Using Simulated Execution in Verifying Distributed Algorithms

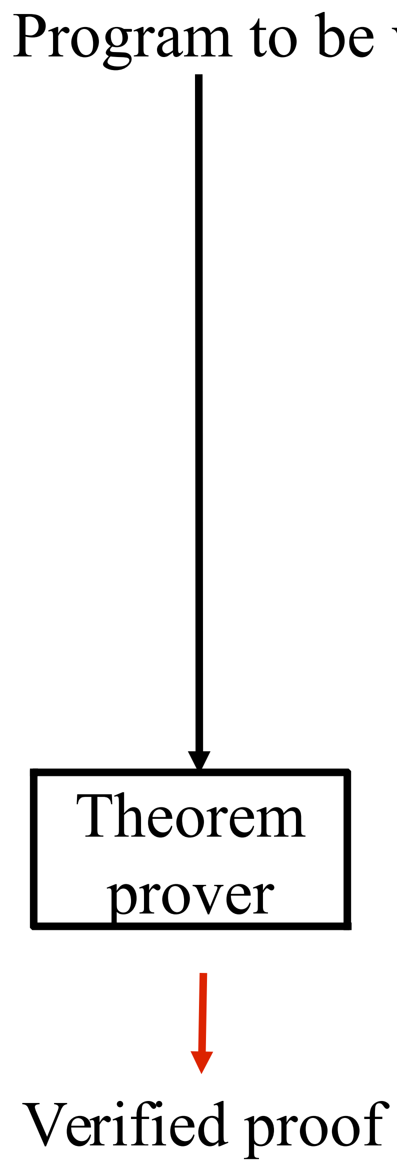
Toh Ne Win, Michael Ernst, Stephen Garland,
Dilsun Kirli Kaynar, Nancy Lynch

"How to help a theorem prover with execution data"

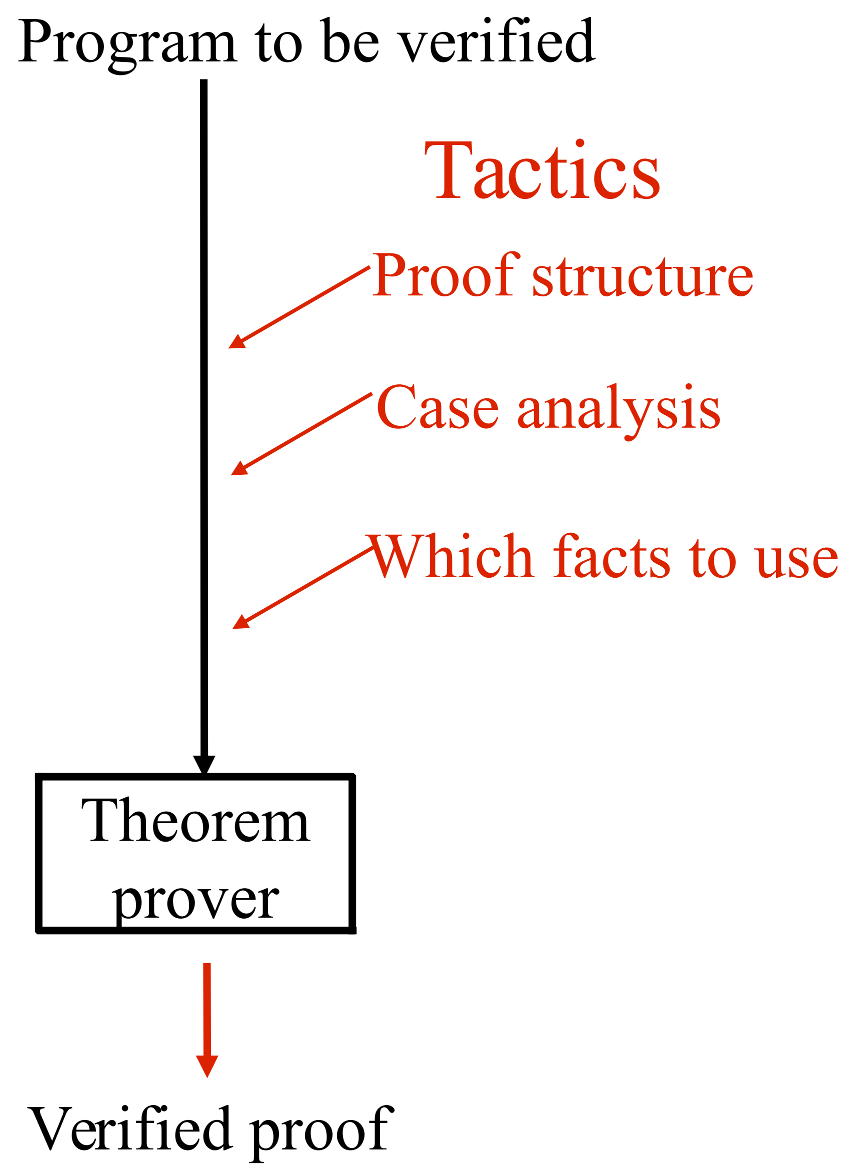
Goal: make theorem provers easier to use

- Why do we want to use a prover?
  - To verify general, infinite state systems
- What's hard about using a prover?
  - They get stuck and need human input

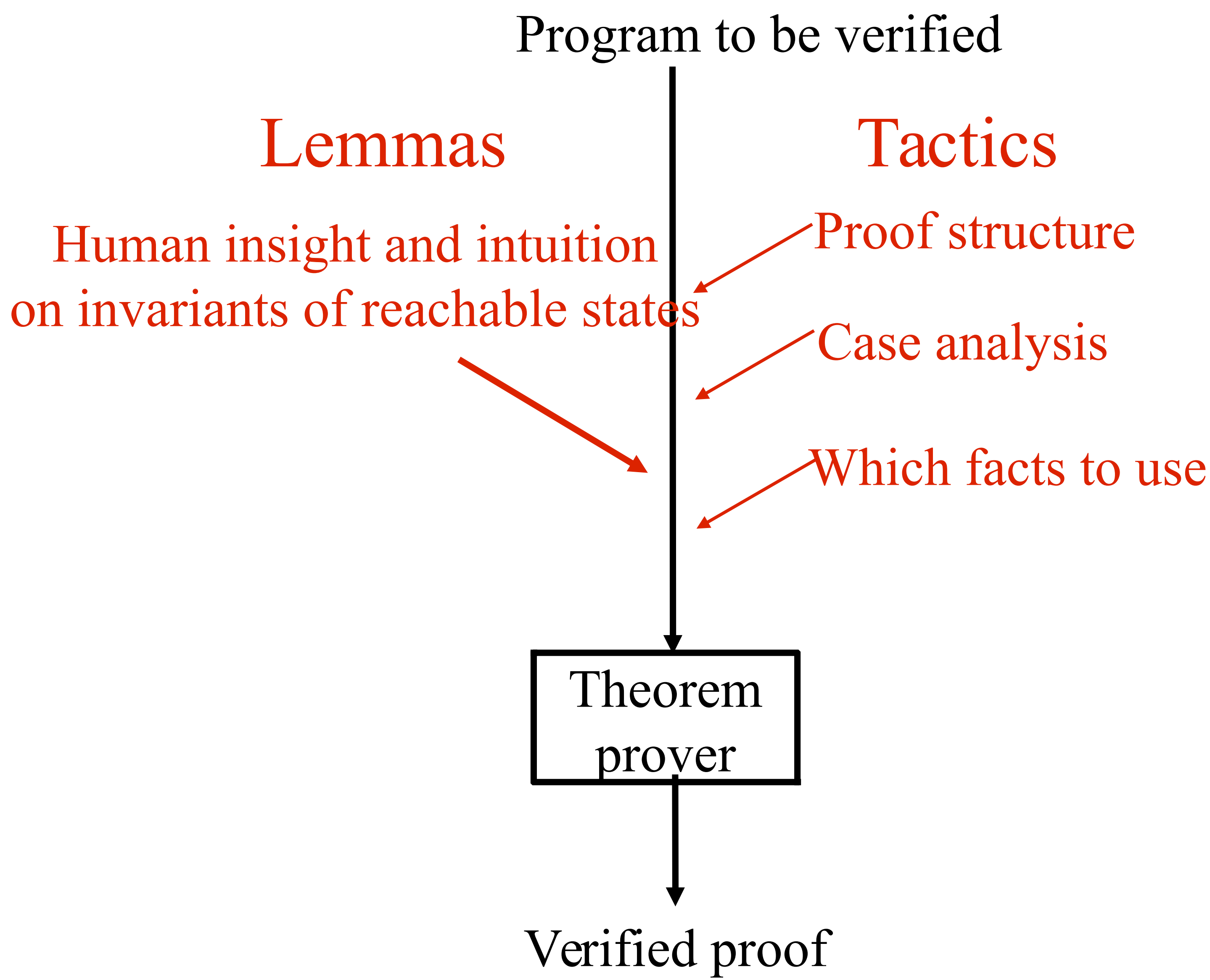
What kind of human input?



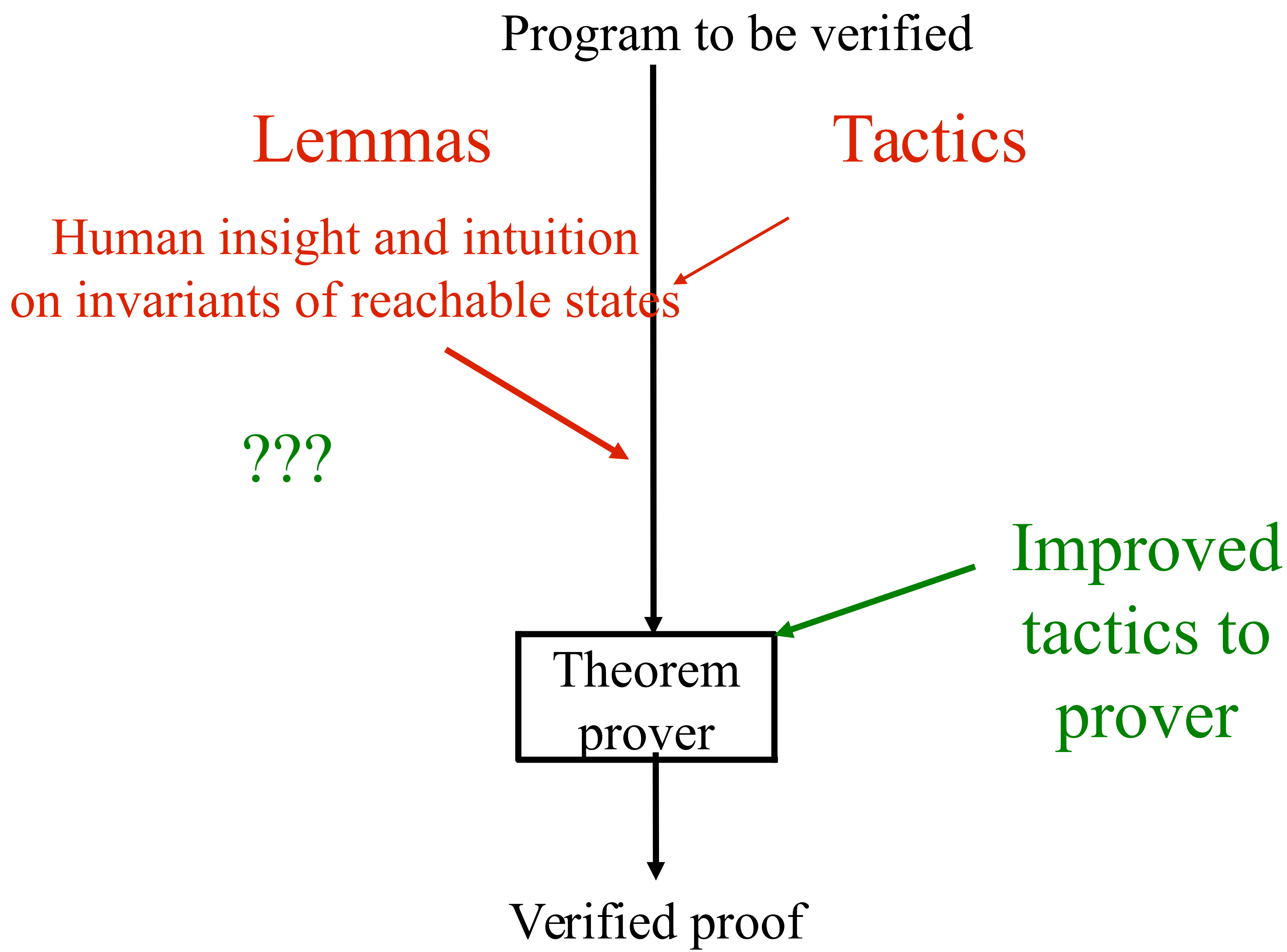
What kind of human input?



What kind of human input?



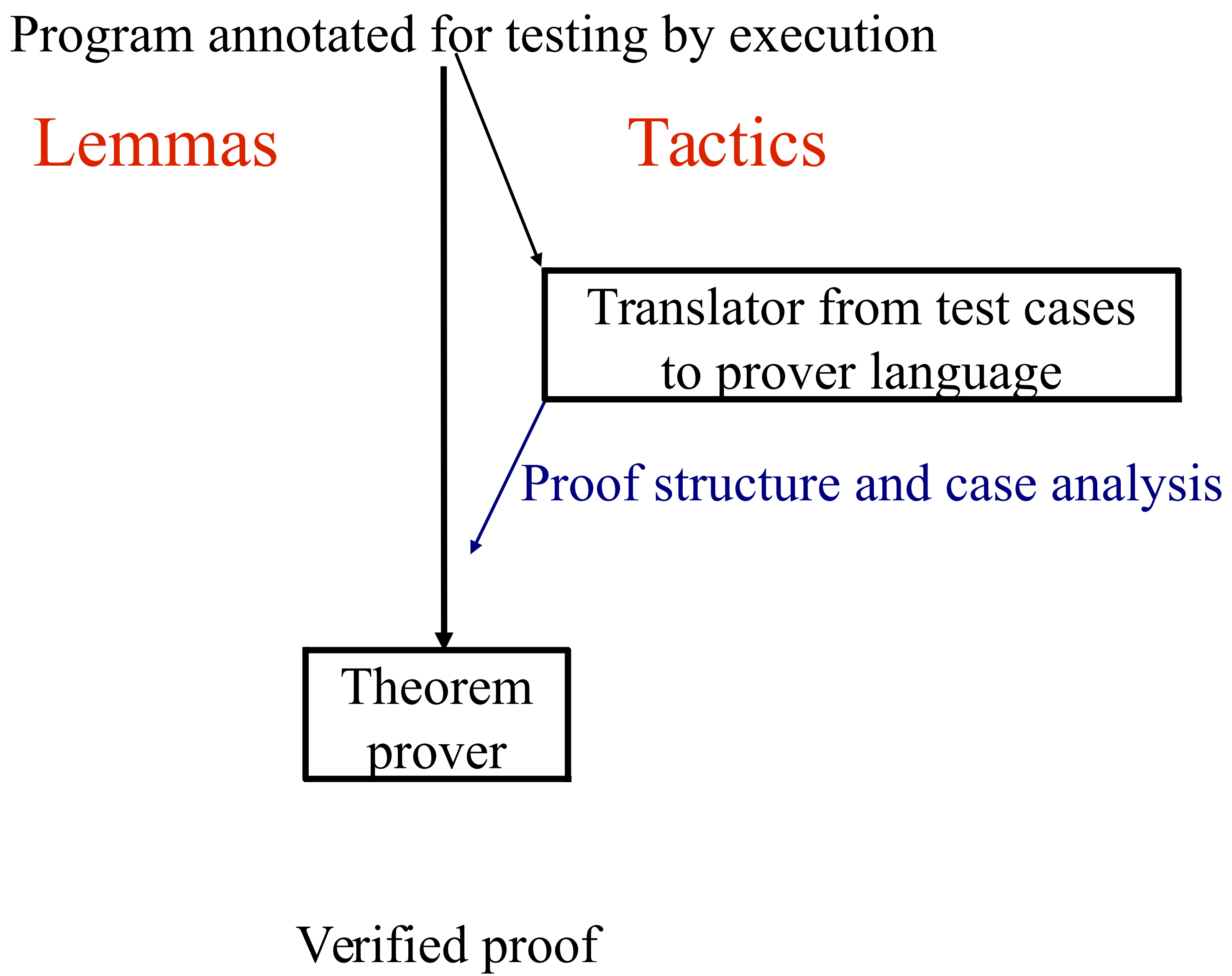
Traditional approaches



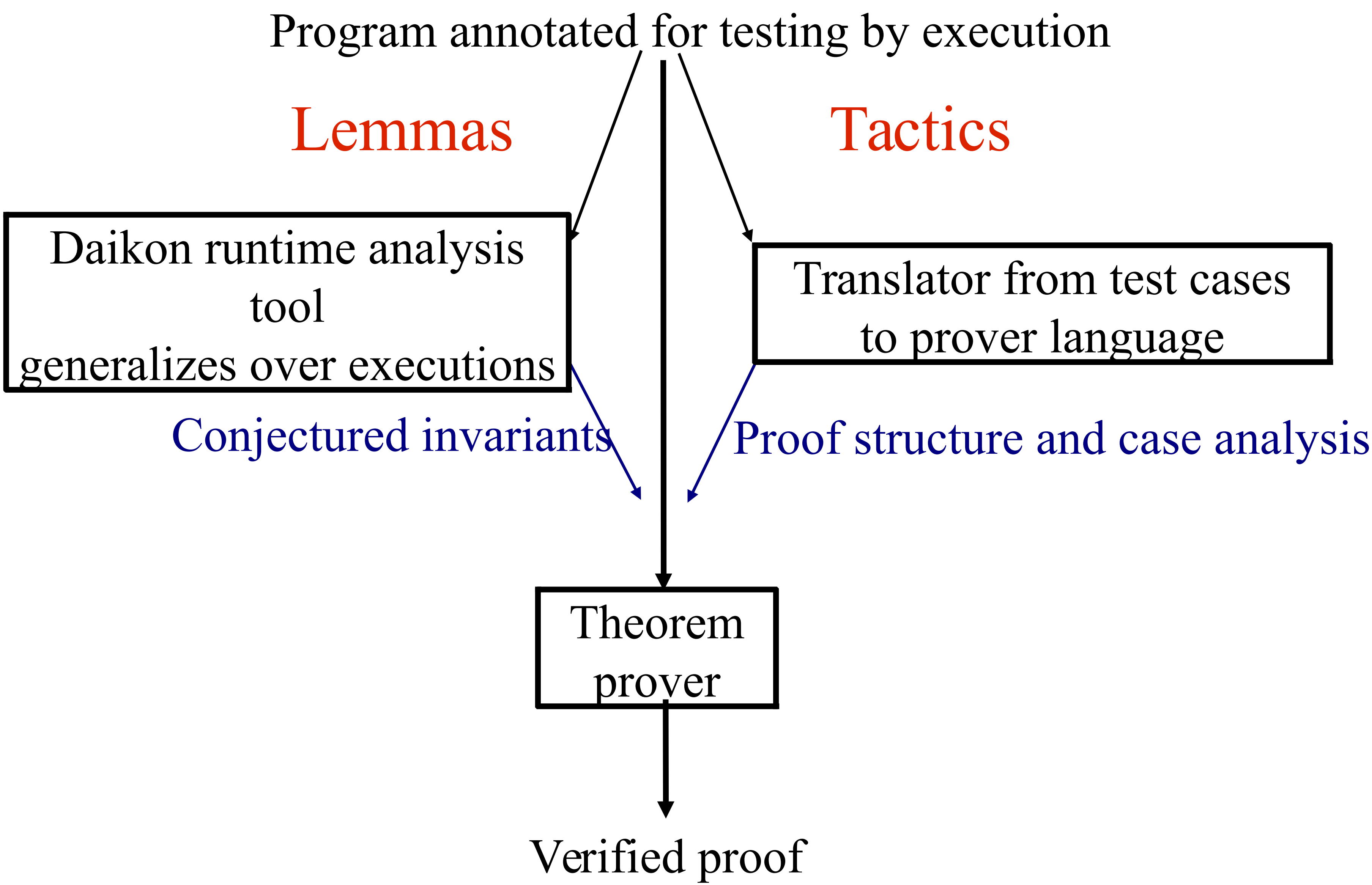
Using execution data to help provers

- Programs are often tested before verification
  - Testing shows errors quickly
  - Verification is expensive in human time
- Execution data is normally thrown away
  - What information can be kept for proofs?

Generating tactics



Generating lemmas



Outline

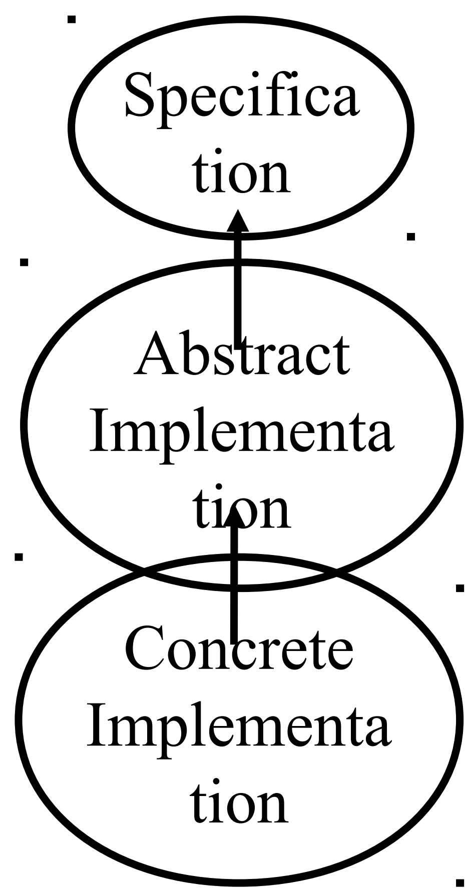
- Motivation: execution-assisted theorem provers
- Formal model: IO automaton
- Case study: Lamport's Paxos protocol
- Lemmas: conjectured invariants
- Tactics: proof outline
- Conclusion

Formal model: IO automaton

- Model for distributed systems [Lynch/Tuttle 89]
  - Labeled (infinite, nondeterministic) state machine
  - First order logic to define transitions

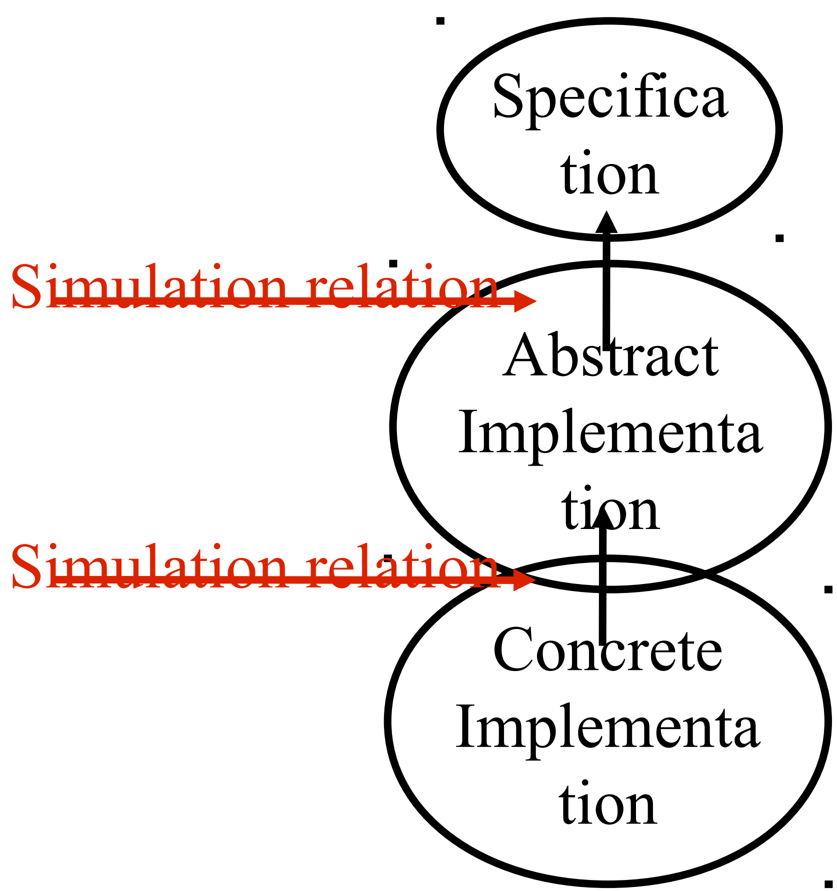
Formal model: IO automaton

- Model for distributed systems [Lynch/Tuttle 89]
  - Labeled (infinite, nondeterministic) state machine
  - First order logic to define transitions
- Multiple levels of abstraction
  - Abstract specification automaton
  - Layered implementation automata



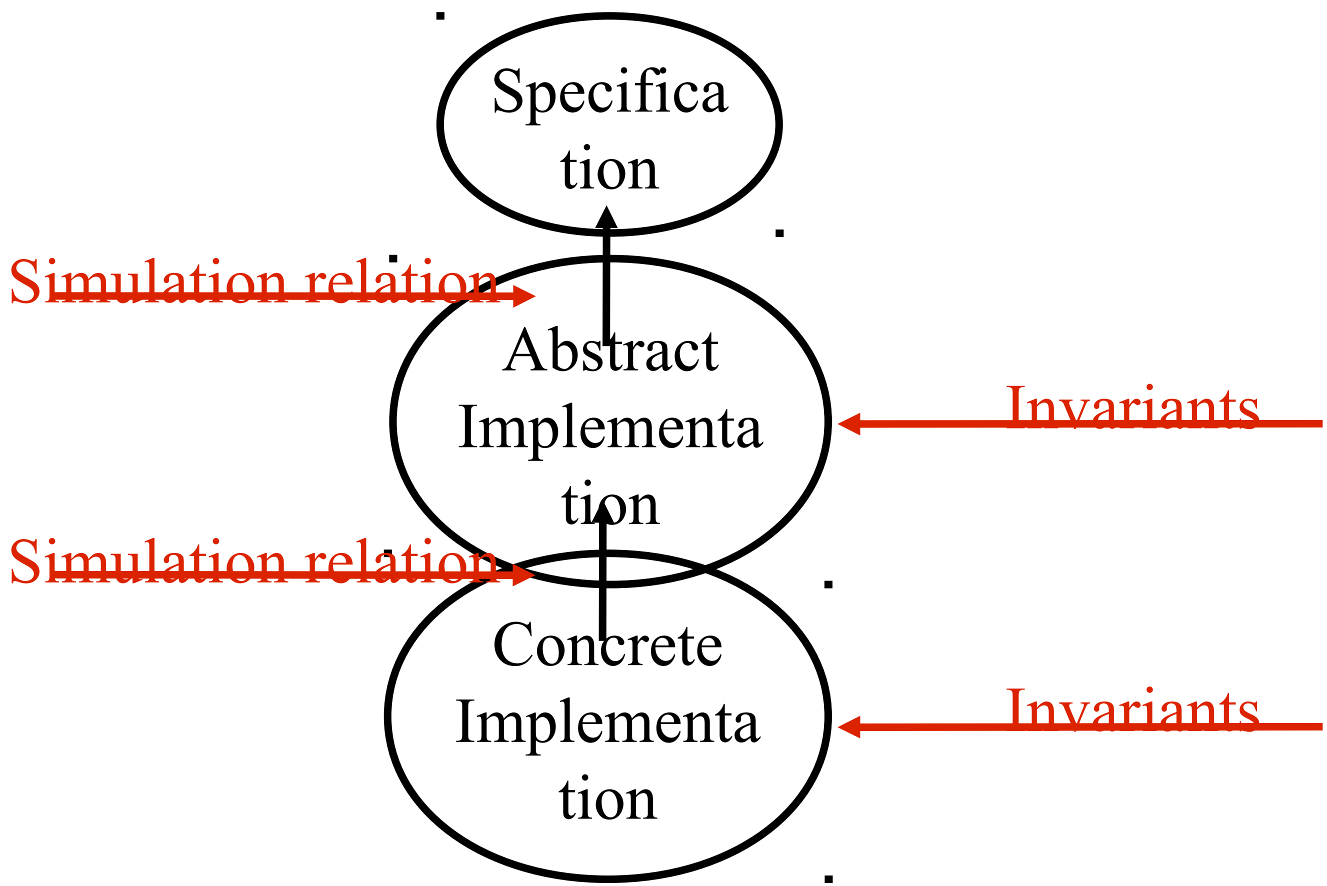
Verification methods

- Simulation relations for refinement



Verification methods

- Simulation relations for refinement
- Invariant assertions for implementations



IOA language and tools

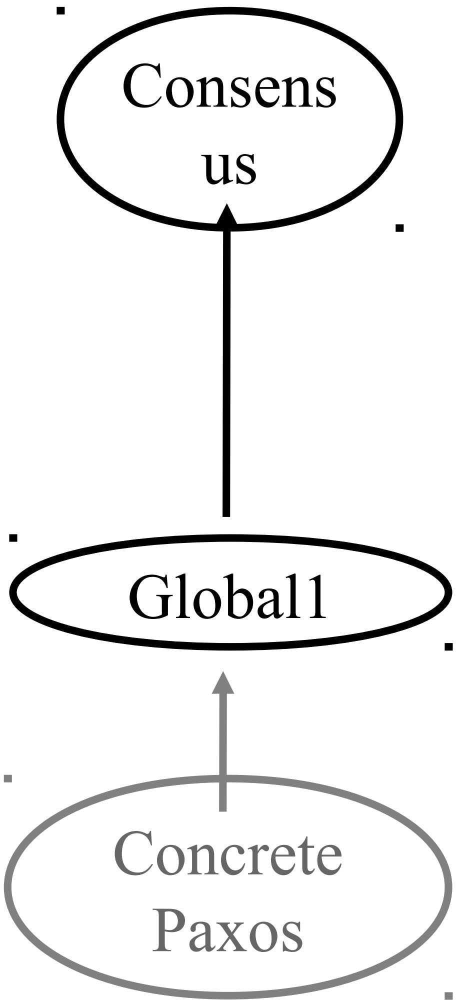
- IOA interpreter
  - Allows simulated execution of one automaton, or of a pair for refinement
  - User-specified scheduling to resolve nondeterminism
- IOA translators to proving languages
  - The Larch Prover
  - Isabelle/HOL

Outline

- Motivation: execution-assisted theorem provers
- IO automaton model
- Case study: Lamport's Paxos protocol
- Lemmas: conjectured invariants
- Tactics: proof outline
- Conclusion

Paxos in IOA

- Specification for consensus
- Globalized implementation using ballots and quorums



Specification for consensus

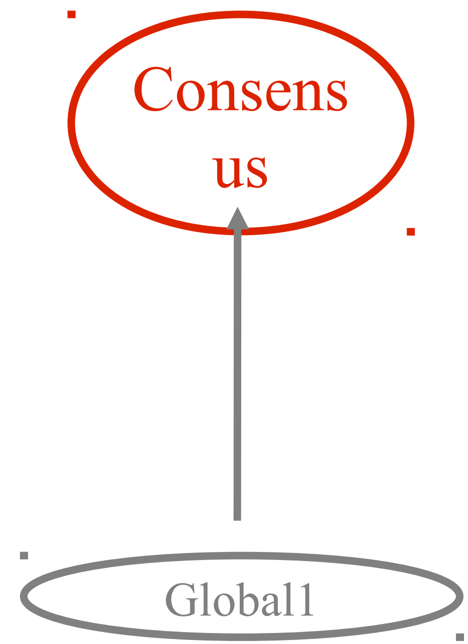
```
automaton Consensus
```

% Inputs and outputs are externally visible.

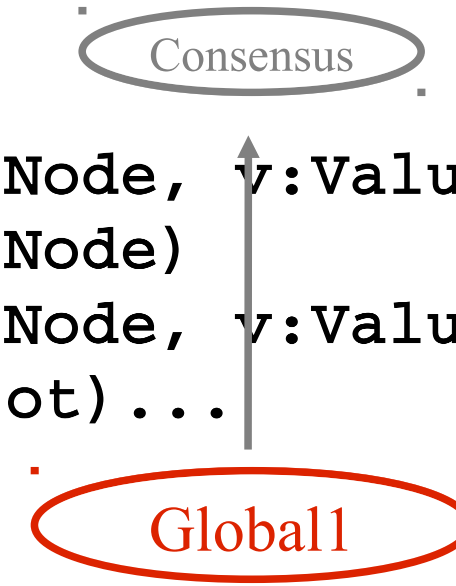
```
signature
  input init          (i:Node, v:Value)
  input fail          (i:Node)
  output decide       (i:Node, v:Value)
  internal chooseVal (v:Value)

states
  proposed, chosen : Set[Value] := {}
  ...

transitions
  internal chooseVal (v)
  pre
    v ∈ proposed;
    chosen = {}
  eff
    chosen := {v}
```



Implementation by Global1



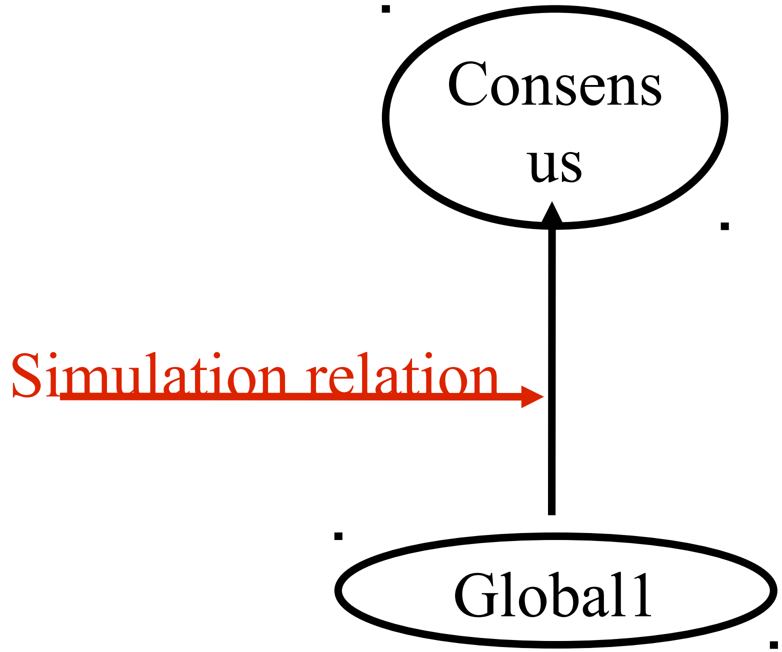

```
Automaton Global1
signature
  input init                          (i:Node, v:Value)
  input fail                          (i:Node)
  output decide                       (i:Node, v:Value)
  internal internalDecide      (b:Ballot)...

states
  succeeded, createdBallots : Set[Ballot]
  ...

internal internalDecide(b:Ballot)
pre
  % The ballot was created.
  b ∈ createdBallots;
  % There was a quorum that voted on the ballot.
  ∃ quorum : Set[Node] (quorum ∈ wquorums ∧
```

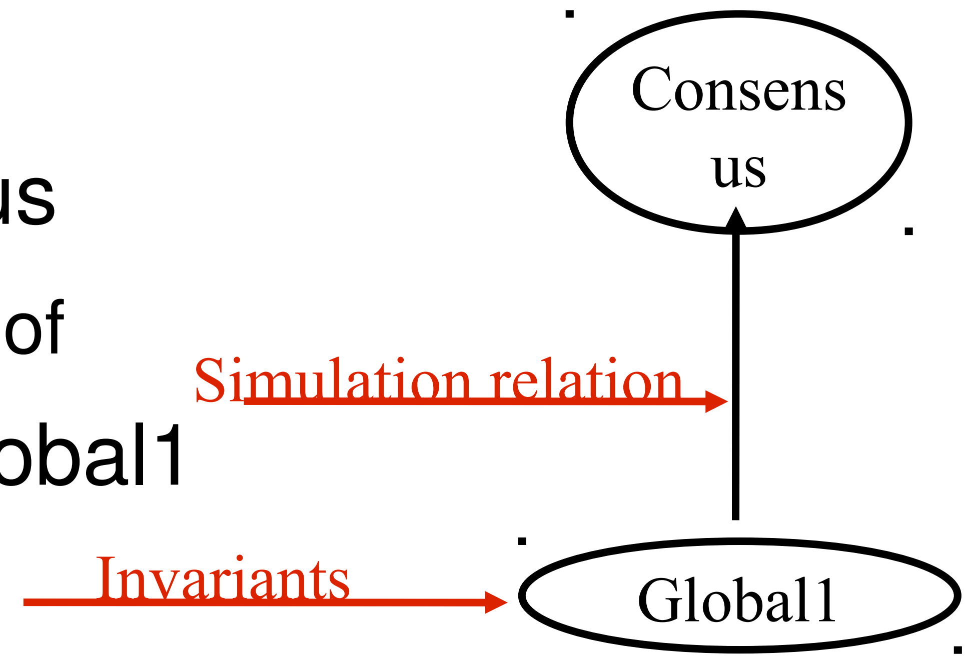
Gameplan for proof

- Show that Global1 implements Consensus
  - Simulation relation proof



Gameplan for proof

- Show that Global1 implements Consensus
  - Simulation relation proof
- Need invariants on Global1

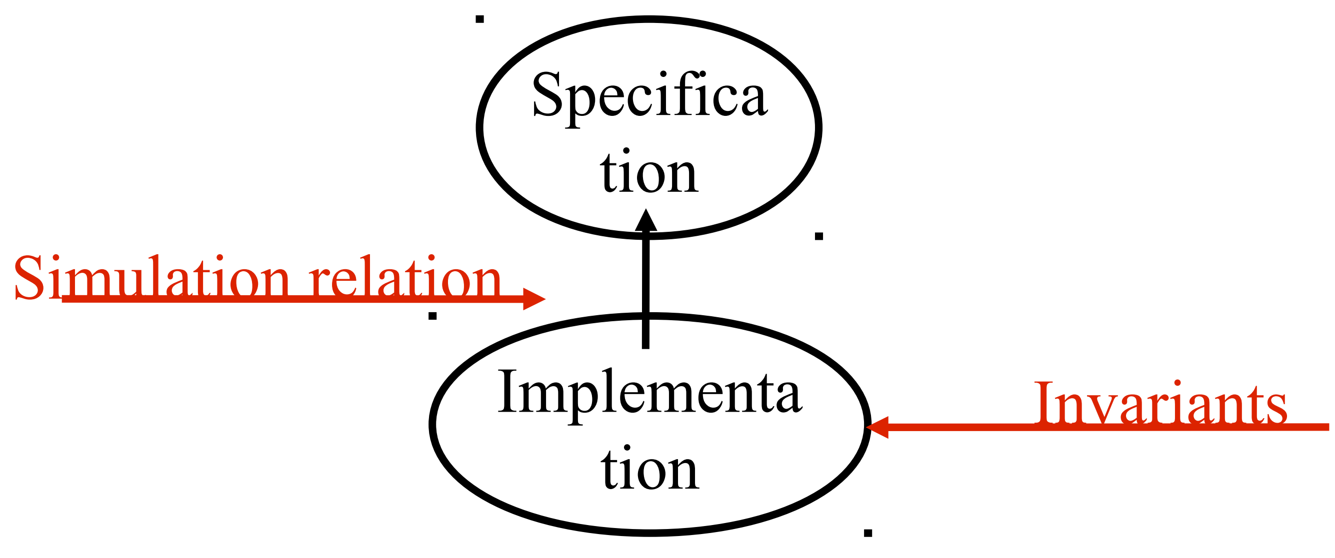


Outline

- Motivation: execution-assisted theorem provers
- IO automaton model
- Case study: Lamport's Paxos protocol
- <u>Lemmas: conjectured invariants</u>
- Tactics: proof outline
- Conclusion

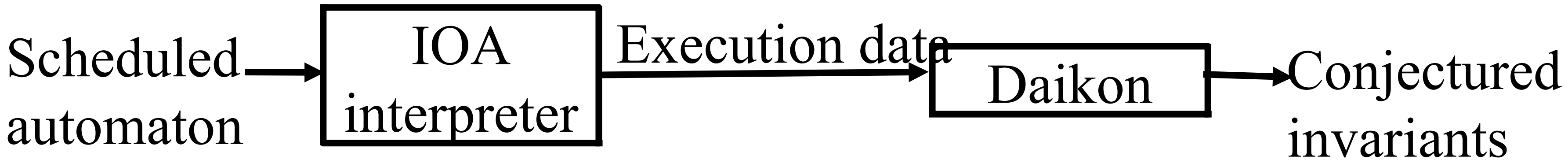
Uses of invariants

- Lemmas in proofs
  - Of simulations relations
  - Of other invariant statements
  - Often needed because the induction hypothesis for a proof must be strong enough



How to conjecture invariants

- Execute automaton using test cases
- Use Daikon tool on execution data
  - Analyzes execution data
  - Outputs properties true for observed executions
  - Invariants in first order logic



Issues with conjectured invariants

- Unsound
  - Statistical analysis reduces false positives
  - Use prover to prove conjectured invariants
- Incomplete
  - Necessary because search space is infinite
- Needs test cases
  - In practice, test cases exist
  - We use randomized scheduling
  - Trial-and-error execution usually enough

Conjectured invariants: example

- Paxos case study
  - Found 4 of 6 invariants needed for simulation relation proof

```
val(nonNull) ⊆ proposed

succeeded ⊆ createdBallots

0 = size(succeeded ∩ dead)

0 = size(voted[aNode] ∩ abstained[aNode])
```

What was not found

- Invariants with
  - Existential quantifiers
    - If a ballot has succeeded, a quorum voted for it

```
(b ∈ succeeded ⇒
     ∃ quorum : Set[Node] (quorum ∈ wquorums ∧
     ∀ n : Node (n ∈ quorum ⇒ b ∈ voted[n]))
```

  - Too many boolean clauses
    - If a ballot has non-nil value, it is the same value as all earlier non-dead ballots

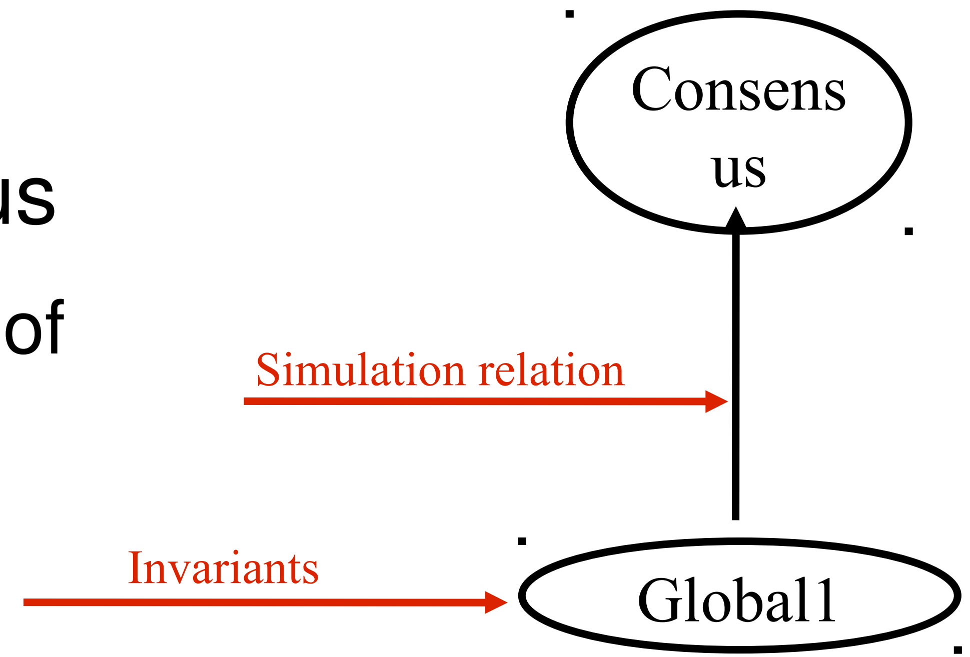
```
(val[b] ~= nil ∧ b' < b) ⇒
     (val[b'] = val[b] ∨ b' ∈ dead)
```

Outline

- Motivation: execution-assisted theorem provers
- IO automaton model
- Case study: Lamport's Paxos protocol
- Lemmas: conjectured invariants
- Tactics: proof outline
- Conclusion

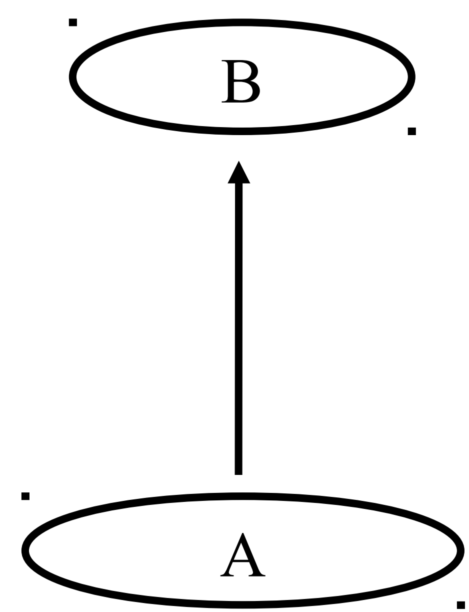
Gameplan for proof

- Show that Global1 implements Consensus
  - Simulation relation proof



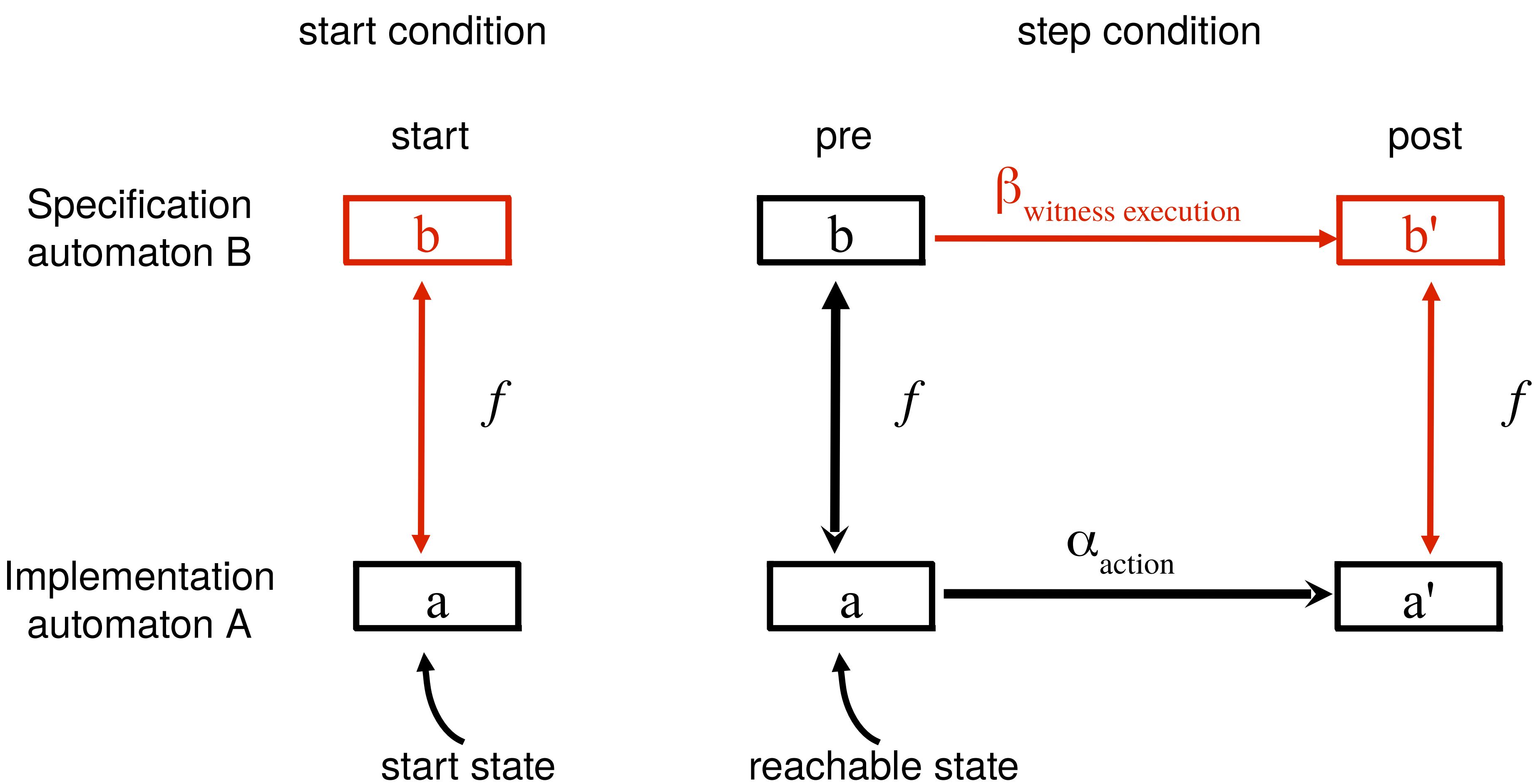
To prove a forward simulation relation

- A implements B if there exists *f* such that *f*:
  - Is a relation on states[A] and states[B]
  - Satisfies a start condition
  - Satisfies a step condition



To prove a forward simulation relation

- A implements B if there exists f such that:



Red = proof obligation

Forward sim: interpreter support

- Paired execution mode of IOA interpreter
  - For testing forward simulations
  - User annotates program for witness executions
- Mechanics of paired execution
  - Execute implementation automaton
  - Use annotations to drive execution of specification automaton
  - Check that f holds

Annotation example

```
for internal internalDecide (b) do
   if (b ∈ Global1.succeeded) then
     ignore
   elseif (Global1.val[b] = nil) then
     ignore
   ...
   else
     fire internal chooseVal
(Global1.val[b].val)
   fi
```

Generating prover tactics from tests

- Translate testing annotations into proof scripts
  - For start condition
    - Pick witness start state b
  - For step condition
    - Tactic: structural induction on action data type
    - Use conditionals ('if') in annotations to perform case splits
    - Pick witness execution β

Forward sim: step example

% Annotation

```
for internal internalDecide (b) do
   if (b ∈ Global1.succeeded) then
     ignore
   elseif (Global1.val[b] = nil) then
     ignore
   ...
   else
     fire internal chooseVal
(Global1.val[b].val)
   fi
```

% Proof

```
prove enabled(internalDecide(b)) =>∃ β ...%
Step condition
resume by cases (b ∈ Global1.succeeded)
```

% case true

```
  resume by specializing β to []
```

Outline

- Motivation: execution-assisted theorem provers
- IO automaton model
- Case study: Lamport's Paxos protocol
- Lemmas: conjectured invariants
- Tactics: proof outline
- <u>Conclusion</u>

Discussion

- Better theorem proving experience
  - Less human effort
  - Lets designers concentrate on high-level proof
- Designers have concept of high-level proof
  - Theorem provers get stuck in details
- Tactics: provide proof structure (82/150 lines)
  - What remains is rephrasing of facts
- Lemmas: provide invariants (4/6)
  - Missing ones syntactically evident in program code

Research directions

- Better conjectured invariants
  - Analyze IOA code statically for invariant templates
    - Find predicates in code, use as left side of implications
- Better proof tactics, more automation
  - Which lemmas are used in all IOA proofs?
  - What ordering of lemmas?
    - E.g., “apply definition of automaton effects only after inducting on the action type”

Conclusion

- Theorem provers need lemmas and tactics
  - Execution data can provide some of both
- Lemmas
  - Generalize over execution data
    - Conjectured invariants
- Tactics
  - Annotations for paired testing provides
    - Proof outline
    - Existential witnesses
- Contribution: easier to use theorem prover