What makes control hard?

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Some of the usual suspects are:

- **non-linearity**
- **high dimensionality**
- **redundancy**
- **noise and uncertainty**

These properties can make an already hard problem harder,
however none of them is a root cause of difficulty.

A control problem can have all these properties and still be easy,
in the sense that there exists a simple strategy that always works:

Push towards the goal!

The problem is hard when this strategy is infeasible, due to **constraints**.

Easy example: Reaching with a redundant arm

joint space configuration q

end effector position $y(q)$

end effector Jacobian $J(q) = \frac{\partial y(q)}{\partial q}$

Jacobian null space $N(q)$

Push hand towards target:

$$u = k\, J(q)^T \left(y^* - y(q)\right)$$

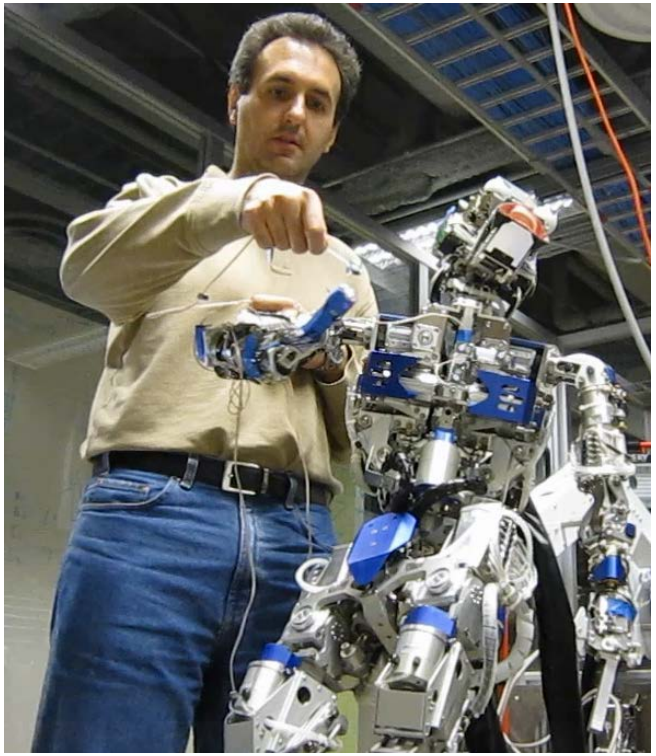
Push hand towards target,
while staying close to default configuration:

$$u = k_1\, J(q)^T \left(y^* - y(q)\right) + k_2\, N(q)\left(q^* - q\right)$$

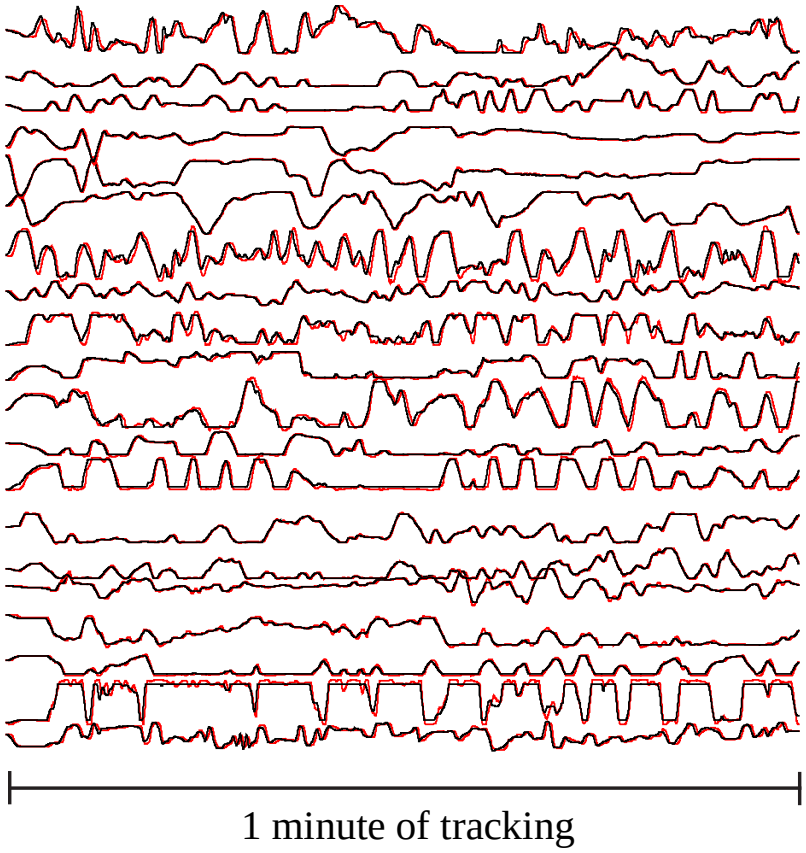
The controller does not need to worry about the path, or the speed profile, or stability, or anything else - it all emerges from the nicely damped dynamics.

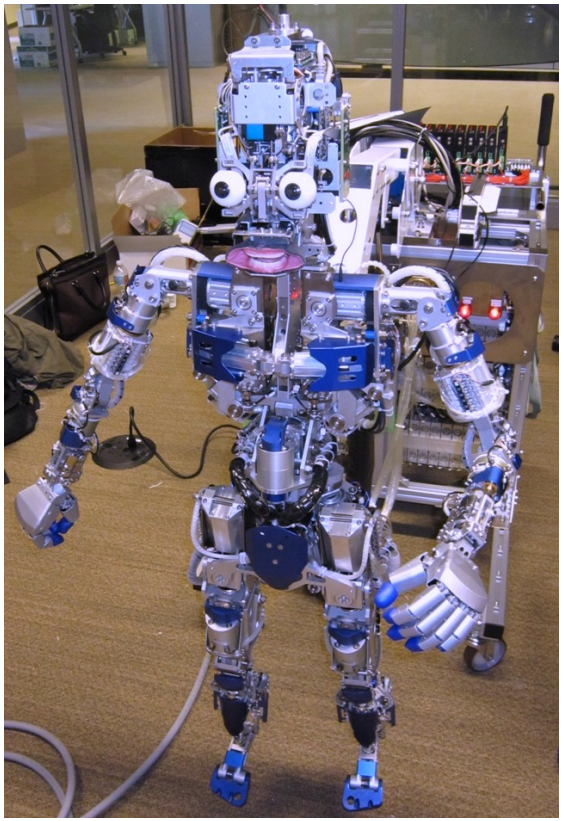
Pneumatic robot (Diego-san)

air pressure similar to muscle activation,
but with longer time constant (~ 80 ms)



Easy example: Trajectory tracking with PD control





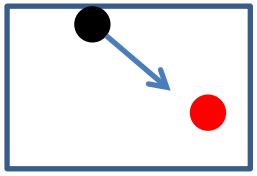
reference trajectory

actual trajectory

Constraints that are (mostly) benign

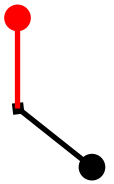
Joint limits

The goal is inside the convex feasible region, so pushing towards the goal will not violate the joint limits.



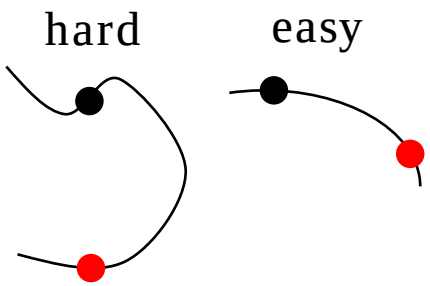
Actuation limits

This is a big problem for under-powered systems, but most robots are sufficiently strong.



Equality constraints

Such constraints restrict the state to a manifold. If the simple push-towards-the-goal action *projected on the manifold* always gets us closer to the goal, then the problem is still easy. Gently curved manifolds are likely to have this property.



Constraints that make the problem hard

Under-actuation

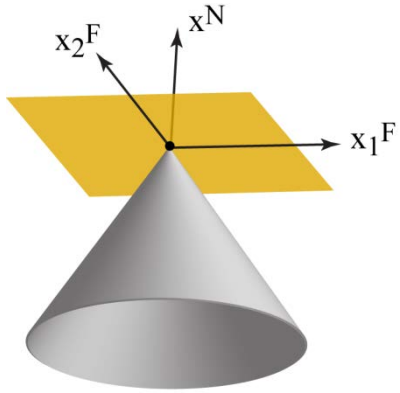
In tasks such as locomotion and object manipulation, some DOFs cannot be controlled directly. These un-actuated DOFs are precisely the ones we would like to control.

Obstacles

Obstacles can turn a control problem into a complicated maze. Solving such problems requires path planning, along with dynamic consistency.

Contact dynamics

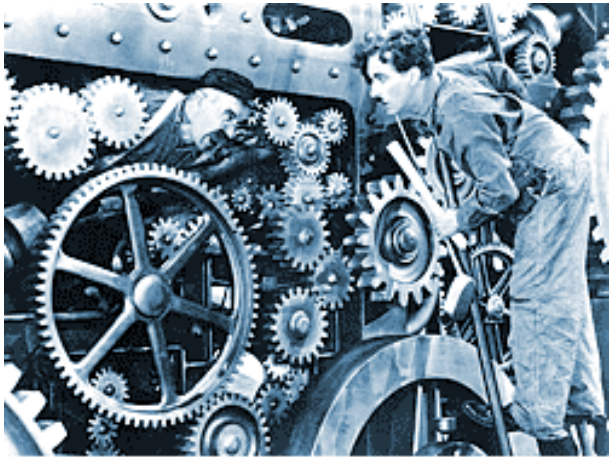
Physical contacts change the plant dynamics qualitatively. Making and breaking contacts is usually required for the task, thus the controller has to operate in many dynamic regimes, and handle abrupt transitions.



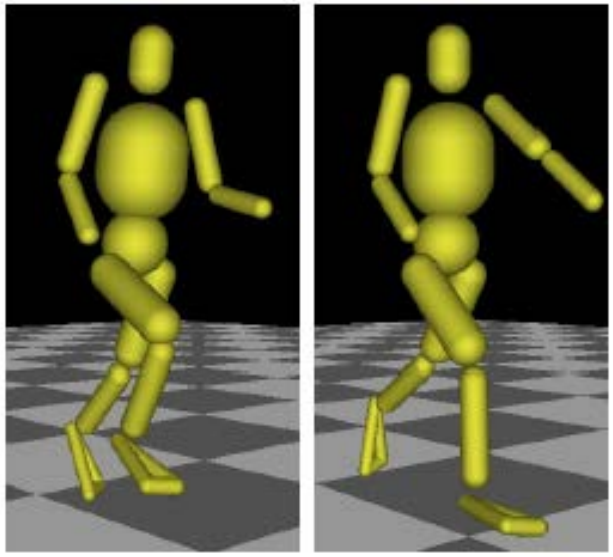
Optimal control approach

plant dynamics
3D humanoid

task-level costs
move CoM forward
use small controls



complex-yet-useful behavior



(Erez and Tassa)

Specify task goals using intuitive cost functions.

For given initial state, find a realizable trajectory (with respect to a model) that minimizes the cumulative cost.

All details of the movement are discovered automatically using numerical optimization.