How Many Elements?
For each of these, how many elements are in the set? If the set has infinitely many elements, say so.
(a) \( A = \{1, 2, 3, 2\} \)
(b) \( B = \{\{\}, \{\}, \{\}, \{\}, \{\}, \{\}, \{\}, \{\}, \{\}, \{\}, \ldots\} \)
(c) \( C = A \times (B \cup \{7\}) \)
(d) \( D = \emptyset \)
(e) \( E = \{\emptyset\} \)
(f) \( F = \mathcal{P}(\{\emptyset\}) \)

Set = Set
Prove the following set identities.
(a) Let the universal set be \( U \). Prove \( \overline{X} = X \) for any set \( X \).
(b) Prove \( (A \oplus B) \oplus B = A \) for any sets \( A, B \).
(c) Prove \( A \cup B \subseteq A \cup B \cup C \) for any sets \( A, B, C \).
(d) Let the universal set be \( U \). Prove \( A \cap \overline{B} \subseteq A \setminus B \) for any sets \( A, B \).

Casting Out Nines
Let \( n \in \mathbb{N} \). Prove that if \( n \equiv 0 \pmod{9} \), then the sum of the digits of \( n \) is a multiple of 9.

Modular Arithmetic
(a) Prove that if \( a \mid b \) and \( b \mid a \), where \( a \) and \( b \) are integers, then \( a = b \) or \( a = -b \).
(b) Prove that if \( n \mid m \), where \( n \) and \( m \) are integers greater than 1, and if \( a \equiv b \pmod{m} \), where \( a \) and \( b \) are integers, then \( a \equiv b \pmod{n} \).
0. Sets All Folks!

Prove \( A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \). Note that while a Venn Diagram is useful, it is not a proof.
GCD

(a) Calculate \( \gcd(100, 50) \).

(b) Calculate \( \gcd(17, 31) \).

(c) Find the multiplicative inverse of 6 modulo 7.

(d) Does 49 have an multiplicative inverse modulo 7?

(e) Find the multiplicative inverse of 7 modulo 311.

(f) Find the multiplicative inverse of 27 modulo 151.

More Number Theory

(a) Prove that if \( n^2 + 1 \) is a perfect square, where \( n \) is an integer, then \( n \) is even.

(b) Prove that if \( n \) is a positive integer such that the sum of the divisors of \( n \) is \( n + 1 \), then \( n \) is prime.

0. Extended Euclidian Algorithm

Find the multiplicative inverse \( y \) of 7 mod 33. That is, find \( y \) such that \( 7y \equiv 1 \pmod{33} \). You should use the extended Euclidean Algorithm. Your answer should be in the range \( 0 \leq y < 29 \).
0. Extended Euclidian Algorithm

Find the multiplicative inverse $y$ of $7 \mod 33$. That is, find $y$ such that $7y \equiv 1 \pmod{33}$. You should use the extended Euclidean Algorithm. Your answer should be in the range $0 \leq y < 29$. 
0. Fool Me Once...
Consider the following sentence:

I only left the light on in my office if I rushed home, but I either rushed home or left early.

(a) Is the sentence a proposition? Explain why or why not.

(b) Define smaller propositions in preparation for converting the sentence to logical notation.
\[ p = ?, q = ?, \ldots \]

(c) Convert the sentence to logical notation.