



## Lecture 2

### Unit Conversion Calculator

Expressions, values, types. Their representation and interpretation.

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with Mangpo and Ali

### *Hack Your Language!*

CS164: Introduction to Programming  
Languages and Compilers, Spring 2013  
[UC Berkeley](#)

# Administrativa

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These are supplementary slides.

Review the material and the sample code to understand interpreters.

# Course grading

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Projects (PA1-9)	45%
Homeworks (HW1-3)	15%
Midterms	20%
Final project	15%
Class participation	5%

## Class participation: many alternatives

- ask and answer questions during lectures or recitations,
- discuss topics on the newsgroup,
- post comments to lectures notes (to be added soon)

# Summary of last lecture

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What's a programming abstraction?

- data types
- operations on them and
- constructs for composing abstractions into bigger ones

Example of small languages that you may design

- all built on abstractions tailored to a domain

What's a language?

- a set of abstractions composable into bigger ones

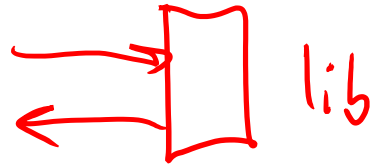
Why small languages?

- see next slide and lecture notes

# MapReduce story

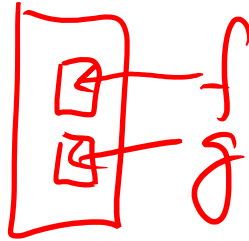
## Evolution of sw reuse and design

- library



MPI

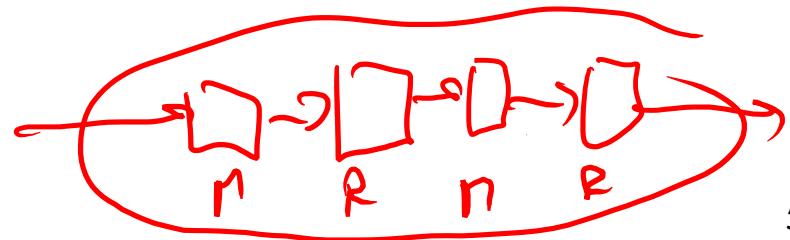
- framework  
parameterizable  
library



MapReduce

- Small language  
composable abstractions

Flume Java



# What's a “true” language

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## Composable abstractions

not composable:

- networking **socket**: an abstraction but can't build a “bigger” socket from an existing socket

composable:

- regexes: `foo|bar*` composes regexes `foo` and `bar*`

# Today

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Programs (expressions), values and types

their representation in the interpreter

their evaluation

Finding errors in incorrect programs

where do we catch the error?

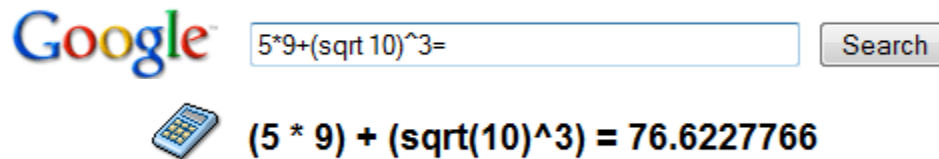
Using unit calculator as our running study

it's an interpreter of expressions with fun extensions

# Recall Lecture 1

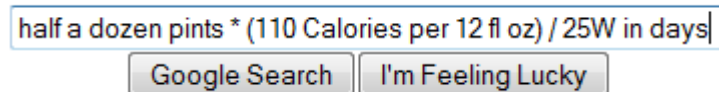
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Your boss asks: “Could our search box answers some semantic questions?” You build a calculator:



Then you remember cs164 and easily add unit conversion.

How long a brain could function on 6 beers --- if alcohol energy was not converted to fat.





# Programs from our calculator language

---

Example:

**34 knots in mph** # speed of S.F. ferry boat

--> *39.126 mph*

Example:

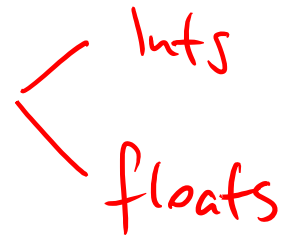
# volume \* (energy / volume) / power = time

**half a dozen pints \* (110 Calories per 12 fl oz) / 25 W in days**

--> *1.704 days*

# Constructs of the Calculator Language

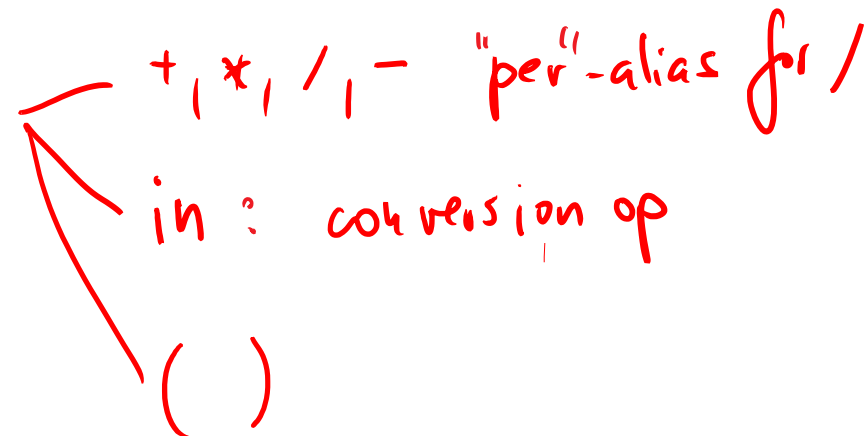
---

- numbers 

- ints
- floats

- units 

modeled as "types"

- operators 

- + , \* , / , - "per"-alias for /
- in : conversion op
- ()

# What do we want from the language

---

- evaluate arithmetic expressions
- ... including those with physical units
- check if operations are legal (area + volume is not)
- convert units

# What additional features may we want

---

what features we may want to add?

- think usage scenarios beyond those we saw
- talk to your neighbor
- we'll add some of these in the next lecture

can we view these as user extending the language?

- new unit types
- time, real time
- RPN - new syntax
- infinite prec. arith.
- eq solving (vars)
- complex, base-2, matrices
- integrals, ...
- pow, and arbitr. op, int div

# Additional features we will implement in Lec3

---

- allow users to extend the language with their units
- ... with new measures (eg Ampere)
- bind names to values
- bind names to expressions (lazy evaluation)

# We'll grow the language a feature at a time

---

1. Arithmetic expressions
2. Physical units for (SI only)
3. Non-SI units
4. Explicit unit conversion

# Sublanguage of arithmetic expressions

---

A programming language is defined as

**Syntax:** set of valid program strings

$2 + 3$       legal  
 $+ 2 3$       illegal

**Semantics:** how the program evaluates

$e_1 + e_2$       performs an addition of  
the values of expressions  
 $e_1$  and  $e_2$

# Syntax

---

The set of syntactically valid programs is 🕒 large.

So we define it recursively:

*a recursive definition of the language*

$$E ::= n \mid E \text{ op } E \mid ( E )$$
$$\text{op} ::= + \mid - \mid * \mid / \mid ^$$

**E** is set of all expressions expressible in the language.

**n** is a number (integer or a float constant)

Examples: 1, 2, 3, ..., 1+1, 1+2, 1+3, ..., (1+3)\*2, ...



# Semantics (Meaning)

---

Syntax defines what our programs look like:

1, 0.01, 0.12131, 2, 3, 1+2, 1+3, (1+3)\*2, ...

But what do they mean? Let's try to define  $e_1 + e_2$

Given the values  $e_1$  and  $e_2$ ,

the value of  $e_1 + e_2$  is the sum of the two values.

We need to state more. What is the range of ints?

Is it  $0..2^{32}-1$  ?

Our calculator borrows Python's unlimited-range integers

How about if  $e_1$  or  $e_2$  is a float?

Then the result is a float.

There are more subtleties, as we painfully learn soon.

# How to represent a program?

---

## concrete syntax

(input program)

"1+2"

"(3+4)\*2"

*a flat string*

## abstract syntax

(internal program representation)

('+', 1, 2)

('\*', ('+', 3, 4), 2)

*} tree built from Python tuples*



AST  
*abstract syntax tree*

# The interpreter

---

## Recursive descent over the abstract syntax tree

```
ast = ('*', ('+', 3, 4), 5) ← (3+4) * 5  
print(eval(ast))
```

```
def eval(e):  
    if type(e) == type(1): return e  
    if type(e) == type(1.1): return e  
    if type(e) == type(()):  
        if e[0] == '+': return eval(e[1]) + eval(e[2])  
        if e[0] == '-': return eval(e[1]) - eval(e[2])  
        if e[0] == '*': return eval(e[1]) * eval(e[2])  
        if e[0] == '/': return eval(e[1]) / eval(e[2])  
        if e[0] == '^': return eval(e[1]) ** eval(e[2])
```

# How we'll grow the language

---

1. Arithmetic expressions ✓
2. Physical units for (SI only)
3. Non-SI units
4. Explicit unit conversion

# Add values that are physical units (SI only)

Example:

$$(2 \text{ m})^2 \rightarrow 4 \text{ m}^2$$

Concrete syntax:

$E ::= n \mid \mathbf{U} \mid E \text{ op } E \mid (E)$

$\mathbf{U} ::= \mathbf{m} \mid \mathbf{s} \mid \mathbf{kg}$

$\text{op} ::= + \mid - \mid * \mid \epsilon \mid / \mid ^$

$\Rightarrow 1\text{m}$  is same as  $1 * \text{m}$

"" empty string

Abstract syntax: represent SI units as string constants

$3 \text{ m}^2$

$(('*', 3, ('^', 'm', 2)))$

parser translates the missing  $*$  into an explicit  $*$

# A question: catching illegal programs

---

Our language now allows us to write illegal programs.

Examples:  $1 + m$ ,  $2\text{ft} - 3\text{kg}$ .

Question: Where should we catch such errors?

- a) in the parser (as we create the AST)
- b) during the evaluation of the AST
- c) parser and evaluator will cooperate to catch this bug
- d) these bugs cannot generally (ie, all) be caught

Answer:

b: parser has only a local (ie, node and its children) view of the AST, hence cannot tell if  $((m))+(kg)$  is legal or not.

# Representing values of units

---

How to represent the value of ('^', 'm', 2)?

A pair (numeric value, Unit)

Unit a map from an SI unit to its exponent:

('^', 'm', 2) → (1, {'m':2})

('\*', 3, ('^', 'm', 2)) → (3, {'m':2})

*evaluates to*

*Python dictionary*  
*'m' is key*  
*2 is value*

# The interpreter

Ex:  $m/m \rightarrow (1, \{\})$  <sup>empty dict.</sup>

```
def eval(e):
    if type(e) == type(1):    return (e, {})
    if type(e) == type('m'): return (1, {e:1})
    if type(e) == type(()):
        if e[0] == '+': return add(eval(e[1]), eval(e[2]))
        ...
def sub((n1,u1), (n2,u2)):
    if u1 != u2: raise Exception("Subtracting incompatible units")
    return (n1-n2,u1)
def mul((n1,u1), (n2,u2)):
    return (n1*n2, mulUnits(u1,u2))
```

$m^2 * kg \rightarrow m^2 * kg$   
 $m^2 * m^3 \rightarrow m^5$

Read rest of code at:

<http://bitbucket.org/bodik/cs164fa09/src/9d975a5e8743/L3-ConversionCalculator/Prep-for-lecture/ConversionCalculator.py>



# How we'll grow the language

---

1. Arithmetic expressions
2. Physical units for (SI only) ✓ [code](#) (link)
3. Non-SI units
4. Explicit unit conversion

You are expected to read the code

It will prepare you for PA1

## Step 3: add non-SI units

---

Trivial extension to the syntax

$E ::= n \mid U \mid E \text{ op } E \mid (E)$

$U ::= m \mid s \mid \text{kg} \mid \text{ft} \mid \text{year} \mid \dots$

But how do we extend the interpreter?

We will evaluate ft to 0.3048 m.

This effectively converts ft to m at the leaves of the AST.

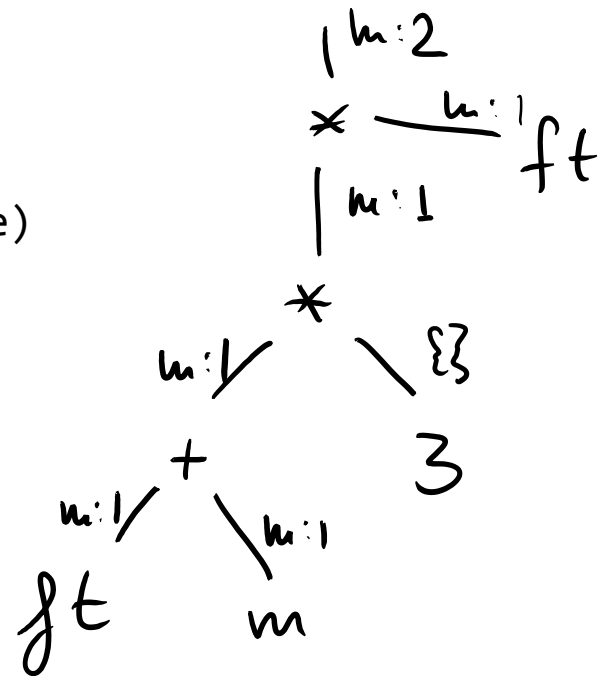
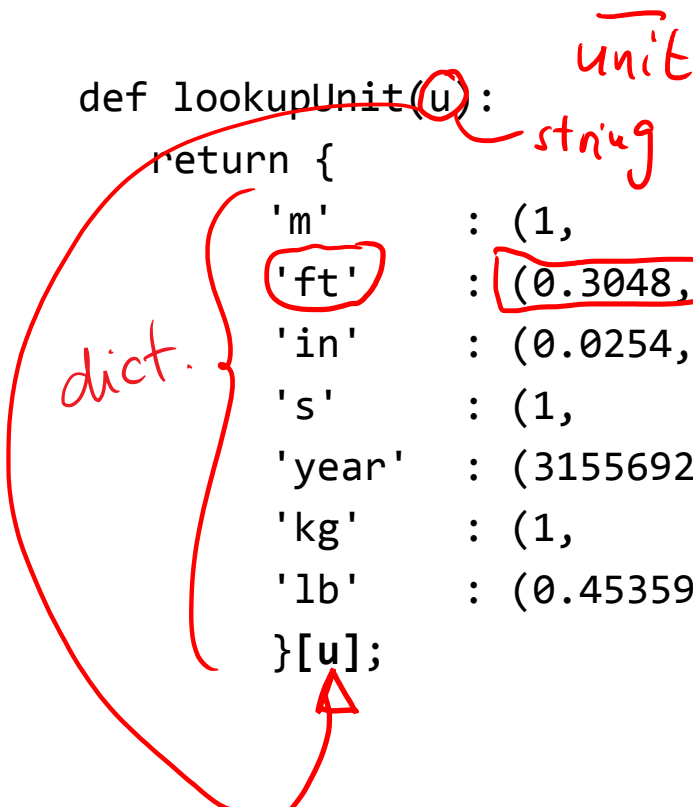
We are canonicalizing non-SI values to their SI unit

SI units are the “normalized type” of our values

# The code

```
def eval(e):  
    if type(e) == type(1): return (e, {})  
    if type(e) == type(1.1): return (e, {})  
    if type(e) == type('m'): return lookupUnit(e)
```

```
def lookupUnit(u):  
    return {  
        'm' : (1, {'m':1}),  
        'ft' : (0.3048, {'m':1}),  
        'in' : (0.0254, {'m':1}),  
        's' : (1, {'s':1}),  
        'year' : (31556926, {'s':1}),  
        'kg' : (1, {'kg':1}),  
        'lb' : (0.45359237, {'kg':1})  
    }[u];
```



how values are propagated up the tree

Rest of code at :

# How we'll grow the language

---

1. Arithmetic expressions
2. Physical units for (SI only) [code \(link\)](#) 44LOC
3. Add non-SI units [code \(link\)](#) 56LOC
  - 3.5 Revisit integer semantics (a coercion bug)
4. Explicit unit conversion

# Coercion revisited

---

To what should "1 m / year" evaluate?

our interpreter outputs 0 m / s

problem: value  $1 / 31556926 * \text{m} / \text{s}$  was rounded to zero

Because we naively adopted Python coercion rules

They are not suitable for our calculator.

We need to define and implement our own.

Keep a value in integer type whenever possible. Convert to float only when precision would otherwise be lost.

Read the code: explains when int/int is an int vs a float

<http://bitbucket.org/bodik/cs164fa09/src/204441df23c1/L3-ConversionCalculator/Prep-for-lecture/ConversionCalculator.py>

# How we'll grow the language

---

1. Arithmetic expressions
2. Physical units for (SI only) [code \(link\)](#) 44LOC
3. Add non-SI units [code \(link\)](#) 56LOC
  - 3.5 Revisit integer semantics (a coercion bug) [code \(link\)](#) 64LOC
4. **Explicit unit conversion**

# Explicit conversion

---

Example:

3 ft/s **in** m/year --> 28 855 653.1 m/year

The language of the previous step:

$E ::= n \mid U \mid E \text{ op } E \mid (E)$

$U ::= m \mid s \mid \text{kg} \mid \text{J} \mid \text{ft} \mid \text{in} \mid \dots$

$\text{op} ::= + \mid - \mid * \mid \varepsilon \mid / \mid ^$

Let's extend this language with “E in C”

# Where in the program can "E in C" appear?

---

## Attempt 1:

$E ::= n \mid U \mid E \text{ op } E \mid (E) \mid E \text{ in } C$

That is, is the construct "E in C" a kind of expression?

If yes, we must allow it wherever expressions appear.

For example in  $(2 \text{ m in ft}) + 3 \text{ km}$ .

For that, E in C must yield a value. Is that what we want?

## Attempt 2:

$P ::= E \mid E \text{ in } C$

$E ::= n \mid U \mid E \text{ op } E \mid (E)$

"E in C" is a top-level construct.

It decides how the value of E is printed.

*correct  
answer*



# Next, what are the valid forms of C?

---

## Attempt 1:

$C ::= U \text{ op } U$

$U ::= m \mid s \mid \text{kg} \mid \text{ft} \mid \text{J} \mid \dots$

$\text{op} ::= + \mid - \mid * \mid \varepsilon \mid / \mid ^$

2 ft in m-mm ?  
NO

Examples of valid programs:

## Attempt 2:

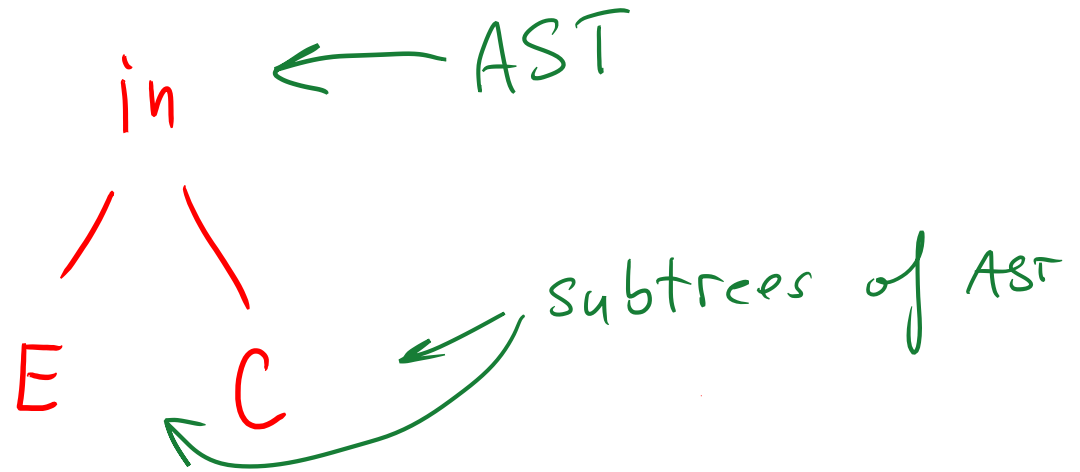
$C ::= C * C \mid C C \mid C / C \mid C ^ n \mid U$

$U ::= m \mid s \mid \text{kg} \mid \text{ft} \mid \text{J} \mid \dots$

# How to evaluate C?

---

Our ideas:



what's the "value" of C?

how is it represented?

we would like to evaluate C with the same function as E. But this seems impossible

# How to evaluate C?

---

What value(s) do we need to obtain from sub-AST C?

1. conversion ratio between the unit C and its SI unit

*— 2 ft/year in m/s*

ex:  $(\text{ft}/\text{year})/(\text{m}/\text{s}) = 9.65873546 \times 10^{-9}$

2. a representation of C, for printing

ex:  $\text{ft} * \text{m} * \text{ft} \rightarrow \{\text{ft}:2, \text{m}:1\}$

let me know  
if you find  
simple design

*we could make other  
sensible choices here*

# How we'll grow the language

---

1. Arithmetic expressions
2. Physical units for (SI only) [code](#) 44LOC
3. Add non-SI units [code](#) 56LOC
  - 3.5 Revisit integer semantics (a coercion bug) [code](#) 64LOC
4. Explicit unit conversion [code](#) 78LOC
  - this step also includes a simple parser: [code](#) 120LOC

You are asked to understand the code.

you will understand the parser code in later chapters

# Where are we?

---

The grammar:

$P ::= E \mid E \text{ in } C$

$E ::= n \mid E \text{ op } E \mid ( E ) \mid U$

$\text{op} ::= + \mid - \mid * \mid \varepsilon \mid / \mid ^$

$U ::= m \mid s \mid \text{kg} \mid \text{ft} \mid \text{cup} \mid \text{acre} \mid 1 \mid \dots$

$C ::= U \mid C * C \mid C C \mid C/C \mid C^n$

After adding a few more units, we have google calc:

**34 knots in mph** --> *39.126 mph*

# What you need to know

---

- Understand the code of the calculator
- Able to read grammars (descriptors of languages)

# Key concepts

---

programs, expressions

are parsed into abstract syntax trees (ASTs)

values

are the results of evaluating the program,  
in our case by traversing the AST bottom up

types

are auxiliary info (optionally) propagated with values during  
evaluation; we modeled physical units as types

## Part 2

---

Grow the calculator language some more.

Allow the user to

- add own units
- reuse expressions



# Review of progress so far

---

Example:

**34 knots in mph** # speed of S.F. ferry boat

--> *39.126 mph*

Example:

# volume \* (energy / volume) / power = time

**half a dozen pints \* (110 Calories per 12 fl oz) / 25 W in days**

--> *1.704 days*

Now we will change the language to be extensible

# How we'll grow the language

---

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this step also includes a simple parser: [code](#) 120LOC
5. Allowing users to add custom non-SI units

# Growing language w/out interpreter changes

---

We want to design the language to be extensible

- Without changes to the base language
- And thus without changes to the interpreter

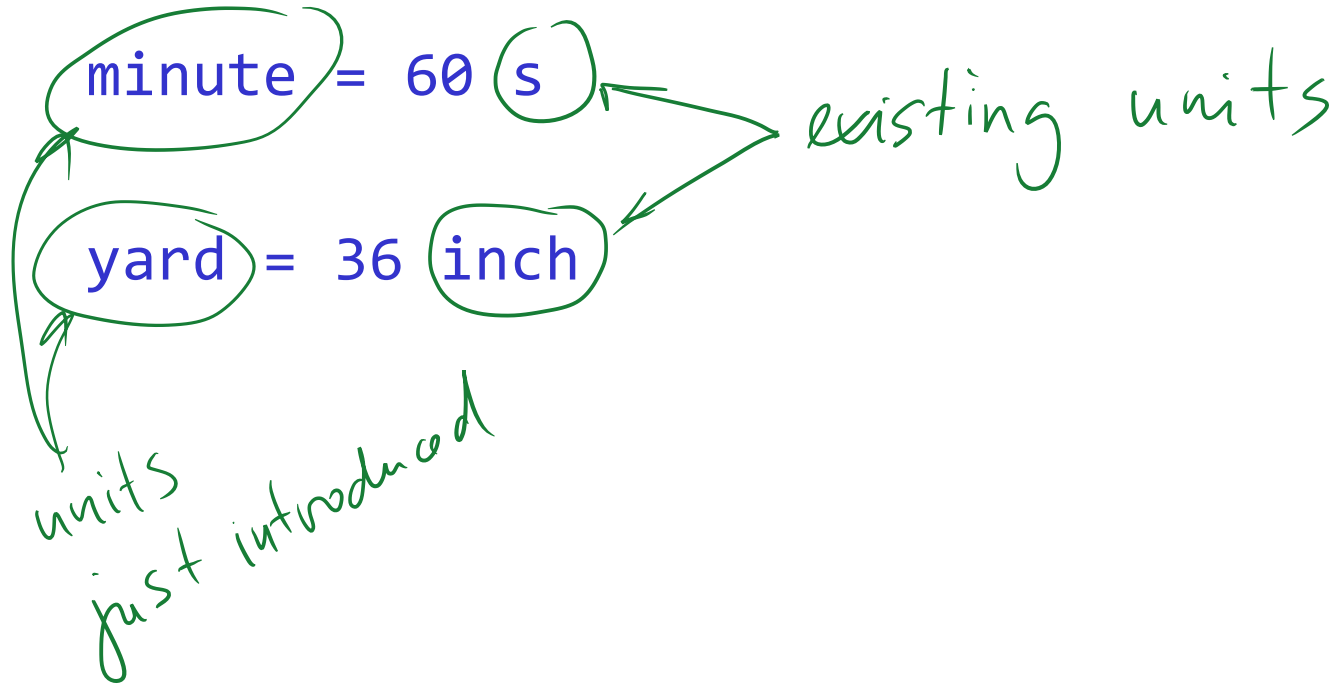
For calc, we want the user to add new units

- Assume the language knows about meters (feet, ...)
- Users may want to add, say, Angstrom and light year

How do we make the language extensible?

# Our ideas

---



# Bind a value to an identifier

---

minute = 60 s

hour = 60 minute

day = 24 hour

month = 30.5 day // maybe not define month?

year = 365 day

km = 1000 m

inch = 0.0254 m

yard = 36 inch

acre = 4840 yard<sup>2</sup>

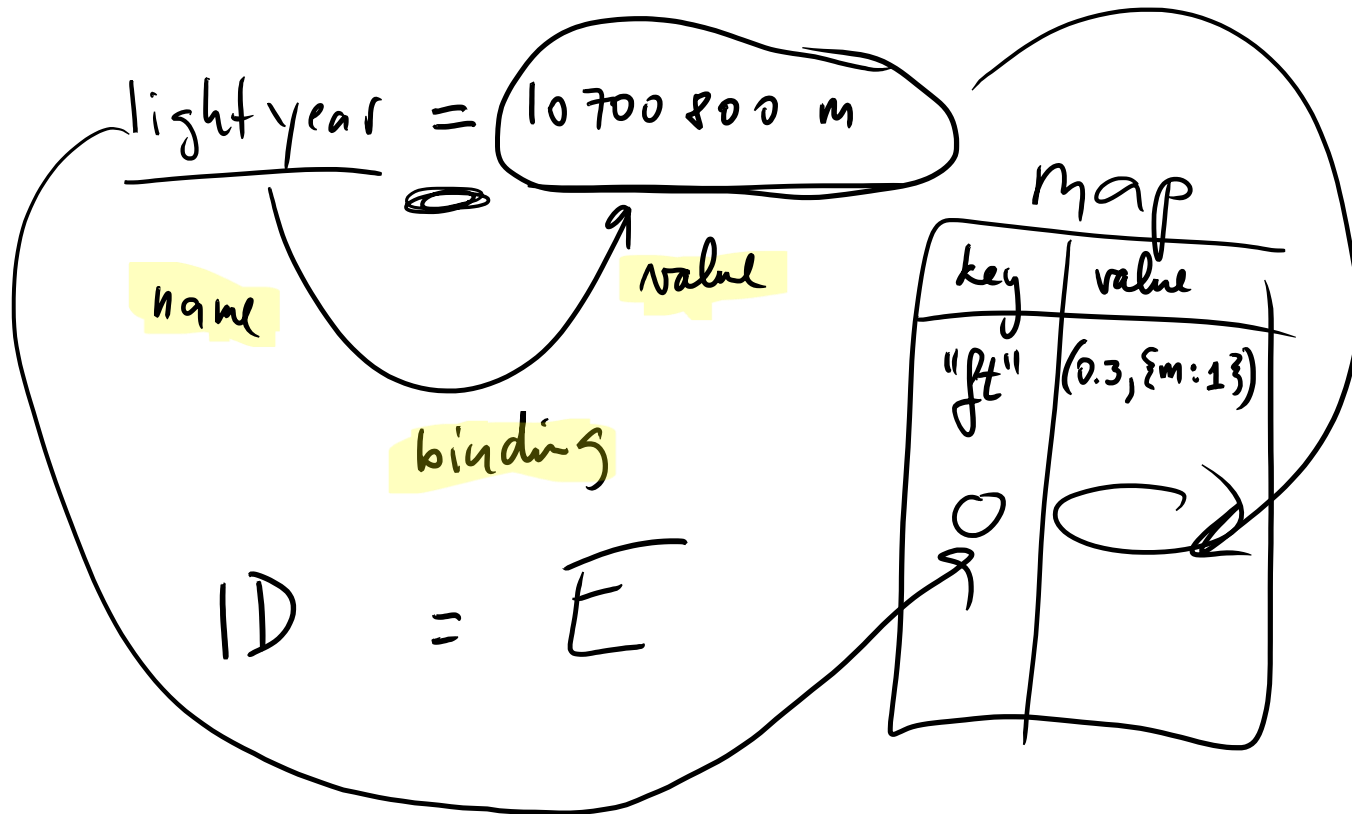
hectare = (100 m)<sup>2</sup>

2 acres in hectare → **0.809371284 hectare**

# Implementing user units

Assume units extends existing measures.

We want the user to add **ft** when **m** or **yard** is known



# How we'll grow the language

---

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this step also includes a simple parser: [code](#) 120LOC
5. Allowing users to add custom non-SI units ✓
6. Allowing users to add custom measures

# How do we add new measures?

---

No problem for Joule, as long you have kg, m, s:

$$J = \text{kg m}^2 / \text{s}^2$$

But other units must be defined from first principles:

Electric current:

- Ampere

Currency:

- USD, EUR, YEN, with BigMac as the SI unit

Coolness:

- DanGarcias, with Fonzie as the SI unit



# Our ideas

---

## Attempt 1:

when we evaluate  $a = 10 b$  and  $b$  is not known, add it as a new SI unit.

This may lead to spuriously SI units introduced due to typos.

## Attempt 2:

ask the user to explicitly declare the new SI unit:

SI Ampere

# Our solution

---

Add into language a construct introducing an SI unit

```
SI A // Ampere
mA = 0.0001 A
SI BigMac
USD = BigMac / 3.57 // BigMac = $3.57
GBP = BigMac / 2.29 // BigMac = £2.29
```

With “SI <id>”, language needs no built-in SI units

```
SI m
km = 1000 m
inch = 0.0254 m
yard = 36 inch
```

# Implementing SI id

---

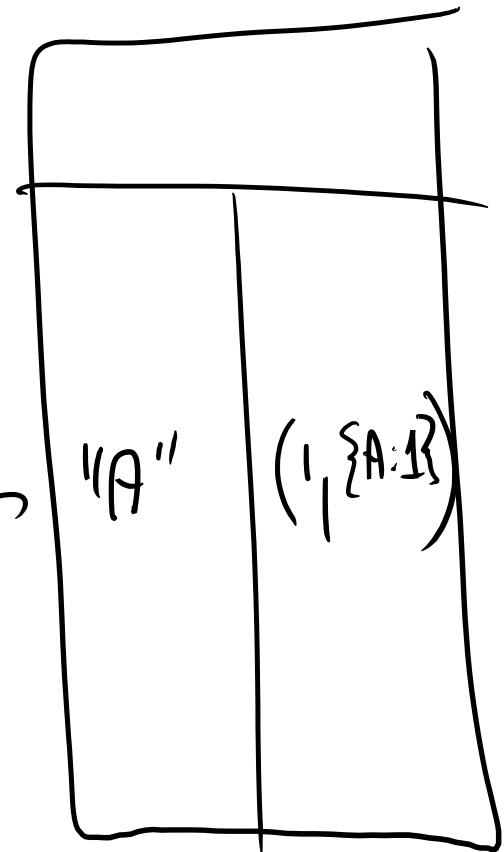
Problem

$$\text{mA} = \text{A} / 1000$$

Solve

declaration: S/W A

$$\text{mA} = 0.001 \text{ A}$$



# How we'll grow the language

---

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5. Allowing users to add custom non-SI units
6. Allowing users to add custom measures [code](#) ✓
7. Reuse of values

# Motivating example

---

Compute # of PowerBars burnt on a 0.5 hour-long run

SI m, kg, s

1b = 0.454 kg; N = kg m / s<sup>2</sup>

J = N m; cal = 4.184 J

powerbar = 250 cal

*we wish to remember it as a constant*

0.5hr \* 170lb \* (0.00379 m<sup>2</sup>/s<sup>3</sup>) in powerbar

--> 0.50291 powerbar

Want to retype the formula after each morning run?

0.5 hr \* 170 lb \* (0.00379 m<sup>2</sup>/s<sup>3</sup>)

# Reuse of values

---

To avoid typing

$$170 \text{ lb} * (0.00379 \text{ m}^2/\text{s}^3)$$

... we'll use same solution as for introducing units:

Just name the value with an identifier.

$$c = 170 \text{ lb} * (0.00379 \text{ m}^2/\text{s}^3)$$

$$28 \text{ min} * c$$

*# ... next morning*

$$1.1 \text{ hour} * c$$

Should time given be in min or hours?

Either. Check this out! Calculator converts automatically!

# How we'll grow the language

---

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this step also includes a simple parser: [code](#) 120LOC
5. Allowing users to add custom non-SI units
6. Allowing users to add custom measures [code](#)
7. Reuse of values (no new code needed) ✓
8. Reuse of expressions (bind names to expressions)

# Another motivating example

---

You want to print the current time left to deadline

`now = 2011 year + 0 month + 18 day + 15 hour + 40 minute`  
--- pretend that `now` is always set to current time of day

Let's try to compute time to deadline

`deadline = 2011 year + 1 month + 3 day // 2/3/2012`

`timeLeft = deadline - now`

`timeLeft in day --> time left`

Wait for current time to advance. Print time left now.

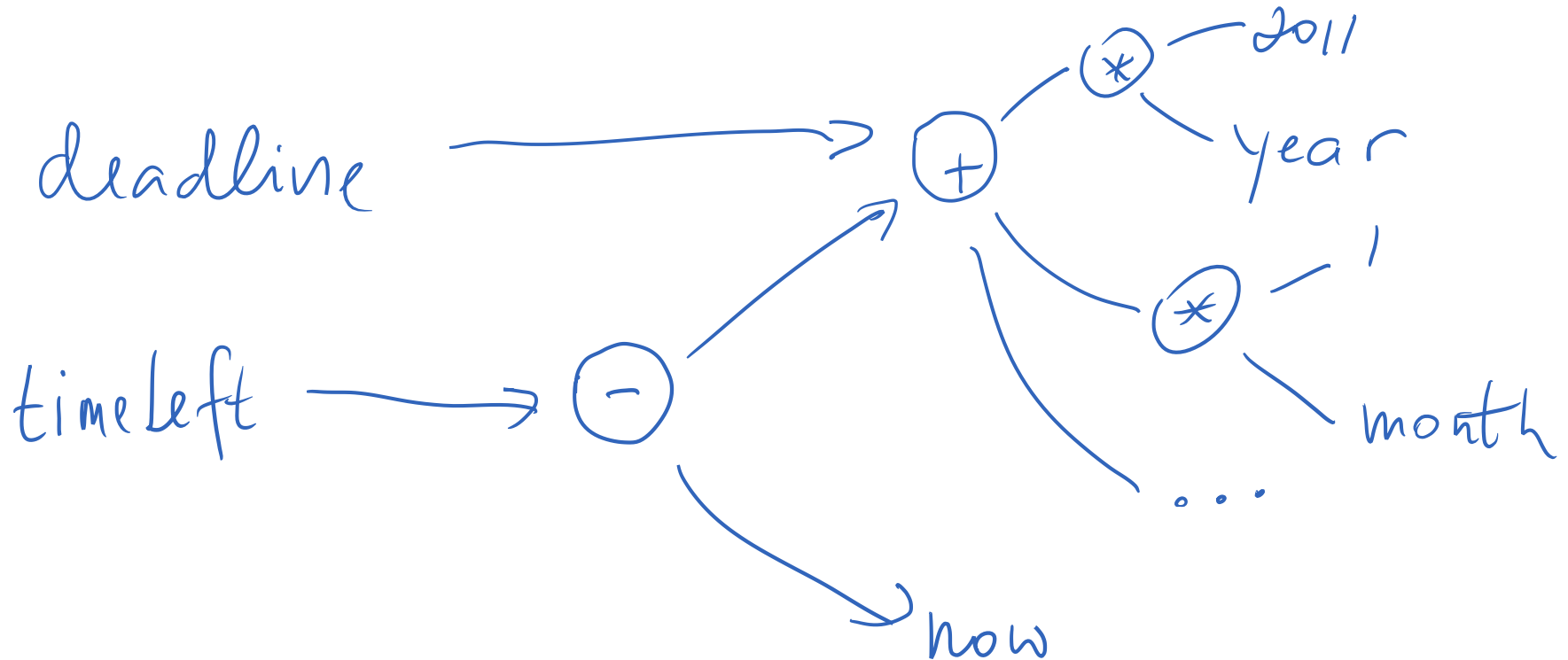
What does the following print?

`timeLeft in day --> updated time left`

How to achieve this behavior?



# timeLeft is bound to an expression



year, month are actually expressions, too

# Naming values vs. naming expressions

---

“Naming an expression” means that we evaluate it lazily when we need its value

# How we'll grow the language

---

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this step also includes a simple parser: [code](#) 120LOC
5. Allowing users to add custom non-SI units
6. Allowing users to add custom measures [code](#)
7. Reuse of values (no new code needed)
8. Reuse of expressions [code](#) (not fully lazy)

# Summary: Calculator is an extensible language

---

Very little built-in knowledge

- Introduce base units with 'SI name'
- Arithmetic performs general unit types and conversion

No need to define all units in terms of SI units

```
cal = 4.184 J
```

Reuse of values by naming the values.

```
myConstant = 170 lb * (0.00379 m^2/s^3)
```

```
0.5 hr * myConstant in powerbar
```

-> Same mechanism as for introduction of non-SI units!

No need to remember units! Both will work fine!

```
0.5 hr * myConstant in powerbar
```

```
30 minutes * myConstant in powerbar
```

# Limitations of calculator

---

## No relational definitions

- We may want to define ft with ‘12 in = ft’
- We could do those with Prolog
  - recall the three colored stamps example in Lecture 1

## Limited parser

- Google parses  $1/2/m/s/2$  as  $((1/2) / (m/s)) / 2$
- There are two kinds of / operators
- Their parsing gives the / operators intuitive precedence
- You will implement his parser in PA6

# What you were supposed to learn

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## Binding names to values

- and how we use this to let the user grow the calculator

## Introducing new SI units required declaration

- the alternative could lead to hard-to-diagnose errors

## names can bind to expressions, not only to values

- these expressions are evaluated lazily