# **Computational Fabrication**

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- Introduction
- Hardware Review
  - From Design to Machine Code
  - **Design Space Representations**
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- Multi-objective Inverse Methods
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- **Course Review**

#### **Performance Space Representations**



#### **Recap: Performance-Driven Design**



**Design Space** 

From Design Space to Performance Space

• Numerical simulation maps points from design space to performance space



**Design Space** 

### **Bounds on Performance**



#### Design Space

### Example I: Color Gamut

 The subset of colors which can be accurately represented within a given color space or by a certain output device.



### Hardware Capabilities Limit Gamut

• Gamut can be directly tied to capabilities of a given hardware



### Example II: Mechanical Properties in Printing Microstructures



Heterogeneous material



What physical properties can be achieved with microstructures?

### Mapping Microstructures to Material Properties



### **Mechanical Properties Gamut**

 Space of bulk material properties that can be achieved with all material microstructures of a given size



# How to Represent Gamut?

- Boundary
  - Mesh/contour
- Volume
  - Grids (e.g., voxels), adaptive grids, points, distance fields



## Why Volumetric Gamut Representations?



# Why Volumetric Gamut Representations?

- Easy to check whether points are inside/outside
- Each cell can store points mapping back to the design space



## How to Represent Gamut in Higher Dimensions?



# How to Represent Gamut in Higher Dimensions?

- These representations are useful but have not been explored much
- Possible representations: points, classifiers



• When design space is low-dimensional



- When design space is low-dimensional we can explicitly compute the mapping for all points in design space
- Example: 2D printers/color



• When design space is high dimensional



• When design space is high dimensional we can use genetic algorithms to expand gamut in all directions





• Microstructure samples



- Microstructure samples
- Compute level set



- Microstructure samples
- Compute level set
- Find random seeds near the level
  set boundary



- Microstructure samples
- Compute level set
- Find random seeds near the level set boundary
- Find gradient towards outside of gamut



- Microstructure samples
- Compute level set
- Find random seeds near the level set boundary
- Find gradient towards outside of gamut
- Discrete and continuous sampling



- Microstructure samples
- Compute level set
- Find random seeds near the level set boundary
- Find gradient towards outside of gamut
- Discrete and continuous sampling
- Update level set

#### Example: Gamut for Microstructures with Cubic Symmetry



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#### **Inverse Methods**



### **Bounds on Performance**



#### Design Space

• Inverse problem is much more difficult



#### **Design Space**

• Inverse problem is much more difficult



#### **Design Space**

• Inverse problem is much more difficult



#### **Design Space**

### Functional Design/ Generative Design



Goal

Printable Object


#### **Inverse: From Performance Space to Design Space**



How do we update the design variables?

- ✓ converge to a good solutions quickly
- ✓ not get stuck in local minima

**Depends on the Design Space!** 





• Each design can be mathematically represented as a point in  $\mathbb{R}^D$ 



**Design Space** 

# **Design Space for Additive Manufacturing**

• Each design can be mathematically represented as a point in  $\mathbb{R}^D$ , where D = number of voxels in a build volume



# **Reducing Design Space**

• Each design can be mathematically represented as a point in  $\mathbb{R}^D$ 



**Design Space** 

# Example



#### Example



X

**Reduced Parameters** 



- Mesh Vertices
- Mesh deformation "knobs"
  - e.g cages





simulated result





simulated result

goal





e.g., Newton's Method

# Inverse Methods: Topology Optimization





- Objectives:
  - Structure should be as stiff as possible (i.e. the compliance should be minimal) when a load is applied
  - The total amount of material should be equal to  $V_{max}$



Initial layout

Large discrete space:  $\{0,1\}^N$ 



Output: Voxels with material assignment (no material, full)

• Design variables

Material property  $\mathbf{C} = \rho \mathbf{C}_0$   $\mathbf{C} = \rho^p \mathbf{C}_0$ 



the method is called SIMP, power-law or density approach.



Large discrete space:  $\{0,1\}^N$ 

• Default boundary conditions: MMB Beam



#### Full domain

• Default boundary conditions: MMB Beam



Half design domain

• Default boundary conditions: MMB Beam



Half design domain

• Default boundary conditions: MMB Beam



Half design domain

• How can we measure compliance?



Compute static equilibrium: KU = F

Measure Energy of the System:  $\mathbf{U}^T \mathbf{K} \mathbf{U}$ 

$$\min_{\mathbf{x}} c(\mathbf{x}) = \mathbf{U}^T \mathbf{K} \mathbf{U} = \sum_{e=1}^N (x_e)^p \mathbf{u}_e^T \mathbf{k}_0 \mathbf{u}_e$$

$$V(\mathbf{x})$$

subject to 
$$\frac{V(\mathbf{x})}{V_0} = f$$
  
 $\mathbf{K}\mathbf{U} = \mathbf{F}$ 

$$0 < x_{\min} \le x \le 1$$

Densities 
$$\min_{\mathbf{x}} c(\mathbf{x}) = \mathbf{U}^T \mathbf{K} \mathbf{U} = \sum_{e=1}^N (x_e)^p \mathbf{u}_e^T \mathbf{k}_0 \mathbf{u}_e$$
subject to 
$$\frac{V(\mathbf{x})}{V_0} = f$$
$$\mathbf{K} \mathbf{U} = \mathbf{F}$$
$$\mathbf{0} < \mathbf{x}_{\min} \le \mathbf{x} \le \mathbf{1}$$

$$\min_{\mathbf{x}} c(\mathbf{x}) = \mathbf{U}^T \mathbf{K} \mathbf{U} = \sum_{e=1}^N (x_e)^p \mathbf{u}_e^T \mathbf{k}_0 \mathbf{u}_e$$

subject to 
$$\frac{V(\mathbf{x})}{V_0} = f$$
  
 $\mathbf{KU} = \mathbf{F}$   
 $\mathbf{0} < \mathbf{x}_{\min} \le \mathbf{x} \le \mathbf{1}$  Valid range for densities

• Minimum compliance problem

Energy of the system  

$$\min_{\mathbf{x}} c(\mathbf{x}) = \mathbf{U}^T \mathbf{K} \mathbf{U} = \sum_{e=1}^N (x_e)^p \mathbf{u}_e^T \mathbf{k}_0 \mathbf{u}_e$$

subject to 
$$\frac{V(\mathbf{x})}{V_0} = f$$
  
 $\mathbf{K}\mathbf{U} = \mathbf{F}$ 

 $0 < x_{\min} \le x \le 1$ 

$$\min_{\mathbf{x}} c(\mathbf{x}) = \mathbf{U}^T \mathbf{K} \mathbf{U} = \sum_{e=1}^N (x_e)^p \mathbf{u}_e^T \mathbf{k}_0 \mathbf{u}_e$$
  
subject to  $\frac{V(\mathbf{x})}{V_0} = f$   
 $\mathbf{K} \mathbf{U} = \mathbf{F}$  Static equilibrium  
 $\mathbf{0} < \mathbf{x}_{\min} \le \mathbf{x} \le \mathbf{1}$ 

$$\begin{split} \min_{\mathbf{x}} c(\mathbf{x}) &= \mathbf{U}^T \mathbf{K} \mathbf{U} = \sum_{e=1}^N (x_e)^p \mathbf{u}_e^T \mathbf{k}_0 \mathbf{u}_e \\ \text{subject to} \quad \boxed{\frac{V(\mathbf{x})}{V_0}} &= f \\ \mathbf{K} \mathbf{U} &= \mathbf{F} \\ \mathbf{0} < \mathbf{x}_{\min} \leq \mathbf{x} \leq \mathbf{1} \end{split}$$

#### Result



• Based on the paper:

"A 99 line topology optimization code in Matlab" by Ole Sigmund, Structural and Multidisciplinary Optimization 21(2), 2001, pp. 120-127

- Code can be find here:
  - <a href="http://www.topopt.mek.dtu.dk/apps-and-software">http://www.topopt.mek.dtu.dk/apps-and-software</a>

#### Challenges



#### Hardware: Object-1000 Plus

- Up to 39.3 x 31.4 x 19.6 in
- 600dpi (~40 microns)
- 5 trillion voxels



#### Software: SIMP Topology Optimization

- Up to millions of elements
- Difficult to handle multiple materials







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# **Multiple Performance Objectives**







Performance metric: **flexibility** 

Performance metric: weight

Performance metric: **stability** 

#### **Multi-Objective Optimization**

min 
$$f_i(x)$$
,  $i = 1, ..., d$   $x \in \mathbb{R}^D$   
Subject to  $g(x) \ge$ ,  $h(x) = 0$ 



#### **Multi-Objective Optimization**

$$\min f_i(x), \qquad i = 1, \dots, d$$
  
Subject to  $g(x) \ge, \quad h(x) = 0$ 

$$F(x) = [f_1(x), \dots, f_d(d)]$$

We know how to do this:

$$\min f\left(x\right)$$
#### **Multi-Objective Optimization**

$$\min f_i(x), \qquad i = 1, \dots, d$$
  
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We know how to do this:

$$\min f\left(x\right)$$

Solution:  $f(x) = \sum_i w_i f_i(x)$ 

#### **Multi-Objective Optimization**

$$\min f_i(x), \qquad i = 1, \dots, d$$
  
Subject to  $g(x) \ge, \quad h(x) = 0$ 

$$F(x) = [f_1(x), \dots, f_d(d)]$$

We know how to do this:

$$\min f\left(x\right)$$

Solution: 
$$f(x) = \sum w_i f_i(x)$$

How do you pick the weights?

#### Do the Weights Mater?

#### Example:

 $f_1(x) = 2x - 5$  $f_2(x) = x + 3$  $0 \le x \le 1$ 

 $f(x) = w_1 f_1(x) + w_2 f_2(x)$ 

#### Do the Weights Mater?

#### Example:

 $f_1(x) = 2x - 5$  $f_2(x) = x + 3$  $0 \le x \le 1$ 

$$f(x) = w_1 f_1(x) + w_2 f_2(x)$$

No matter what weights you pick arg  $\min f(x) = 0$ 

#### When Objectives are Conflicting



## Experiment

(A)



3 carrots 8 candies



6 carrots 6 candies



## Experiment





7 carrots3 candies



6 carrots 6 candies

(E)



5 carrots 7 candies





4 carrots 9 candies

## **Definition: Dominance**





3 carrots8 candies

(D)



7 carrots3 candies





6 carrots 6 candies

(E)



5 carrots 7 candies



6 carrots 4 candies



4 carrots 9 candies

## **Definition: Dominance**





3 carrots8 candies

(D)



7 carrots3 candies





6 carrots 6 candies

(E)



5 carrots 7 candies



6 carrots 4 candies



4 carrots 9 candies

A solution  $x_1$  is said to dominate the other solution  $x_2$ , if both the following conditions are true:

- 1. The solution  $x_1$  is no worse than  $x_2$  in all objectives.
- 2. The solution  $x_1$  is strictly better than  $x_2$  in at least one objective.

A point is Pareto optimal if it in not dominated by any point: called non-dominated point

## Let's Plot this



4 carrots 9 candies

#### **Pareto Front**



## **For Minimization**



## **Space of Optimal Solutions**



## **Space of Optimal Solutions**



#### The Geometry of the Front



Not a straight line!

Solution:  $f(x) = \sum_i w_i f_i(x)$   $\otimes$ 

## The Front Can Have Gaps



#### The Front Can Have Non-Convex Regions



## **Pareto Front Discovery**

Main Challenge:

- Converge to optimal solutions
- Diverse set that describes the full front





**Performance Space** 

#### Problem: Each Single Objective Optimization is not SIMPLE!



Move many points in parallel towards the front at the same time?

















#### **Elitist Non-dominated Sorting GA or NSGA-II**





Figure 13: Population at generation 100.

# Finding the Full (Continuous) Front



## Set of Manifolds



## Example



Schulz et al 2018

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#### **Advanced Performance-Driven Design**



# Advanced Performance-Driven Design

- Performance Evaluation Speed-Up
  - Sensitivity Analysis
  - Precomputation + Interpolation
  - Replace Simulation with ML
- Design space exploration and Optimization
  - Expert Systems
  - Data-Driven Search
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#### **Application: Interactive Garment Design**









#### Linear Sensitivity Analysis

• What does this derivative tells us about our surface ?



## Linear Sensitivity Analysis



Source: Umetani et al 2011

### **Sensitivity Modes**

# Sensitivity Mode



#### **Result: Interactive Garment Design**



# Advanced Performance-Driven Design

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## **Application: Performance-Driven Design in CAD Systems**



## **Precomputation and Interpolation**



precomputed data



#### **Results**





#### **Stress Distribution**





#### **Results**



## InstantCAD



precomputed data

output

# Advanced Performance-Driven Design

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#### **Application: Fast Simulation for Control**



# **Replacing Simulation with Machine Learning**

#### Simulation Results FluidFall #1

Ground Truth



Model Rollout Input: position & velocity at the first frame



# Advanced Performance-Driven Design

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## Application: Design of Robots with Ground Locomotion



#### Fast Gait

**Slower Gait** 

#### **Geometry Change**

#### **Application: Design of Robots with Ground Locomotion**



#### **Real Time Feedback**





#### **Optimization**



# Assembly



## **Physical Robots Created**



# **Advanced Performance-Driven Design**

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## Application: Design by Composition



## Design and Fabrication by Example



Dataset



#### Dataset



# Snapping

- Constraints:
  - Data driven: similar connections
- Optimization:
  - user interaction





## **Adding Physical Connectors**



**Working Model** 

**Snapped Configuration** 

**Connected Configuration** 

## **Searching for Connections**





#### **Snapped configuration**

**Linked Elements** 

## **Searching for Connections**







#### **Working Model**

## **Searching for Connections**



## **Physical Connectors: An Example**



#### Extracted Directly From Data!
# Designing a Go-Kart



# Advanced Performance-Driven Design

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## **Application: Machine Knitting**



Source: Ministry of Supply

# Solution: Knit Graph Representation

$$\mathcal{N} \equiv \{n, \ldots\} \qquad \bigcirc$$
  
$$\mathcal{R} \equiv \{(n_i, n_j), \ldots\} \qquad \cdots$$
  
$$C \equiv \{(n_i, n_j), \ldots\} \qquad \uparrow$$





**Graph Properties:** 



#### Source: Narayanan et al 2018

### From Geometry to Knitting Instructions



### From Geometry to Knitting Instructions



Source: Narayanan et al 2018

### Results

# Stanford Bunny





# Advanced Performance-Driven Design

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# **Application: Color Reproduction via Multi-Layer Printing**



Original painting (Sunlight)

Our reproduction (Sunlight)

# **Bidirectional Layout–Spectrum Mapping**



Requires more than tens of thousands paired training data

### **Contoning Dataset**



# **Physical Reproductions**





# **Comparison with Color Contoning**



Ours

Original

Color Contoning [Babaei et al. 2017]

# Advanced Performance-Driven Design

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# **Application: Hybrid Copter Design**



### **Advantages of Hybrid Copters**



#### Copter mode



✓ Flexibility

#### Plane mode

- ✓ High speed
- ✓ Energy efficiency

#### Interactive Hybrid-Copter Design



🤕 🕂 🐔 🗁 Examples 📄 Hybrid UAV 】 🛅 Hybrid UAV

# Interactive Hybrid-Copter Design



# **Traditional Controller Design**

#### Quad-plane controller



#### Tail-sitter controller





## **Our Approach: NN Controller**



### **Reinforcement Learning**



### **Reinforcement Learning**









# **Real Flight Tests**



# **Our Approach: NN Controller**



# **Real Flight Tests**



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#### **Computational Design Stack**



#### **Computational Design Stack**



# Additive Manufacturing Processes

- Thermoplastic Extrusion
  - Fused deposition modeling (FDM)
- UV Curable Resins/thermosets
  - Stereolithography (SLA) & DLP Printing
  - Photopolymer Inkjet Printing
- Powders
  - Selective laser sintering (SLS)
  - Binder jetting/3D Printing
- Sheets
  - Laminated object manufacturing (LOM



Figure 2. Various 3D printing techniques. a) Selective laser sintering (SLS), b) Fused deposition modeling (FDM, also termed "thermoplastic extrusion"), c) Photopolymer inkjet printing, d) Binder ketting, also trademarked as 3DP, e) Laminated object manufacturing (LOM), f) Stereo-Hibbrgraph (SL), Images countrey of CustomPartNetzon.

#### **Computational Design and Fabrication Pipeline**



#### Hardware

#### Machine Code



#### Performance

# **3D Printing Software Pipeline**





• For a discrete z value, compute an intersection of a plane with a model



#### **Computational Design Stack**



#### Hardware

#### **Machine Code**



#### Performance

## **Design Space**

- Each design can be mathematically represented as a point in  $\mathbb{R}^{D}$ 



#### **Design Space**

### Parametric Design and CAD



# **Procedural Modeling**



Source: Converting 3D Furniture Models to Fabricable Parts and Connectors, Lau et al., Siggraph 2011
## **Deformation Methods**



## **Computational Design Stack**



#### Hardware

#### Machine Code



### Performance

## **Design Driven By Performance**



# Simulation

- Mechanical
  - dynamic
  - static
- Acoustic
- Thermal
- Electromagnetic
- etc.



## Performance Space Representations: Gamut



Inverse: From Performance Space to Design Space

• Inverse problem is much more difficult



## From Performance Space to Design Space



## **Optimization in Reduced Space**



Rest shape

# **Topology Optimization**



## **Multi-Objective Optimization: Pareto Front**



8 candies



4 candies

3 candies

7 candies



3 carrots 9 candies

## Multi-Objective Optimization: Pareto Front



## **Pareto Front**



## **Speeding up Simulation**



## **Expert Systems for Computational Design**



## **Data-driven Systems for Computational Design**



## **Computational Design Stack**



#### Hardware

#### Machine Code



### Performance