



Ras Bodik  
with Ali & Mangpo

## Lecture 8

### Grammars and Parsers

grammar and derivations, recursive descent  
parser vs. CYK parser, Prolog vs. Datalog

### *Hack Your Language!*

CS164: Introduction to Programming  
Languages and Compilers, Spring 2013

[UC Berkeley](#)

# Outline

---

*Grammars*: a concise way to define program syntax

*Parsing*: recognize syntactic structure of a program

Parser 1: *recursive descent* (backtracking)

Parser 2: CYK (dynamic programming algorithm)

**Note:** this file includes useful hidden slides which do not show in the PowerPoint Slide View.

# Why parsing?

---

Parsers making sense of these sentences:

*This lecture is dedicated to my parents, Mother Teresa and the pope.*

The (missing) serial comma determines whether M.T.&p. associate with “my parents” or with “dedicated to”.

*Seven-foot doctors filed a law suit.*

does “seven” associate with “foot” or with “doctors”?

**if  $E_1$  then { if  $E_2$  then  $E_3$  else  $E_4$  }**

typical semantics associates “else  $E_4$ ” with the closest if (ie, “if  $E_2$ ”)

In general, programs and data exist in text form

which needs to be understood by parsing (and converted to tree form)

# The cs164 parsing story

---

From string generation to Earley parser

1. Write a **random expression generator**.
2. **Invert** this generator into a parser by inverting print into scan and random() into askOracle(). The oracle constructs the parse tree.
3. Rewrite this parser in Prolog, which serves as your oracle.  
This gives you the ubiquitous **recursive descent parser**. Time =  $O(2^n)$
4. Observe that this Prolog parser has no negation. It's in a **Datalog** subset of Prolog (more or less).
5. Datalog programs are evaluated bottom-up (dynamic programming).  
Rewriting the Prolog parser into Datalog gives us the **CYK parser**.  $O(n^3)$
6. Datalog evaluation can be optimized with the Magic Set Transformation, which gives us **Earley Parser**. (Covered in Lecture 9.)  $O(n)$  with a suitable grammar. Earley is the basis for all efficient modern parsers.

# Grammars

# Grammar: a recursive definition of a language

---

**Language:** a set of (desired) strings

**Example:** the language of Regular Expressions (RE).

RE can be defined as a grammar:

base case: any input character  $c$  is regular expression;

inductive case: if  $e_1, e_2$  are regular expressions, then the following four are also regular expressions:

$e_1 \mid e_2$      $e_1 e_2$      $e_1^*$      $(e_1)$

red

Example:

a few strings in this language:

$a$     $a|a$     $a|b^*$

a few strings not in this language:

$*x$     empty string ( $\epsilon$ )     $*|x$   
 $(a)$

# Terminals, non-terminals, productions

---

The *grammar* notation:

$R ::= c \mid R R \mid R|R \mid R^* \mid (R)$

*terminals* (**red**): input characters

also called the alphabet of the language

*non-terminals*: substrings in the language

these symbols will be rewritten to terminals

*start non-terminal*: starts the derivation of a string

convention: always the first nonterminal mentioned

*productions*: rules governing string derivation

RE has five:  $R ::= c$ ,  $R ::= R R$ ,  $R ::= R|R$ ,  $R ::= R^*$ ,  $R ::= (R)$

It's *grammar*, not *grammer*.

---

“Not all writing is due to bad *grammer*.” (sic)

Saying “*grammer*” is a lexical error, not a syntactic (ie, grammatic) one.

In the compiler, this error is caught by the lexer.

lexer fails to recognize “*grammer*” as being in the lexicon.

In cs164, you learn which part of compiler finds errors.

lexer, parser, syntactic analysis, or runtime checks?



# Deriving a string from a grammar

---

How is a string derived in a grammar:

S

1. write down the start non-terminal S
2. rewrite S with the rhs of a production  $S \rightarrow rhs$
3. pick a non-terminal N
4. rewrite N with the rhs of a production  $N \rightarrow rhs$
5. if no non-terminal remains, we have generated a string.
6. otherwise, go to 3.

Example:

grammar G:  $E ::= \underline{T} \mid \underline{T + E}$      $T = \underline{F} \mid \underline{F * T}$      $F = \underline{a} \mid \underline{(E)}$

derivation of a string from  $L(G)$ :  $\underline{E} \rightarrow T + E \rightarrow F + E \rightarrow a + E$   
 $\rightarrow a + T \rightarrow a + F \rightarrow a + a$

Left- and right-recursive grammars

# Grammars vs. languages

ba baq  
baaa

Write a grammar for the language all strings  $ba^i$ ,  $i > 0$ .

grammar 1:  $S ::= Sa \mid ba$

grammar 2:  $S ::= baA$       $A ::= aA \mid \epsilon$

← empty string

A language can be described with multiple grammars

$L(G)$  = language (strings) described by grammar  $G$

in our example,  $L(\text{grammar 1}) = L(\text{grammar 2})$

Left recursive grammar:

$X ::= Xa$

Right-recursive grammar:

$X ::= Za$   
 $Z ::= Xb$

both l-rec and r-rec:

$S ::= bA \mid baA$   
 $A ::= aA \mid \epsilon$

# Why care about left-/right-recursion?

---

Some parser can't handle left-recursive grammars.

It may get them into infinite recursion.

Same principle as in Prolog programs that do not terminate.

Luckily, we can rewrite a l-rec grammar into a r/r one.

while describing the same language

Example 1:

$S ::= Sa \mid a$  can be rewritten to  $S ::= aS \mid a$

# The typical expression grammar

---

A grammar of expressions:

$$G_1: \quad E ::= n \mid E + E \mid E * E \mid (E)$$

$G_1$  is l-rec but can be rewritten to  $G_2$  which is not

$$G_2: \quad \begin{aligned} E &::= T \mid T + E \\ T &::= F \mid F * T \\ F &::= n \mid (E) \end{aligned}$$

In addition to removing left recursion, nonterminals **T** (a term) and **F** (a factor) introduce desirable precedence and associativity. More in L9.

Is  $L(G_1) = L(G_2)$ ?

That is, are these same sets of string? Yes.

# The parsing problem

# What the parser does

---

The *syntax-checking* parsing problem:

given an input string  $s$  and grammar  $G$ , check if  $s \in L(G)$

The *parse-tree* parsing problem:

given an input string  $s \in L(G)$ , return the parse tree of  $s$

Really  
A Poor Man's Parser  
↑



# Generate-and-test “parser”

---

We want to test if  $s \in L(G)$ . Our “algorithm”:

- print a string  $p \in L(G)$ , check if  $s = p$ , repeat

The plan:

Write a function  $\text{gen}(G)$  that prints a string  $p \in L(G)$ .

If  $L(G)$  is finite,  $\text{gen}(G)$  will eventually print all strings in  $L(G)$ .

Does this algorithm work?

*Depends if you are willing to wait. ☺*

*Also,  $L(G)$  may be infinite.*

This parser is useful only for instructional purposes  
in case it's not clear already

# gen(G)

---

Grammar G and its language L(G):

G:  $E ::= a \mid E + E \mid E * E$

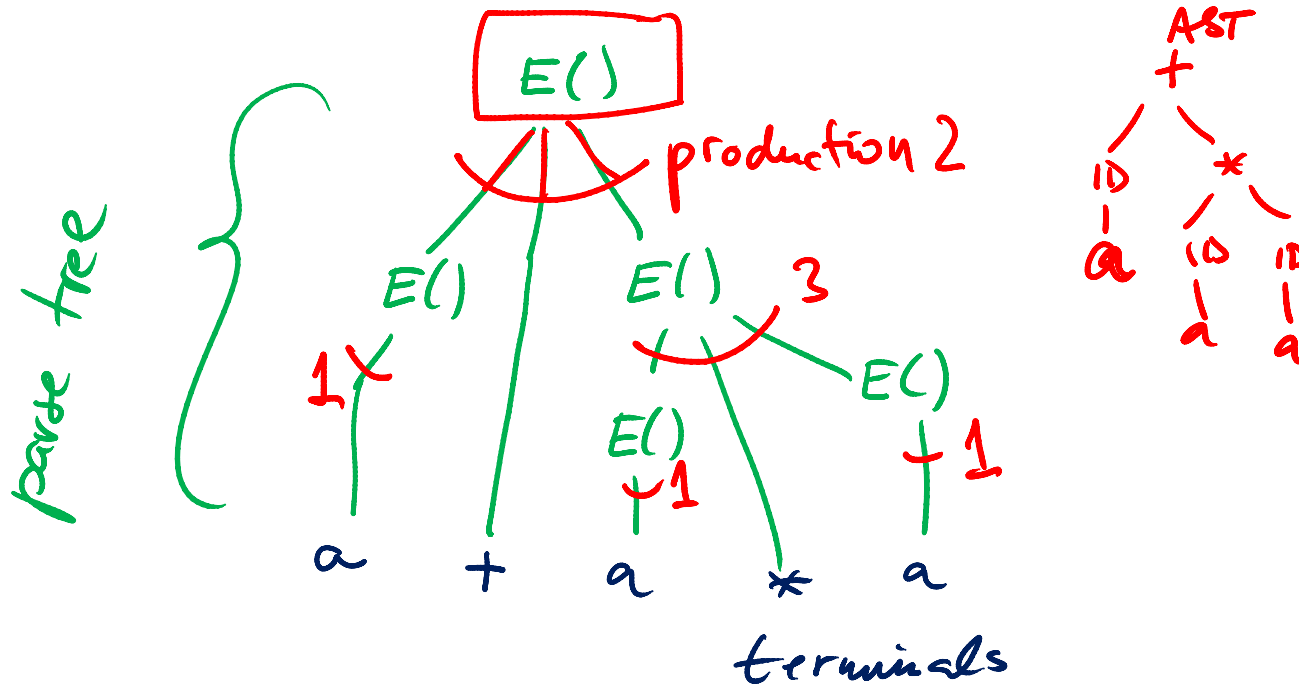
$L(G) = \{ a, a+a, a*a, a*a+a, \dots \}$

For simplicity, we hardcode G into gen()

```
def gen() { E(); print EOF }
def E() {
  switch (choice()):
  case 1: print "a"
  case 2: E(); print "+"; E()
  case 3: E(); print "*"; E()
}
```

# Visualizing string generation with a **parse tree**

The tree that describe string derivation is parse tree.



Are we generating the ~~string~~ top-down or bottom-up?

Top-down. Can we do it other way around? Sure. See CYK.

# Parsing

---

Parsing is the inverse of string generation:

given a string, we want to find the parse tree

If parsing is just the inverse of generation, let's obtain the parser mechanically from the generator!

```
def gen() { E(); print EOF }
def E() {
  switch (choice()):
  case 1: print "a"
  case 2: E(); print "+"; E()
  case 3: E(); print "*"; E()
}
```

a + a \* b

# Generator vs. parser

---

```
def gen() { E(); print EOF }
def E() { switch (choice()) {
    case 1: print "a"
    case 2: E(); print "+"; E()
    case 3: E(); print "*"; E() }}
```

```
def parse() { E(); scan(EOF) }
def E() { switch (oracle()) {
    case 1: scan("a")
    case 2: E(); scan("+"); E()
    case 3: E(); scan("*"); E() }}

def scan(s) { if rest of input starts with s,
    consume s; else abort }
```

Reconstruct the Parse Tree

# Parse tree

---

Parse tree: shows how the string is derived from  $G$

*leaves*: input characters

*internal nodes*: non-terminals

*children of an internal node*: production used in derivation

Why do we need the parse tree?

We evaluate it to obtain the AST, or sometimes to directly compute the value of the program.

Test yourself: construct the AST from a parse tree.

# Example: evaluate an expression on parse tree

**Input:**  $2 * (4 + 5)$

**Grammar:**

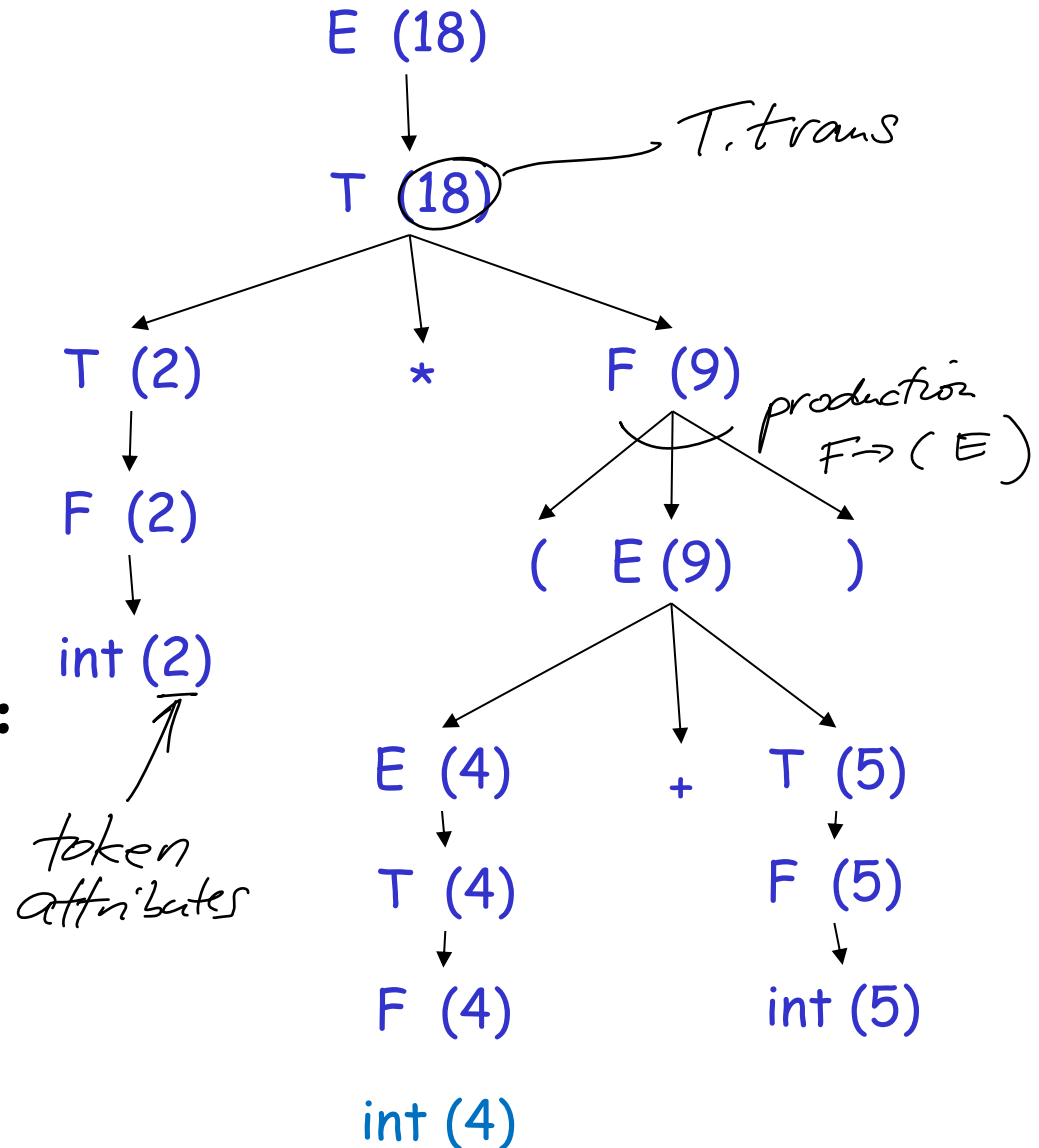
$E ::= T \mid T + E$

$T ::= F \mid F * T$

$F ::= n \mid (E)$

**Parse Tree**

**(annotated with values):**





# Parse tree vs. abstract syntax tree

---

Parse tree = *concrete* syntax tree

- contains all syntactic symbols from the input
- including those that the parser needs “only” to discover
  - intended nesting: parentheses, curly braces
  - statement termination: semicolons

Abstract syntax tree (AST)

- abstracts away these artifacts of parsing,
- abstraction compresses the parse tree
  - flattens parse tree hierarchies
  - drops tokens

# Add parse tree reconstruction to our parser

---

```
def parse() {  root = E(); scan(EOF);
                return root }

def E() {
    switch (oracle()) {
    case 1: scan("a")
            return ("a",)
    case 2: left = E()
            scan("+")
            right = E()
            return ("+", left, right)
    case 3: // analogous
    }}

```

*Python tuple*

# Recursive Descent Parser

(by implementing the oracle with Prolog)

# How to implement our oracle? (hidden slide)

---

Recall *amb*: the nondeterministic evaluator from cs61A

(amb 1 2 3 4 5) evaluates to 1 or .. or 5

Which option does amb choose? One leading to success.

in our case, success means parsing successfully

How was amb implemented?

backtracking

Our parser with amb:

```
def E() { switch (amb(1,2,3)) {  
    case 1: scan("a")  
    case 2: E(); scan("+"); E()  
    case 3: E(); scan("*"); E() }}
```

Note: amb may not work with any left-recursive grammar

# How do we implement the oracle

---

We could implement it with coroutines.

We'll use use **logic programming** instead.

After all, we already have oracle functionality in our Prolog

We will define a parser as a logic program

backtracking will give it exponential time complexity

# Backtracking parser in Prolog

---

Example grammar:

$E ::= a$

$E ::= a + E$

We want to parse a string  $a+a$ , using a query:

$?- \text{parse}([a,+,a]).$

true  $\Rightarrow a+a$  is in  $L(G)$

Backtracking Prolog parser for this grammar

$e([a|Out], Out).$

$e([a,+|R], Out) :- e(R,Out).$

$\text{parse}(S) :- e(S,[]).$

# How does this parser work? (1)

---

Let's start with simple Prolog queries:

?- [H | T] = [a,+,a].

H = a,

T = [+, a].

?- [<sup>a</sup>a, ~~u~~, b, +, c] = [a, + | Rest].

Rest = [b, +, c].

# How does this parser work? (2)

*for concise*

$a(I,0) :- I=[a|T],$   
 $0=T.$

Let's start with this (incomplete) grammar:

$e([a|T], T).$

$E ::= a$

Sample queries:

$e([a,+,a], Rest).$  } corresponds to  $scan('a')$   
 $--> Rest = [+ , a]$

$e([a], Rest).$   
 $--> Rest = []$

$e([a], []).$

$--> true$  // parsed successfully

*scan (EOF) at end parse*

*like doing*



# Parser for the full expression grammar

---

$E = T \mid T + E$        $T = F \mid F * T$        $F = a$

$e(\text{In}, \text{Out}) \text{ :- } t(\text{In}, \text{Out}).$

$e(\text{In}, \text{Out}) \text{ :- } t(\text{In}, [+|R]), e(\text{R}, \text{Out}).$

$t(\text{In}, \text{Out}) \text{ :- } f(\text{In}, \text{Out}).$

$t(\text{In}, \text{Out}) \text{ :- } f(\text{In}, [*|R]), t(\text{R}, \text{Out}).$

$f([a|\text{Out}], \text{Out}).$

$\text{parse}(S) \text{ :- } e(S, []).$

$?- \text{parse}([a, +, a, *, a], T). \text{ --> } true$

# Construct also the parse tree

---

$E = T \mid T + E$        $T = F \mid F * T$        $F = a$

$e(\text{In}, \text{Out}, e(T1))$                     :-  $t(\text{In}, \text{Out}, T1)$ .  
 $e(\text{In}, \text{Out}, e(T1, +, T2))$  :-  $t(\text{In}, [+|R], T1), e(R, \text{Out}, T2)$ .  
 $t(\text{In}, \text{Out}, e(T1))$                     :-  $f(\text{In}, \text{Out}, T1)$ .  
 $t(\text{In}, \text{Out}, e(T1, *, T2))$  :-  $f(\text{In}, [*|R], T1), t(R, \text{Out}, T2)$ .  
 $f([a|Out], \text{Out}, a)$ .

$\text{parse}(S, T) \text{ :- } e(S, [], T)$ .

$?- \text{parse}([a, +, a, *, a], T)$ .

$T = e(e(a), +, e(e(a), *, e(a)))$

*buggy  
see Lg  
for a correct  
version.*

# Construct also the AST

---

$E = T \mid T + E$        $T = F \mid F * T$        $F = a$

```
e(In,Out,T1)           :- t(In, Out, T1).  
e(In,Out,plus(T1,T2)) :- t(In, [+|R], T1), e(R,Out,T2).  
t(In,Out,T1)           :- f(In, Out, T1).  
t(In,Out,times(T1,T2)):- f(In, [*|R], T1), t(R,Out,T2).  
f([a|Out],Out, a).
```

```
parse(S,T) :- e(S,[],T).
```

```
?- parse([a,+,a,*,a],T).  
T = plus(a, times(a, a))
```

# Running time of the backtracking parser

---

We can analyze either version. They are the same.

amb:

```
def E() { switch (oracle(1,2,3)) {  
    case 1: scan("a")  
    case 2: E(); scan("+"); E()  
    case 3: E(); scan("*"); E() } }
```

Prolog:

$e(\text{In}, \text{Out}) :- \text{In} == [\text{a} | \text{Out}] .$

$e(\text{In}, \text{Out}) :- e(\text{In}, \text{T1}), \text{T1} == [+ | \text{T2}], e(\text{T2}, \text{Out})$

$e(\text{In}, \text{Out}) :- e(\text{In}, \text{T1}), \text{T1} == [* | \text{T2}], e(\text{T2}, \text{Out})$

# Recursive descent parser

---

This parser is known as recursive descent parser (rdp)

The parser for the calculator (Lec 2) is an *rdp*.

Study its code. *rdp* is *the* way to go when you need a small parser.

Crafting its grammar carefully removes exponential time complexity.

Because you can avoid backtracking by facilitating making choice between rules based on immediate next input. See the calculator parser.

# Summary

# Summary

---

## Languages vs grammars

a language can be described by many grammars

## Grammars

string generation vs. recognizing if string is in grammar

random generator and its dual, oracular recognizer

## Parse tree:

result of parsing is parse tree

## Recursive descent parser

runs in exponential time.