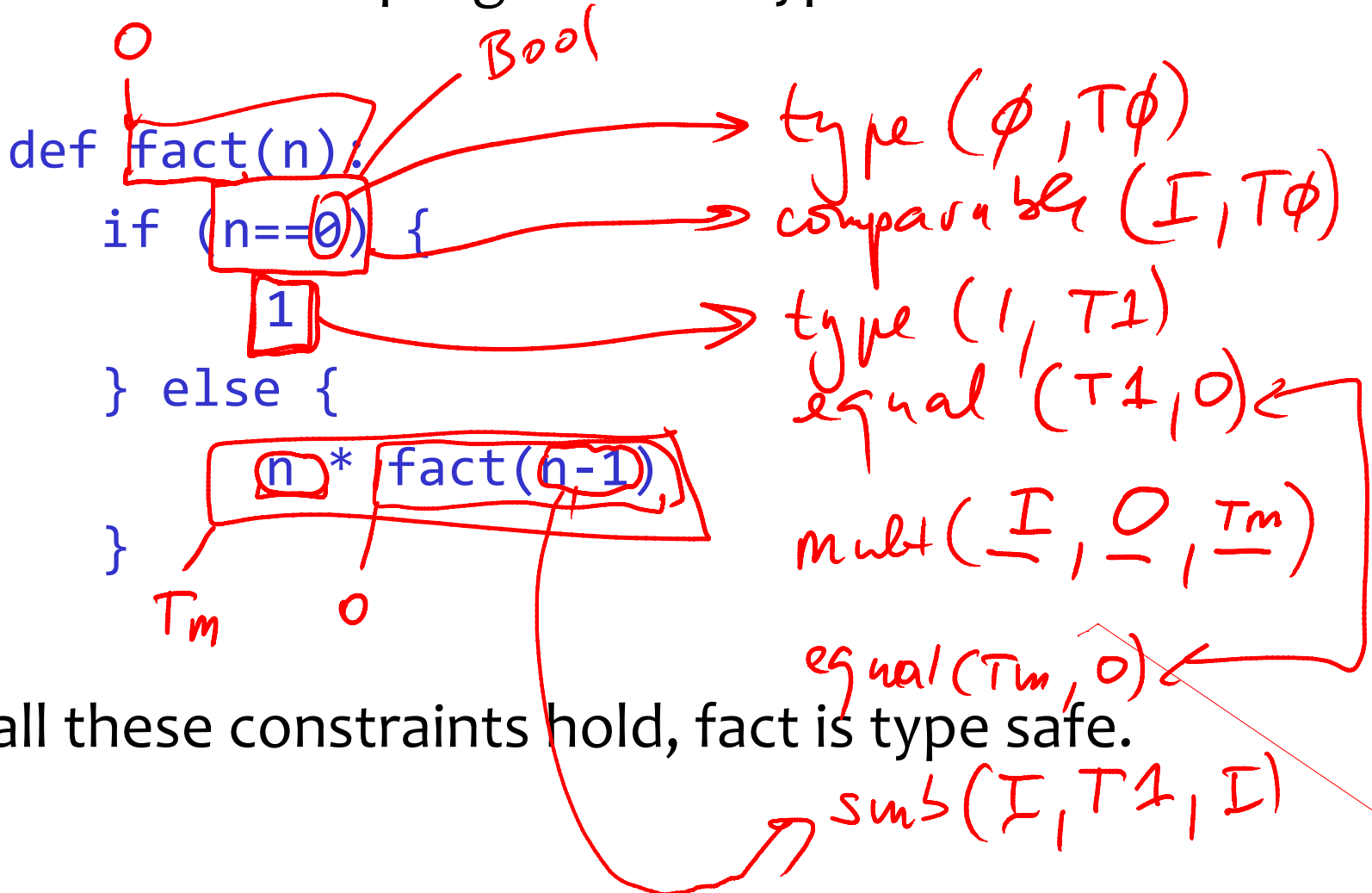
Conveniently, we will use Prolog:

```
type(0,int).           % 0 is an int value
type(1,int).
mult(int,int,int).    % E * E
mult(float,float,float).
sub(int,int,int).     % E - E
sub(float,float,float).
comparable(int,int).  % E == E
comparable(float,float).
```

These rules hold for all programs in our language.

# Collect constraints from the program

Now translate a program into type constraints.



If all these constraints hold, fact is type safe.

# Constraints for the factorial function

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$I \rightarrow 0$

```
fact(fun(I,0)) :-      % I is the type of n
    type(0,T0),        % T0 is the type of value 0
    type(1,T1),        % T1 is the type of value 1
    comparable(I,T0), % is n==0 legal?
    T1=0,              % type(1) must equal ret type
    mult(I,0,0),
    sub(I,T1,I).      % (2)
```

# Solving type constraints

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We ask Prolog to solve these constraints.

```
?- fact(fun(I,0)).
```

```
I = int
```

```
O = int
```

There is a solution to these constraints, so fact is type safe when if called with the parameter type I=int.

We also learn that it will return value of type O=int.

# Notes

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$\text{fun}(I,O)$  is our (Prolog) way of denoting the function type. The usual notation is  $I \rightarrow O$

How do we know that the return type of  $\text{fact}(n-1)$  is  $O$ ?

We have decided that  $\text{fact}$  has the same type in each invocation, hence the type  $\text{fact}(n-1)$  must be the same as that of  $\text{fact}(n)$ , which we denoted  $O$ .



# ML

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A language that has influenced modern static languages, such as Scala.

ML is based on unification type system, like we used in our fact example.

Let's look at ML's type more closely.

# Function definition are composed of cases

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Function definition:

```
fun fact 0 = 1
  | fact n = n * fact(n-1);
```

If the definition type checks, the compiler accepts the definition, and prints out:

```
val fact = fn : int -> int
```

# Lists

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The cons operator

`1::2::3::[]`

is the same as

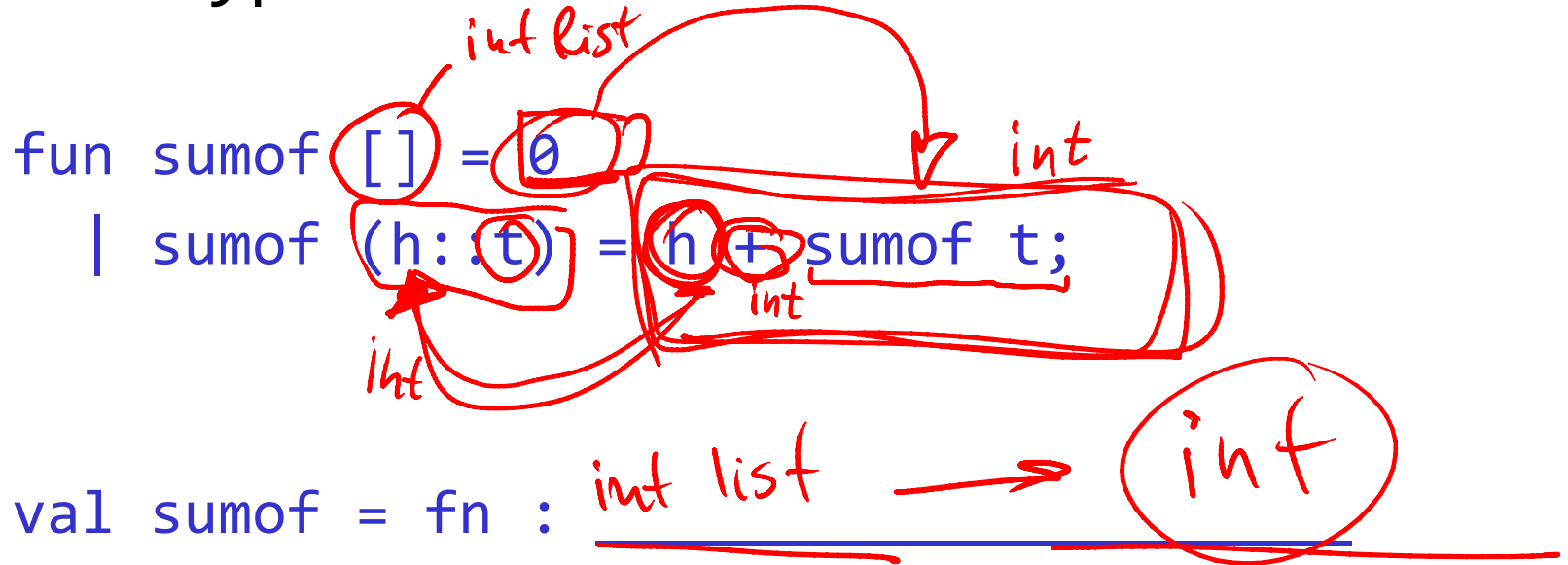
`[1,2,3]`

The `::` operator is a binary function with infix syntax.

# Type inference for lists

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Let's type this recursive function:



# More list type inference

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What's different in this function definition?

```
fun map(f, []) = []  
  | map(f, (h::t)) = f(h)::(map(f, t));
```

Two usage examples

$\text{map } (\alpha \rightarrow \beta \times \alpha \text{ list}) \rightarrow (\beta \text{ list})$

```
map (sqrt, [1.0, 2.0, 3.0])
```

```
map (rev, [[1, 2, 3], [4, 5, 6], [7, 8, 9]])
```

# Polymorphic types

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What would be the type of function :: ?

'a' \* 'a' list -> 'a list

$\alpha$       $\alpha$               $\alpha$

:: is a polymorphic function

# Let's work out the type inference

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```
fun map(f, []) = []  
  | map(f, (h::t)) = f(h)::(map(f, t));
```

# Another version of map

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```
fun map f [] = []
```

```
  | map f (h::t) = (f h)::(map f t);
```

```
val map = fn : ('a -> 'b) -> ('a list -> 'b list)
```

```
val sqrtall = map (sqrt);
```

```
val sqrtall = fn : real list -> real list
```

```
sqrtall [1.0,4.0,9.0];
```

```
val it = [1.0, 2.0, 3.0] : real list
```