

# Hierarchical Feedback and Learning for Multi-joint Arm Movement Control

Weiwei Li, Emanuel Todorov and Xiuchuan Pan

**Abstract**—This paper presents a general method for hierarchical feedback control of redundant systems, and applies it to the problem of arm movement control. A high-level feedback controller, designed using optimal control techniques, operates on a simplified virtual plant. A low-level controller is responsible for performing a feedback transformation of the physical plant into the desired virtual plant. The method is applied in the context of reaching with two realistic models of the human arm: a 2-DOF, 6-muscle model, and a 7-DOF, 14-muscle model. Simulation results demonstrate the effectiveness of the proposed scheme.

## I. INTRODUCTION

A distinguishing feature of biomechanical plants is their massive redundancy, which makes the motor system very flexible, but at the same time requires a well designed controller that can choose intelligently among the many possible alternatives. Optimal control theory provides a principled approach to this problem — it postulates that the movement patterns being chosen are the ones optimal for the task at hand. Here we develop a general method for hierarchical feedback control of redundant biomechanical movement systems, inspired by the hierarchical organization of the sensorimotor system.

Sensorimotor control occurs simultaneously on multiple levels [1], [9], and involves a large number of parallel feedback loops. In such a system, it is important to understand how the low-level loops cooperate with high-level loops in order to produce integrated movement. In robotics, Brooks [2] constructed low-level control circuits driving a mobile robot; these circuits were then coordinated by a high-level controller that achieved the reasonable locomotion. The Operational Space Framework [7] and Virtual Model Control [11] are also closely related to hierarchical control.

The structure of biological movements examined in this paper involves two-level feedback control hierarchies (Fig. 1): a low-level controller is responsible for transforming high-level commands into appropriate muscle activations, and also for transforming the dynamics of the real plant into the virtual dynamics assumed by the high level. The high-level controller performs goal-directed movement control,

but is partly isolated from the complexity of the physical plant. As shown in Fig. 1, the low-level controller receives information about the plant state  $x$ , as well as a high-level control signal  $u_y$ , and generates appropriate muscle activations  $u_x$ . It also produces a more abstract state representation  $y$  that is sent to the high-level controller. The high-level controller receives  $y$  and generates abstract control command  $u_y$  to coordinate the low-level unit.

Here we focus on the study of hierarchical optimal control for human arm movements. The optimal feedback controller on the high-level will be designed using an iterative Linear-Quadratic-Gaussian (iLQG) method derived in our recent paper [12]. The simplified virtual model assumed on the high-level may not be a perfect model of the augmented plant, therefore, a learning process is also included which can continuously calibrate the expectations on the high level. The low-level has to activate muscles so as to accomplish desired effects in the task-relevant space. Since there are more muscles than task variables, the low-level faces a redundancy problem which will be solved using static optimization. The control of biological movement systems is a very complex problem, because such systems have nonlinear dynamics and high-dimensionality, preventing the use of many traditional methods for controller design. The goal of the present paper is to combine the optimality principle and hierarchical strategy in order to develop a new framework for the sensorimotor system, which hopefully could better reflect the real control problem that the brain faces.

The paper is organized as follows. In section 2 the general hierarchical feedback control scheme for the nonlinear system is formulated. Section 3 presents the numerical results for two realistic human arm models, in the context of reaching task. Discussion and concluding remarks are drawn in section 4.

This work is supported by NIH Grant R01-NS045915.

Weiwei Li is with Department of Mechanical and Aerospace Engineering, University of California San Diego, La Jolla, CA 92093-0411 USA [wli@mechanics.ucsd.edu](mailto:wli@mechanics.ucsd.edu)

Emanuel Todorov is with Faculty of Department of Cognitive Science, University of California San Diego, La Jolla, CA 92093-0515 USA [todorov@cogsci.ucsd.edu](mailto:todorov@cogsci.ucsd.edu)

Xiuchuan Pan is with Department of Cognitive Science, University of California San Diego, La Jolla, CA 92093-0515 USA [xpan@cogsci.ucsd.edu](mailto:xpan@cogsci.ucsd.edu)

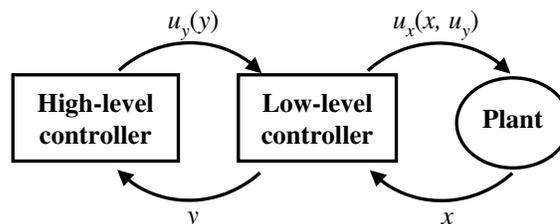


Fig. 1. Illustration of hierarchical control structure

## II. PROBLEM FORMULATION

Consider a continuous nonlinear dynamical system with state  $x(t) \in R^n$  and control signal  $u(t) \in R^m$ , whose dynamics is

$$\dot{x} = f(x, u), \quad x(0) = x_0. \quad (1)$$

Let  $l(x(t), u(t), t)$  be the instantaneous cost rate,  $h(x(T))$  the final cost and  $T$  a specified final time, then the performance criterion to be minimized is

$$J = h(x(T)) + \int_0^T l(x(t), u(t), t) dt, \quad (2)$$

Here we focus on hierarchical control approach to solve for this general nonlinear optimal control problem. Fig. 1 shows that the decision making of this problem could be divided into two levels which are coordinated each other.

### A. High-level dynamic model

The high-level dynamics with state  $y(t)$  and control signal  $u_y(t)$  is defined as

$$\dot{y} = g(y, u_y) + H(y; w), \quad (3)$$

where  $H(y; w)$  denotes the uncertainty of the high-level virtual model which will be identified by the online estimation, and  $w$  is the weighting coefficient. We should notice that designing the virtual dynamics should be simple and sufficiently close to the true dynamics, so that the optimal control methods become feasible, the transformation between high-level and low-level will be more reasonable, and it can describe the input and output behavior of the plant as accurately as possible.

The optimal feedback controller on the high-level will be designed using iterative Linear-Quadratic-Gaussian (iLQG) method, which was derived in our recent paper [12]. And the objective of the high-level is to find the control  $u_y$  that minimizes the following performance criterion

$$J = h(y(x(T))) + \int_0^T [q(y(x(t))) + r\|u_y(t)\|^2] dt, \quad (4)$$

where  $r$  is a constant,  $y(x)$  denotes that state  $y$  is a function of  $x$  which is the state of low-level dynamic model. And  $q(y(x))$  is a cost function that quantifies the notion of a ‘‘task’’. Redundancy means that the function  $q(y(x))$  depends on the state  $x$  only through a reduced set of (more abstract) variables  $y(x)$ . In the task of reaching, for example, performance depends only on the position of the hand relative to the target.

### B. Low-level dynamic model

The low-level dynamics is modelled as the real plant described in (1), which is written as the following again

$$\dot{x} = f(x, u_x), \quad x(0) = x_0,$$

The low-level loop will be designed as solving a constrained quadratic optimization problem:

$$\min_{u_x} \frac{1}{2} \int_0^T \|u_x\|^2 dt, \quad (5)$$

subject to

$$u_y = \mathcal{P}(x) u_x, \quad (6)$$

$$0 \leq u_x \leq 1, \quad (7)$$

where (6) establishes an actual relationship between the high-level control command  $u_y$  and low-level control  $u_x$ , we will explain how to compute  $\mathcal{P}(x)$  in the next section. The low-level loop needs to determine how to control the real plant in such a way to achieve the specified command signal  $u_y$  obtained from the high-level.

## III. NUMERICAL RESULTS

The general approach to approximately optimal hierarchical control of redundant systems outlined in the previous section is studied numerically using the following two realistic models of the human arm.

### A. Application to reaching movements with a 2-DOF arm model

The 2DOF arm model contains 2 links, 6 muscles with varying moment arms, muscle length-velocity-tension curves based on the Virtual Muscle model, and activation dynamics modelled as a nonlinear low-pass filter [8]. The forward dynamics can be computed as

$$\ddot{\theta} = \mathcal{M}(\theta)^{-1}(\tau_{mus} - \mathcal{C}(\theta, \dot{\theta}) - \mathcal{G}(\theta)), \quad (8)$$

where  $\theta$  is the joint angle (shoulder:  $\theta_1$ , elbow:  $\theta_2$ ),  $\mathcal{M}(\theta)$  is inertia matrix,  $\mathcal{C}(\theta, \dot{\theta})$  is centripetal and coriolis forces,  $\mathcal{G}(\theta)$  is the gravity, and joint torque  $\tau_{mus}$  is given by

$$\tau_{mus} = \mathbf{M}(\theta) \mathbf{T}(a, l(\theta), v(\theta, \dot{\theta})), \quad (9)$$

where  $\mathbf{M}(\theta)$  is muscle moment arms,  $\mathbf{T}(a, l, v)$  is muscle tension. Muscle activation  $a$  is modelled as

$$\dot{a} = \alpha(u_x - a), \quad (10)$$

where  $\alpha$  is the muscle decay constant. Combining (8)-(10), the low-level dynamic system can be obtained where the state is given by  $x = (\theta_1 \ \theta_2 \ \dot{\theta}_1 \ \dot{\theta}_2 \ a)^T$ .

The high-level dynamics is defined in end-effector space (hand space), and modelled using end-effector equations of motion. The construction of the end-effector dynamic model is achieved by expressing the relationships of hand position  $p$ , velocities  $v$ , as well as the operational forces  $f$  acting on the hand. Therefore, initially, we could use a linear model to describe the high-level dynamics as  $\dot{y} = Ay + Bu_y$ , where we could choose  $y = [p \ v]^T$  or  $[p \ v \ f]^T$ . Based on the characteristics of the transformation from the joint space to the end-effector space [13], the mapping between the state of real plant  $x$  and the state of the high-level dynamics  $y$  can be established as  $y = \Phi(x)$ , then  $\mathcal{P}(x)$  in (6) is solved by

$$\mathcal{P}(x) = B^T \Phi_x f_u \quad (11)$$

where  $\Phi_x$  denotes the partial derivation of  $\Phi$  over  $x$ , and  $f_u$  denotes the partial derivation of  $f$  over  $u$ .

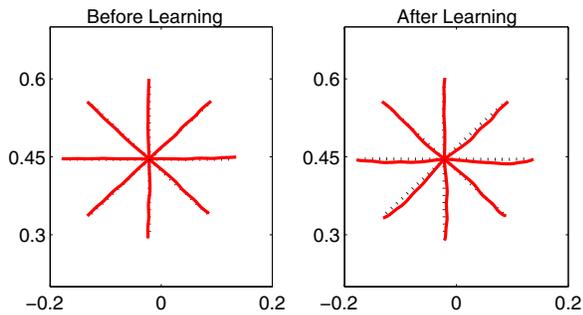


Fig. 2. Reaching trajectories in hand space obtained before and after learning with  $y$  containing hand position and velocity. Dotted lines — trajectories obtained by applying the high-level feedback controller to the virtual dynamics. Solid lines — trajectories obtained by applying the hierarchical control scheme to the real plant.

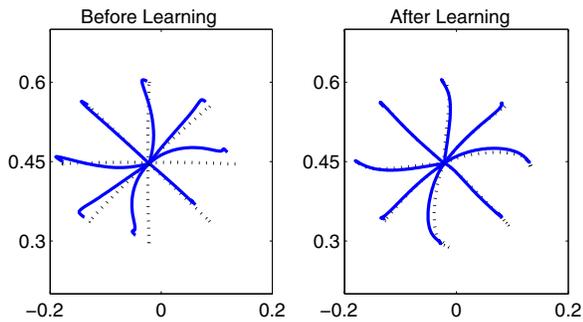


Fig. 3. Reaching trajectories in hand space obtained before and after learning with  $y$  containing hand position, velocity and net muscle force. Dotted lines — trajectories obtained by applying the high-level feedback controller to the virtual dynamics. Solid lines — trajectories obtained by applying the hierarchical control scheme to the real plant.

In order to demonstrate the effectiveness of our design, we apply this hierarchical control scheme to the center-out reaching task which is commonly studied in the Motor Control — the targets are arranged in a circle with 15cm radius around the starting position. We study the simulation results using two different representations of high-level dynamics. In the first case (Fig. 2),  $y$  is a 4-dimensional vector containing the hand position and velocity. In the second case (Fig. 3),  $y$  is a 6-dimensional vector containing the hand position, velocity and net muscle force. Fig. 2 and Fig. 3 illustrate the reaching trajectories in hand space. The solid lines are the actual trajectories of the hand, obtained by applying the two-level hierarchy to the real detailed arm model. The dotted lines are the trajectories that would have resulted from applying the feedback control law to the virtual dynamical system.

We notice that, in Fig. 3, the “expected trajectories” before learning are straight, because we do not include nonlinearities in the initial virtual model. However, after the system identification stage, the virtual model is improved, and it now contains nonlinear terms. As a result, both the virtual and real trajectories become curved, and more importantly, they get closer to each other. Therefore, the high-level dynamics model that was learned is a good

approximation to the dynamics of the real plant. Furthermore, the comparison between between Fig. 2 and Fig. 3 shows that the trajectories in Fig. 3 are more curved because we include more nonlinearities in the high-level dynamics. Overall we conclude that our hierarchical control scheme is reasonably close to the behavior of the optimal controller when the high-level state contains hand position and velocity.

### B. Application to reaching movements with a 7-DOF arm model

A large number of research and studies has reported on developing biomechanical models of human upper limb, including three chained mechanisms — the shoulder girdle, the elbow and the wrist. The proper description and simulation of the musculoskeletal structure of the upper limb is necessary to predict realistic human movement. Engin et al. in 1987 presented a shoulder model with quantitative descriptions of the individual joint sinus cones [3]. Högfors et al. in 1987 applied the optimization techniques to predict muscle forces as functions of the static arm position and the external loads, which provided an improved description of the shoulder model [6]. A dynamic shoulder model was proposed by van der Helm et al. by means of the finite element method [5]. Based on the high resolution images obtained from VHP dataset, in 2001, Garner and Pandy [4] developed a complete biomechanical model of the arm, which included the three-dimensional movements of all the bones of the upper limb, the model enclosed seven anatomical joints from the shoulder girdle down to the wrist, and used thirteen degree-of-freedom to describe the relative position and orientation of seven upper-extremity bones.

To simplify the problem, we approximate the arm segments (upper arm, forearm and palm) by rigid cylinders. Our model (Fig. 4) uses seven degree-of-freedom to describe the relative movements of the segments: the shoulder is modelled as a three degree-of-freedom joint (only the glenohumeral joint was taken into account here), with abduction-adduction,

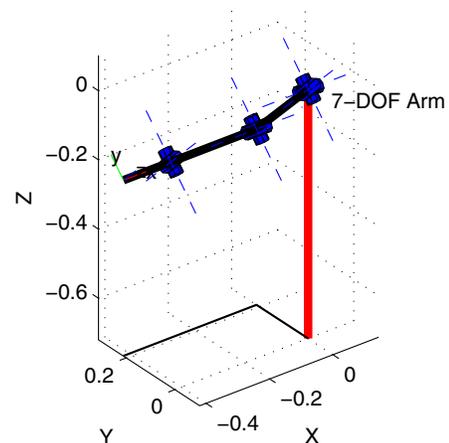


Fig. 4. 7DOF human arm

flexion-extension and axial rotation; the elbow is modelled as a two degree-of-freedom joint, with flexion-extension and pronation-supination movements; the wrist is also modelled as a two degree-of-freedom mechanism, with radiocarpal flexion-extension and radiocarpal radial-ulnar deviation. This 7DOF arm model contains 14 muscles, and muscle activation dynamics modelled as a nonlinear low-pass filter [8]. Currently, we are using Robotics toolbox to simulate the kinematics and dynamics of serial-link upper limb as shown in Fig. 4.

We apply the hierarchical control scheme to the 7DOF arm model. In this case the state of the high-level virtual dynamics is 6-dimensional, and contains hand position and velocity expressed in hand space. We use an initial linear model of the  $y$  dynamics, where the velocity is the derivative of position, and the control signal  $u_y$  is the derivative of velocity. For that model, we build the high-level controller for 24 training movements (different from the testing one shown in the figure). We then apply the control law to the real plant, collect data, and improve the high-level virtual model by fitting a quadratic polynomial. The control law is then optimized with respect to the new dynamics model, and applied to the real plant. Fig. 5 illustrates the reaching trajectories in hand space obtained after learning. In each plot, the solid lines are the actual trajectories of the hand, obtained by applying the two-level hierarchy to the real detailed arm model. The dotted lines are the “expected” trajectories that would have resulted from applying the feedback control law to the virtual dynamical system. After learning the expected and actual trajectories are close, and both arrive to the desired target position.

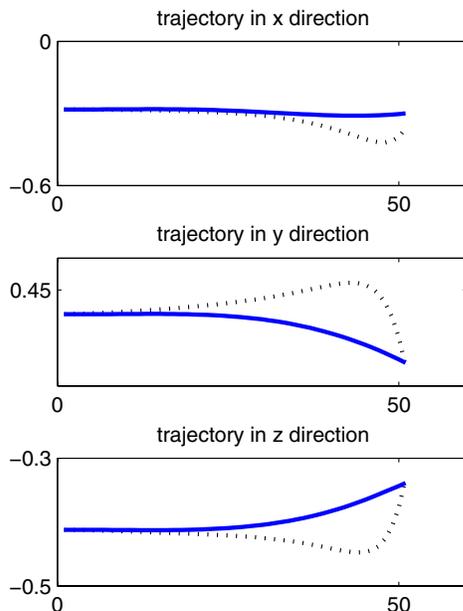


Fig. 5. Reaching trajectories in hand space obtained after learning. Dotted lines — trajectories obtained by applying the high-level feedback controller to the virtual dynamics. Solid lines — trajectories obtained by applying the hierarchical control scheme to the real plant.

#### IV. CONCLUSION AND FUTURE WORK

Here we present a general approach for hierarchical feedback control of redundant biological movement systems, inspired by the hierarchical structure of the sensorimotor system as well as the need to simplify complex control problems in biological Motor Control. We illustrate its application in the context of reaching with two models of the human arm. The simulation results demonstrate the effectiveness of the method, and confirm that the learning process can improve the system performance.

However, there exists several limitations in our model. First, the Robotics toolbox uses Euler’s method to represent 3D orientation, where the rotation is performed in a specific order about three predefined axes. One potential problem of using Euler angles is that they suffer from “gimbal lock”: when two axes effectively line up, this results in a temporary loss of one degree-of-freedom. Because of this limitation, the modelling of shoulder complex becomes especially difficult. Therefore, we are considering other simulation engines (such as SD/FAST, ODE or DynaMechs) which support a 3DOF ball-joint model, to implement a more realistic and physically-based simulation of the human arm. Another computational aspect of our future work is the continuous improvement of the virtual dynamics, to compensate the unavoidable transformation errors between the high-level and low-level.

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