Medical image segmentation via min s-t cuts with side constraints

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Abstract

Graph cut algorithms (i.e., min s-t cuts) [3][10][15] are useful in many computer vision applications. In this paper we develop a formulation that allows the addition of side constraints to the min s-t cuts algorithm in order to improve its performance. We apply this formulation to foreground/background segmentation and provide empirical evidence to support its usefulness. From our experiments on medical image segmentation, the graph cut with constraints achieve significantly better performance than that without any constraint. Although the constrained min s-t cut problem is generally NP-hard, our approximation algorithm that uses linear programming relaxation and a simple rounding technique as a heuristic produces good results in a few seconds with our unoptimized code.

1 Introduction

Graph cut algorithms (i.e., min s-t cut) [3][10][15] achieve good performance in many different computer vision applications, and different kinds of prior knowledge and constraints have been incorporated to further improve their performance. For example, different shape or partial labeling constraints [4][7][16][2][12] have been added to the original graph cuts segmentation algorithm and have improved its performance.

In this paper, we formulate the constrained problem as min s-t cuts with side constraints. Because in general, the constrained min s-t cut problem is NP-hard, we use linear programming relaxation and a simple rounding technique to find approximation solutions. We model different semantically meaningful segmentation constraints as side constraints and compare their performance in contour tracking of organs in CT images. From our experiments, the graph cut with side constraints can achieve significantly better performance than that without any constraints. In addition, the approximation algorithm produces good approximation solutions in our experiments.

2 Related Work

Many early computer vision problems, such as foreground/background segmentation and stereo, can be formulated as energy minimization problems [10]. Previous works [10][6] showed that certain energy functions can be exactly minimized by finding min s-t cuts on capacitated graphs and provided details on how to construct a capacitated graph with respect to a given energy function. Furthermore, the min s-t cut problem can be formulated as an 0-1 linear integer programming problem as described in [13], which is a building block of our proposed problem formulation.

Different constraints such as shape constraints [7][16][2][12] or partial labeling constraints [4] have been incorporated into graph cut algorithms. There are two classes of constraints: hard constraints that must be satisfied by any solution and soft constraints that may or may not be satisfied. For example, Zeng et al. [16] considered hard constraints and proved that topology cut, a min cut algorithm for topology preserving segmentation, is an NP-hard problem, and therefore they provides an approximation algorithm for it. Boykov and Jolly [4] considered other hard constraints such as the ways that some pixels must be foreground pixels or background pixels. In contrast with hard constraints, soft constraints typically change the capacities of the edges in graphs based on shape priors [7][2][12], which can then be exactly solved with min s-t cut algorithms. The shape priors can be a single template [7][12] or a shape model learned from training examples [2]. In this paper, we focus on adding hard constraints to graph cut algorithms for medical image segmentation.

A good source of different constrained flow problems can be found in [1]. For example, network flow including added side constraints that further restrict arc flows to minimum cost flow problems arise in many applications, such as constrained shortest path and multicommodity flow problems. In this paper, we apply a similar idea to min s-t cut problems. We formulate foreground/background segmentation as min s-t cuts with side constraints and provide empirical evidence to support its practical use in segmentation.

3 Constrained Min s-t Cut Problem with Side Constraints

3.1 Problem Formulation

In many computer vision applications, we are interested in adding hard constraints to help to find better solutions. For example, we may know the upper bound of the size of a foreground object in a foreground/background segmentation problem.

Formally, we consider a constrained min s-t problem defined as

$$\min\ cx\ s.t.\ \begin{bmatrix} A \\ W \end{bmatrix} x \leq \begin{bmatrix} b \\ d \end{bmatrix},\ x \in B^n \qquad (1)$$

where the elements of x are decision variables and A, b, c are from a given min s-t cut problem. Intuitively speaking, the elements of x are binary labels on nodes (i.e, 0 for background and 1 for foreground) and edges in the capacitated graph, A and b reflect the relationships among labels in the graph, and c gives the capacities of edges. W and d are additional hard linear constraints that reflect prior knowledge, and we will refer them as side constraints. For example, we can represent the upper bound constraint as $\sum_{i \in V - \{s,t\}} x_i \leq u$ where u is some positive constant, V is the vertex set of the given capacitated graph, s and t are the source and sink nodes, respectively. The reason why $\sum_{i \in V - \{s,t\}} x_i \leq u$ represents this constraint is that x_i for $i \in V$ is 0 or 1 and $\sum_{i \in V - \{s,t\}} x_i$ is the size of the foreground object. Similarly, we can represent the lower bound constraint as $-\sum_{i \in V - \{s,t\}} x_i \leq -l$ where l is some positive constant. Note that we only consider linear hard constraints in (1); nonlinear hard constraints can be translated into linear constraints by introducing new variables called slack variables and additional linear constraints.

We encode size constraints as side constraints. We try two different methods for formulating size constraints: 1) global bounds: we provide global upper and lower bounds on the contours of the object to be segmented derived from training data on which the true contours are drawn 2) local parts: using the slice above a given slice, we provide upper and lower bounds for each row of the object to be segmented.

Because (1) is 0-1 integer programming that is generally NP-hard [11], we use linear programming relaxation and a simple rounding technique (i.e., if a fractional number is close to 1, we round it to 1 otherwise we round it to 0) as a heuristic to get approximation solutions, which is a standard approximation approach to solve NP hard problems [13].

3.2 The Image Segmentation Problem

The problem we want to solve is the segmentation of CT images for the purpose of obtaining 3D reconstruction of the separate organs. 3D reconstruction is needed for radiation treatment planning, in which the goal is to maximize the radiation toward the cancerous tumor and minimize its effect on other anatomical entities, and is also useful for medical education.

We are given a 3D CT scan of a particular area of a patient's body, such as the abdomen. We wish to find the contours that bound each of a set of required organs such as a kidney or spleen. We evaluate the use of both global bounds and local parts techniques to improve the segmentation results.

4 Experiments

In our experiments, we used the graph cut based active contour algorithm in [14] for foreground/background segmentation and modified their program so that (1) can be solved. The metrics we use to evaluate our segmentation results were inspired by common precision and recall measures in information retrieval and are similar to the segmentation accuracy measure used in the PASCAL visual object class challenge 2007 [5]. The measures we used to evaluate the foreground/background segmentation are $\frac{I}{S}$ and $\frac{I}{T}$ where S is the area of the estimated foreground segment, T is the area of the ground truth foreground segment and I is the area of the intersection of S and T. In this paper, we will call $\frac{I}{S}$ the precision and $\frac{I}{T}$ the recall of each segmentation result, since $\frac{I}{S}$ tells us how much of the segmented region actually belongs to the organ and $\frac{I}{T}$ tells us how much of the actual organ the segmented region covers. Intuitively, the higher these two measures are, the better the segmentation performance is. For the medical images we use in the experiments, we have ground truth segmentations given by medical doctors. The goals of our experiments are to show how graph cuts with additional constraints performs compared with graph cuts without any constraints, how different size constraints perform and how constraints and parameters (i.e., upper or lower bounds) can be estimated from training images.

Table 1. Quantitative comparisons

	w.o	w.o	w	w
	precision	recall	precision	recall
Kidney (local parts)	0.9011(0.076)	0.3130(0.0003)	0.9951(0.000)	0.9110(0.003)
Kidney (global bounds)	0.9011(0.076)	0.3130(0.0003)	0.9816(0.0002)	0.9399(0.0005)
Spleen (local parts)	0.9147(0.0023)	0.8248(0.0621)	0.8348(0.0015)	0.9266(0.0038)
Spleen (global bounds)	0.9147(0.0023)	0.8248(0.0621)	0.8783(0.0117)	0.9014(0.0024)

For the global bounds technique, the upper and lower bounds on size are estimated from multiple training images and are entered manually for each organ. The segmentations performed on each slice use the same upper and lower bounds. For the local parts technique, a tracking approach in which the segmentation from the previous image is used as training data for the current image is employed. In this approach, each row of the designated organ in the training image provides separate upper and lower bound size constraints, which taken together can be thought of as a shape constraint.

Quantitative comparisons of both techniques can be found in Table 1, which gives the means and variances of precision and recall with and without constraints. The first two rows of Table 1 give the results for tracking a kidney over 6 image slices, while the second two rows give the results for tracking a spleen over 10 image slices. For the kidney experiments, size constraints (both global bounds and local parts) significantly improved both precision and recall. For the spleen experiments, recall was significantly improved, but precision went down at the same time. Performance was better for global bounds constraints than local parts constraints in all cases. Our explanation for this observation is that although local part constraints are more expressive than global bounds constraints, they may be too restrictive. In particular, putting a constraint on each row of an organ does not allow its shape to change from slice to slice.

Figure 1 illustrates the use of constraints to improve segmentation results. The figure shows how the graph cut algorithm with constraints tracks the contour of a left kidney more accurately than that without constraints. Because there are no strong edges around the ground truth shapes of the kidneys in the testing images, the graph cut algorithm with no constraints failed. In contrast, the graph cut algorithm with local part constraints accurately tracks the contour.

From these results, we can see that size constraints can be used to significantly improve the performance of foreground/background segmentation and that shape constraints are useful when the shape of the organ being tracked is relatively constant from slice to slice. In addition, our approximation algorithm produced good approximation results in these experiments.

5 Conclusions and Future Work

In this paper, we have formulated the encoding of prior knowledge into graph cut algorithms [3][10][15] with hard constraints as min s-t cut problems with side constraints. We developed two different constraints in practice and applied these constraints to foreground/background segmentation and contour tracking. From our experiments, the graph cut algorithm with constraints achieves significantly better performance than that without constraints. Although the constrained min s-t cut problem is generally NP-hard, our approximation algorithm produced good approximation results in these experiments and took only a few seconds to execute.

In this paper, we manually designed the constraints. Finding different linear constraints that model shape constraints in practice merits further research. In particular, we want to learn interesting semantic constraints that can be represented as linear constraints and learned from training images. How to solve the constrained s-t cut problem efficiently and effectively is another important future endeavor. Dynamic flow problems are very common in computer vision, especially for video segmentation and tracking. Different methods such as flow recycling [9] and cuts recycling [8] were proposed for speeding up the computational process. We are interested in applying these concepts to our new formulations. We also want to investigate different approximation algorithms and find the best one for the constrained min s-t cut problem. Finally, we plan to evaluate our proposed formulation and approximation algorithm on contour tracking of different organs in a larger set of CT images.

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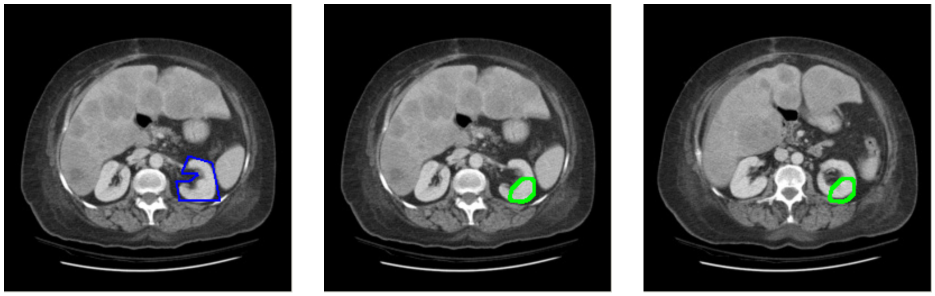
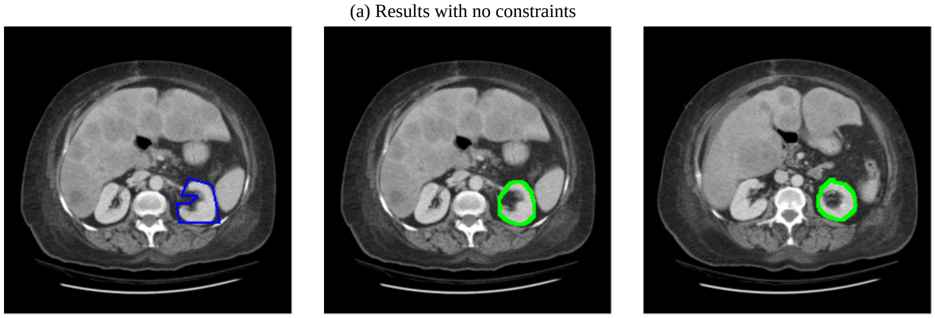


(a) Results with no constraints



(b) Results with size constraints

Figure 1. A comparison between graph cut without any constraints and that with multiple part size constraints on a contour tracking example. (a) Results without constraints on testing images. The first image shows the initial contour in the first testing image. (b) Results with size constraints on testing images. The first image shows the same initial contour as that of the first image in the first row.

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