

## Lecture 6

## Logic Programming

 rule-based programming with PrologRas Bodik Hack Your Language!
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## Today

Why study Prolog

- our abstraction stack

Introduction to Prolog

- facts and queries
- generalization, instantiation, and the Prolog semantics
- rules
- functors: AST and a simple interpreter
- working with lists
- a simple AST rewrite engine


## Reading

## Compulsory:

Adventures in Prolog, a tutorial (Chapters 1-11)

## Optional:

The Art of Prolog, on reserve in Engineering Library
(starting Friday evening, Feb 8).

Why logic programming?

## Our abstraction stack

Parsing (PA4), Analysis of Programs, Types

- we'll express each with Prolog rules
- (not all will require backtracking)

Logic programming (PA3)

- a convenient layer over backtracking
- enables rule-based programming, inference

Coroutines (PA2)

- enable lazy iterators and backtracking
- ex: regex matching: for all matches of patt1, match patt2

Closures and lexical scoping (PA1)

- enable, for example, iterators
- ex: d3.select(someNodes). $\underline{\text { each(aClosure) }}$


## Infrastructure

## Software

## Software:

install SWI Prolog
Usage:
?- [likes]. \# loads file likes.pl
Content of file likes.pl:
likes(john,mary).
likes(mary,jim).
After loading, we can ask a query:
?- likes(X,mary). \#who likes mary?
X = john;
false. \# no one else

## Facts, queries, and variables

## Facts and queries

(Database of) two facts:
likes(john, mary). \# relation: facts with same name, eg likes likes(mary, jim). \# relation is a.k.a. predicate

Boolean queries (answers are true or false) ?- likes(john,jim). \# sometimes we use syntax likes( $a, b$ )? false

Existential queries (is there X s/t likes( $\mathrm{X}, \mathrm{jim}$ ) holds?)
?- likes(X,jim). \# variables start with capital letters mary
\# atoms start with capital letters

## goals, facts, and queries

Syntactically, facts and queries look similar
goal: likes(jim, mary)
notice that there is no dot at the end of goal
fact: likes(jim, mary).
states that the goal likes(jim,mary) is true
query: likes(jim, mary)? sometimes we write ?- goal. asks whether the goal likes(jim, mary) is true

## Terminology

Ground terms (do not contain variables)
father(a,b). \# fact ( $a$ is father of $b$ )
?- father(a,b). \# query (is a father of b?)
Non-ground terms (contain variables)
$\begin{array}{ll}\forall \text { likes( } X, X) . & \text { \# fact: everyone likes him } \\ \exists \text { ?- likes(Y,mary). \# query: who likes mary? }\end{array}$

Variables in facts are universally quantified for whatever $X$, it is true that $X$ likes $X$
Variables in queries are existentially quantified
does there exist an X such that X likes mary?

## Example

A single fact
likes(X,X).

## Queries:

?- likes(a,b).
false.
?- likes(a,a).
true.
?- likes(X, a).
$X=a$.
?- likes $(X, Y)$.
$\mathrm{X}=\mathrm{Y} . \quad<-$ Answers to queries need not be fully grounded
?- likes(A,A).
true.

# Generalization and Deduction via Substitution for Variables 

## Generalization (a deduction rule)

## Facts

father(abraham,isaac).
Query
?- father(abraham,X).

This query is a generalization of the fact

We answer the query by finding a substitution $\{X=i s a a c\}$. This substitution turns the query into a fact that exists in the database, leading to true. Well, the answer also shows the substitution.

## Instantiation (another deduction rule)

Rather than writing
plus(0,1,1). plus(0,2,2). ...
We write
plus(o,X,X).
\# $0+x=x$
plus(X,0,X). \# $x+0=x$

Query
?- plus $(0,3,3)$. \# this query is instantiation of plus( $0, X, X$ ). yes
We answer by finding a substitution $\{X=3\}$.

## Prolog semantics

How the Prolog interpreter answers a query:

$$
\begin{array}{ll}
p(a, a) . & \text { \# fact } 1 \\
p(a, b) . & \text { \# fact } 2 \\
p(b, c) . & \text { \# fact } 3
\end{array}
$$

?- $\mathrm{p}(\mathrm{a}, \mathrm{A})$. \# This query raises one goal, $\mathrm{p}(\mathrm{a}, \mathrm{A})$
A =a; \# going top down across facts, the goal matches 1
A = b. \# and when asked for next match, it matches 2.

We'll generalize this algorihm when we add rules.

## Rules

## Rules

Rules define new relationships in terms of existing ones
parent $(X, Y)$ :- father $(X, Y)$.
parent $(\mathrm{X}, \mathrm{Y})$ :- mother $(\mathrm{X}, \mathrm{Y})$.
grandfather(X,Y) :-father(X,Z), parent(Z,Y).
Assume facts
father(john,mary).
mother(mary,jim).
Now ask the query
?- grandfather $(\mathrm{X}, \mathrm{Y})$.
X = john,
$\mathrm{Y}=\mathrm{jim}$;
false.

## The Prolog semantics

## Prolog algorithms in the presence of rules:

father(john,mary).
mother(mary,jim).
grandfather(fim, fum). \#1
parent $(X, Y)$ :- father $(X, Y)$.
parent $(X, Y)$ :- mother $(X, Y)$. grandfather( $\mathrm{X}, \mathrm{Y}$ ) :-father( $\mathrm{X}, \mathrm{Z}$ ), parent(Z,Y). \# 2
?- grandfather $(X, Y)$.
$\mathrm{X}=\mathrm{fim}, \mathrm{Y}=\mathrm{fum}$; \# matches fact 1 from relation 'grandfather'
$\mathrm{X}=\mathrm{john}, \mathrm{Y}=\mathrm{jim}$; \# matches head (lhs) of rule 2, which then \# creates two new goals from the rhs of 2.
false.
\# lhs = left-hand-side

## So Prolog can do a simple inference!

1801 - Joseph Marie Jacquard uses punch cards to instruct a loom to weave "hello, world" into a tapestry. Redditers of the time are not impressed due to the lack of tail call recursion, concurrency, or proper capitalization.

1972 - Alain Colmerauer designs the logic language Prolog. His goal is to create a language with the intelligence of a two year old. He proves he has reached his goal by showing a Prolog session that says "No." to every query.

## Database programming

A database programming rule
brother(Brother, Sib) :-
parent(P, Brother),
parent(P, Sib),
male(Brother),
Brother \= Sib. \# same as l=(Brother,Sib)

This rule assumes that we have defined relations parent and male. (The $\=$ relation is a built-in.)

## Database programming

In cs164, we will translate SQL-like queries to Prolog. But Prolog can also express richer (recursive) queries:

descendant $(Y, X)$ :- parent( $X, Z$ ), descendant( $Y, Z$ ).

## Order of rules and clauses matters

1) Given a goal, Prolog matches facts and rules topdown as they appear in the file.
ex: on slide 19, \#1 matches before \#2 matches.
2) If the rhs of a rule raises multiple goals, they are answered left-to-right.
ex: on slide 19, match 2, father $(X, Z)$ is resolved before parent(Z,Y).

## Test yourself

Make sure you understand why these three variants of descendants have different behaviors:
v1:
descendant $(Y, X)$ :- parent $(X, Y)$.
descendant $(\mathrm{Y}, \mathrm{X})$ :- parent( $\mathrm{X}, \mathrm{Z}$ ), descendant( $\mathrm{Y}, \mathrm{Z})$.
v2:
descendant( $\mathrm{Y}, \mathrm{X}$ ) :- parent( $\mathrm{X}, \mathrm{Z})$, descendant( $\mathrm{Y}, \mathrm{Z})$.
descendant( $\mathrm{Y}, \mathrm{X}$ ) :- parent( $\mathrm{X}, \mathrm{Y}$ ).
v3:
descendant $(\mathrm{Y}, \mathrm{X})$ :- parent $(\mathrm{X}, \mathrm{Y})$.
descendant $(\mathrm{Y}, \mathrm{X})$ :- descendant( $\mathrm{Y}, \mathrm{Z})$, parent $(\mathrm{X}, \mathrm{Z})$.

## Compound terms

## Compound terms

Compound term = functors and arguments.
Name of functor is an atom (lower case), not a Var. example: cons(a, cons(b, nil))

A rule:

car(Head, List) :- List = cons(Head,Tail).<br>car(Head, cons(Head,Tail)). \# equivalent to the above

Query:
?- car(Head, cons(a, cons(b, nil)).

## A simple interpreter

A representation of an abstract syntax tree
int(3)
plus(int(3),int(2))
plus(int(3),minus(int(2),int(3)))
An interpreter
eval(int(X),X).
eval(plus(L,R),Res):-
eval(L,Lv),
eval(R, Rv),
Res is Lv + Rv.
eval(minus(L,R),Res):-
\# same as plus

## Working with lists

## Lists

Lists are just compounds with special, clearer syntax.

Cons is denoted with a dot '.’

| .$(\mathrm{a},[])$ | is same as | $[\mathrm{a} \mid[]]$ is same as |
| :--- | :--- | :--- |
| .$(\mathrm{a},(\mathrm{b},[\mathrm{C}])$ | $[\mathrm{a}]$ |  |
| .$(\mathrm{a}, \mathrm{X})$ | $[\mathrm{a} \mid[\mathrm{b} \mid[[]]]$ | $[\mathrm{a}, \mathrm{b}]$ |
|  | $[\mathrm{a} \mid \mathrm{X}]$ | $[\mathrm{a} \mid \mathrm{X}]$ |

## predicate "Am I a list?"

Let's test whether a value is a list
list([]).
$\operatorname{list}([X \mid X s])$ : : list(Xs).

Note the common Xs notation for a list of X's.

## Let's define the predicate member

## Desired usage:

?- member(b, [a,b,c]).
true

## Lists

$\operatorname{car}([X \mid Y], X)$.
cdr([X|Y],Y).
cons(X,R,[X|R]).
meaning ...

- The head (car) of $[X \mid Y]$ is $X$.
- The tail (cdr) of $[\mathrm{X} \mid \mathrm{Y}]$ is Y .
- Putting $X$ at the head and $Y$ as the tail constructs (cons) the list $[X \mid R]$.

From: http://www.csupomona.edu/-jrrisher/www/prolog_tutorial

## An operation on lists: <br> $$
\left.\begin{array}{l} {[a, b, c] \rightarrow-\rightarrow} \\ {[a,[b,[c,[]] \rightarrow} \\ \cdot(a, \cdot(b, \cdot(c, u, i))) \end{array}\right\} \begin{aligned} & {[] \text { is }} \\ & \text { sis } \\ & \text { susa } \end{aligned}
$$

member (X, $[X \mid R])$.
member $(X,[Y \mid R])$ :- member $(X, R)$.

One can read the clauses the following way:
$X$ is a member of a list whose first element is $X$. $X$ is a member of a list whose tail is $R$ if $X$ is a member of $R$.

## List Append

$$
\begin{aligned}
& \text { append([], List, List). } \\
& \text { append([H|Tail],X, } \mathrm{H} \mid \text { NewTail] }):- \\
& \quad \text { append(Tail,X,NewTail). } \\
& \text { ?- append }([a, b],[c, d], X) . \\
& X=[a, b, c, d] . \\
& ?-\text { append([a,b],X,[a,b,c,d]). } \\
& X=[c, d] .
\end{aligned}
$$

This is "bidirectional" programming
Variables can act as both inputs and outputs

## More on append

?- append (Y,X,[a,b,c,d]).
$Y=[]$,
$X=[a, b, c, d]$;
$Y=[a]$,
$X=[b, c, d]$;
$Y=[a, b]$,
$X=[c, d]$;
$Y=[a, b, c]$,
$X=[d]$;
$Y=[a, b, c, d]$,
X = [] ;
false.

## Exercise for you

Create an append query with infinitely many answers.

$$
\begin{aligned}
& \text { ?- append(Y,X,Z). } \\
& Y=[] \text {, } \\
& \text { X = Z ; } \\
& Y=\left[\_G 613\right] \text {, } \\
& \text { Z = [_G613|X] ; } \\
& Y=\left[\_G 613, \quad\right. \text { G619] } \\
& \text { Z = [_G613, _G619|X] ; }
\end{aligned}
$$

## Another exercise: desugar AST

Want to rewrite each instance of $2^{*} x$ with $x+x$ : rewrite(times(int(2),R), plus(Rr, Rr)) :!, rewrite(R,Rr). rewrite(times(L,int(2)), plus(Lr,Lr)) :!, rewrite(L,Lr). rewrite(times(L,R),times(Lr, Rr)) :!, rewrite(L,Lr),rewrite(R,Rr). rewrite(int(X),int(X)).

## And another exercise

Analyze a program:

1) Translate a program into facts.
2) Then ask a query which answers whether a program variable is a constant at the of the program.
Assume the program contains two statement kinds

$$
\mathrm{S}::=\mathrm{S}^{*}|\operatorname{def} \mathrm{ID}=\mathrm{n}| \text { if }(\mathrm{E}) \mathrm{ID}=\mathrm{n}
$$

You can translate the program by hand

## Some other cool examples to find in tutorials

compute the derivative of a function
this is example of symbolic manipulation
solve a math problem by searching for a solution:
"Insert +/- signs between 12345 so that the result is 5."

