Computational Fabrication

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Course Schedule

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- Introduction
- Hardware Review
 - From Design to Machine Code
 - **Design Space Representations**
- Performance-Driven Design
- Break
- **Performance Space Representation**
- **Inverse Methods**
- Multi-objective Inverse Methods
- Advanced Performance-Driven Design
- **Course Review**

Performance Space Representations



Recap: Performance-Driven Design



Design Space

From Design Space to Performance Space

• Numerical simulation maps points from design space to performance space



Design Space

Bounds on Performance



Design Space

Example I: Color Gamut

 The subset of colors which can be accurately represented within a given color space or by a certain output device.



Hardware Capabilities Limit Gamut

• Gamut can be directly tied to capabilities of a given hardware



Example II: Mechanical Properties in Printing Microstructures



Heterogeneous material



What physical properties can be achieved with microstructures?

Mapping Microstructures to Material Properties



Mechanical Properties Gamut

 Space of bulk material properties that can be achieved with all material microstructures of a given size



How to Represent Gamut?

- Boundary
 - Mesh/contour
- Volume
 - Grids (e.g., voxels), adaptive grids, points, distance fields



Why Volumetric Gamut Representations?



Why Volumetric Gamut Representations?

- Easy to check whether points are inside/outside
- Each cell can store points mapping back to the design space



How to Represent Gamut in Higher Dimensions?



How to Represent Gamut in Higher Dimensions?

- These representations are useful but have not been explored much
- Possible representations: points, classifiers



• When design space is low-dimensional



- When design space is low-dimensional we can explicitly compute the mapping for all points in design space
- Example: 2D printers/color



• When design space is high dimensional



• When design space is high dimensional we can use genetic algorithms to expand gamut in all directions





• Microstructure samples



- Microstructure samples
- Compute level set



- Microstructure samples
- Compute level set
- Find random seeds near the level
 set boundary



- Microstructure samples
- Compute level set
- Find random seeds near the level set boundary
- Find gradient towards outside of gamut



- Microstructure samples
- Compute level set
- Find random seeds near the level set boundary
- Find gradient towards outside of gamut
- Discrete and continuous sampling



- Microstructure samples
- Compute level set
- Find random seeds near the level set boundary
- Find gradient towards outside of gamut
- Discrete and continuous sampling
- Update level set

Example: Gamut for Microstructures with Cubic Symmetry



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Inverse Methods



Bounds on Performance



Design Space

• Inverse problem is much more difficult



Design Space

• Inverse problem is much more difficult



Design Space

• Inverse problem is much more difficult



Design Space

Functional Design/ Generative Design



Goal

Printable Object


Inverse: From Performance Space to Design Space



How do we update the design variables?

- ✓ converge to a good solutions quickly
- ✓ not get stuck in local minima

Depends on the Design Space!





• Each design can be mathematically represented as a point in \mathbb{R}^D



Design Space

Design Space for Additive Manufacturing

• Each design can be mathematically represented as a point in \mathbb{R}^D , where D = number of voxels in a build volume



Reducing Design Space

• Each design can be mathematically represented as a point in \mathbb{R}^D



Design Space

Example



Example



X

Reduced Parameters



- Mesh Vertices
- Mesh deformation "knobs"
 - e.g cages





simulated result





simulated result

goal





e.g., Newton's Method

Inverse Methods: Topology Optimization





- Objectives:
 - Structure should be as stiff as possible (i.e. the compliance should be minimal) when a load is applied
 - The total amount of material should be equal to V_{max}



Initial layout

Large discrete space: $\{0,1\}^N$



Output: Voxels with material assignment (no material, full)

• Design variables

Material property $\mathbf{C} = \rho \mathbf{C}_0$ $\mathbf{C} = \rho^p \mathbf{C}_0$



the method is called SIMP, power-law or density approach.



Large discrete space: $\{0,1\}^N$

• Default boundary conditions: MMB Beam



Full domain

• Default boundary conditions: MMB Beam



Half design domain

• Default boundary conditions: MMB Beam



Half design domain

• Default boundary conditions: MMB Beam



Half design domain

• How can we measure compliance?



Compute static equilibrium: KU = F

Measure Energy of the System: $\mathbf{U}^T \mathbf{K} \mathbf{U}$

$$\min_{\mathbf{x}} c(\mathbf{x}) = \mathbf{U}^T \mathbf{K} \mathbf{U} = \sum_{e=1}^N (x_e)^p \mathbf{u}_e^T \mathbf{k}_0 \mathbf{u}_e$$

$$V(\mathbf{x})$$

subject to
$$\frac{V(\mathbf{x})}{V_0} = f$$

 $\mathbf{K}\mathbf{U} = \mathbf{F}$

$$0 < x_{\min} \le x \le 1$$

Densities
$$\min_{\mathbf{x}} c(\mathbf{x}) = \mathbf{U}^T \mathbf{K} \mathbf{U} = \sum_{e=1}^N (x_e)^p \mathbf{u}_e^T \mathbf{k}_0 \mathbf{u}_e$$
subject to
$$\frac{V(\mathbf{x})}{V_0} = f$$
$$\mathbf{K} \mathbf{U} = \mathbf{F}$$
$$\mathbf{0} < \mathbf{x}_{\min} \le \mathbf{x} \le \mathbf{1}$$

$$\min_{\mathbf{x}} c(\mathbf{x}) = \mathbf{U}^T \mathbf{K} \mathbf{U} = \sum_{e=1}^N (x_e)^p \mathbf{u}_e^T \mathbf{k}_0 \mathbf{u}_e$$

subject to
$$\frac{V(\mathbf{x})}{V_0} = f$$

 $\mathbf{KU} = \mathbf{F}$
 $\mathbf{0} < \mathbf{x}_{\min} \le \mathbf{x} \le \mathbf{1}$ Valid range for densities

• Minimum compliance problem

Energy of the system

$$\min_{\mathbf{x}} c(\mathbf{x}) = \mathbf{U}^T \mathbf{K} \mathbf{U} = \sum_{e=1}^N (x_e)^p \mathbf{u}_e^T \mathbf{k}_0 \mathbf{u}_e$$

subject to
$$\frac{V(\mathbf{x})}{V_0} = f$$

 $\mathbf{K}\mathbf{U} = \mathbf{F}$

 $0 < x_{\min} \le x \le 1$

$$\min_{\mathbf{x}} c(\mathbf{x}) = \mathbf{U}^T \mathbf{K} \mathbf{U} = \sum_{e=1}^N (x_e)^p \mathbf{u}_e^T \mathbf{k}_0 \mathbf{u}_e$$

subject to $\frac{V(\mathbf{x})}{V_0} = f$
 $\mathbf{K} \mathbf{U} = \mathbf{F}$ Static equilibrium
 $\mathbf{0} < \mathbf{x}_{\min} \le \mathbf{x} \le \mathbf{1}$

$$\begin{split} \min_{\mathbf{x}} c(\mathbf{x}) &= \mathbf{U}^T \mathbf{K} \mathbf{U} = \sum_{e=1}^N (x_e)^p \mathbf{u}_e^T \mathbf{k}_0 \mathbf{u}_e \\ \text{subject to} \quad \boxed{\frac{V(\mathbf{x})}{V_0}} &= f \\ \mathbf{K} \mathbf{U} &= \mathbf{F} \\ \mathbf{0} < \mathbf{x}_{\min} \leq \mathbf{x} \leq \mathbf{1} \end{split}$$

Result



• Based on the paper:

"A 99 line topology optimization code in Matlab" by Ole Sigmund, Structural and Multidisciplinary Optimization 21(2), 2001, pp. 120-127

- Code can be find here:
 - http://www.topopt.mek.dtu.dk/apps-and-software

Challenges



Hardware: Object-1000 Plus

- Up to 39.3 x 31.4 x 19.6 in
- 600dpi (~40 microns)
- 5 trillion voxels



Software: SIMP Topology Optimization

- Up to millions of elements
- Difficult to handle multiple materials







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Multiple Performance Objectives







Performance metric: **flexibility**

Performance metric: weight

Performance metric: **stability**

Multi-Objective Optimization

min
$$f_i(x)$$
, $i = 1, ..., d$ $x \in \mathbb{R}^D$
Subject to $g(x) \ge$, $h(x) = 0$



Multi-Objective Optimization

$$\min f_i(x), \qquad i = 1, \dots, d$$

Subject to $g(x) \ge, \quad h(x) = 0$

$$F(x) = [f_1(x), \dots, f_d(d)]$$

We know how to do this:

$$\min f\left(x\right)$$
Multi-Objective Optimization

$$\min f_i(x), \qquad i = 1, \dots, d$$

Subject to $g(x) \ge, \quad h(x) = 0$

$$F(x) = [f_1(x), \dots, f_d(d)]$$

We know how to do this:

$$\min f\left(x\right)$$

Solution: $f(x) = \sum_i w_i f_i(x)$

Multi-Objective Optimization

$$\min f_i(x), \qquad i = 1, \dots, d$$

Subject to $g(x) \ge, \quad h(x) = 0$

$$F(x) = [f_1(x), \dots, f_d(d)]$$

We know how to do this:

$$\min f\left(x\right)$$

Solution:
$$f(x) = \sum w_i f_i(x)$$

How do you pick the weights?

Do the Weights Mater?

Example:

 $f_1(x) = 2x - 5$ $f_2(x) = x + 3$ $0 \le x \le 1$

 $f(x) = w_1 f_1(x) + w_2 f_2(x)$

Do the Weights Mater?

Example:

 $f_1(x) = 2x - 5$ $f_2(x) = x + 3$ $0 \le x \le 1$

$$f(x) = w_1 f_1(x) + w_2 f_2(x)$$

No matter what weights you pick arg $\min f(x) = 0$

When Objectives are Conflicting



Experiment

(A)



3 carrots 8 candies



6 carrots 6 candies



Experiment





7 carrots3 candies



6 carrots 6 candies

(E)



5 carrots 7 candies





4 carrots 9 candies

Definition: Dominance





3 carrots8 candies

(D)



7 carrots3 candies





6 carrots 6 candies

(E)



5 carrots 7 candies



6 carrots 4 candies



4 carrots 9 candies

Definition: Dominance





3 carrots8 candies

(D)



7 carrots3 candies





6 carrots 6 candies

(E)



5 carrots 7 candies



6 carrots 4 candies



4 carrots 9 candies

A solution x_1 is said to dominate the other solution x_2 , if both the following conditions are true:

- 1. The solution x_1 is no worse than x_2 in all objectives.
- 2. The solution x_1 is strictly better than x_2 in at least one objective.

A point is Pareto optimal if it in not dominated by any point: called non-dominated point

Let's Plot this



4 carrots 9 candies

Pareto Front



For Minimization



Space of Optimal Solutions



Space of Optimal Solutions



The Geometry of the Front



Not a straight line!

Solution: $f(x) = \sum_i w_i f_i(x)$ \otimes

The Front Can Have Gaps



The Front Can Have Non-Convex Regions



Pareto Front Discovery

Main Challenge:

- Converge to optimal solutions
- Diverse set that describes the full front





Performance Space

Problem: Each Single Objective Optimization is not SIMPLE!



Move many points in parallel towards the front at the same time?

















Elitist Non-dominated Sorting GA or NSGA-II





Figure 13: Population at generation 100.

Finding the Full (Continuous) Front



Set of Manifolds



Example



Schulz et al 2018

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Advanced Performance-Driven Design



Advanced Performance-Driven Design

- Performance Evaluation Speed-Up
 - Sensitivity Analysis
 - Precomputation + Interpolation
 - Replace Simulation with ML
- Design space exploration and Optimization
 - Expert Systems
 - Data-Driven Search
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Application: Interactive Garment Design









Linear Sensitivity Analysis

• What does this derivative tells us about our surface ?



Linear Sensitivity Analysis



Source: Umetani et al 2011

Sensitivity Modes

Sensitivity Mode



Result: Interactive Garment Design



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Application: Performance-Driven Design in CAD Systems



Precomputation and Interpolation



precomputed data



Results





Stress Distribution





Results



InstantCAD



precomputed data

output

Advanced Performance-Driven Design

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Application: Fast Simulation for Control



Replacing Simulation with Machine Learning

Simulation Results FluidFall #1

Ground Truth



Model Rollout Input: position & velocity at the first frame



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Application: Design of Robots with Ground Locomotion



Fast Gait

Slower Gait

Geometry Change

Application: Design of Robots with Ground Locomotion



Real Time Feedback





Optimization



Assembly



Physical Robots Created



Advanced Performance-Driven Design

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Application: Design by Composition



Design and Fabrication by Example



Dataset



Dataset



Snapping

- Constraints:
 - Data driven: similar connections
- Optimization:
 - user interaction





Adding Physical Connectors



Working Model

Snapped Configuration

Connected Configuration

Searching for Connections





Snapped configuration

Linked Elements

Searching for Connections







Working Model

Searching for Connections



Physical Connectors: An Example



Extracted Directly From Data!
Designing a Go-Kart



Advanced Performance-Driven Design

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Application: Machine Knitting



Source: Ministry of Supply

Solution: Knit Graph Representation

$$\mathcal{N} \equiv \{n, \ldots\} \qquad \bigcirc$$

$$\mathcal{R} \equiv \{(n_i, n_j), \ldots\} \qquad \cdots$$

$$C \equiv \{(n_i, n_j), \ldots\} \qquad \uparrow$$





Graph Properties:



Source: Narayanan et al 2018

From Geometry to Knitting Instructions



From Geometry to Knitting Instructions



Source: Narayanan et al 2018

Results

Stanford Bunny





Advanced Performance-Driven Design

- Performance Evaluation Speed-Up
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Application: Color Reproduction via Multi-Layer Printing



Original painting (Sunlight)

Our reproduction (Sunlight)

Bidirectional Layout–Spectrum Mapping



Requires more than tens of thousands paired training data

Contoning Dataset



Physical Reproductions





Comparison with Color Contoning



Ours

Original

Color Contoning [Babaei et al. 2017]

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Application: Hybrid Copter Design



Advantages of Hybrid Copters



Copter mode



✓ Flexibility

Plane mode

- ✓ High speed
- ✓ Energy efficiency

Interactive Hybrid-Copter Design



🤕 🕂 🐔 🗁 Examples 📄 Hybrid UAV 】 🛅 Hybrid UAV

Interactive Hybrid-Copter Design



Traditional Controller Design

Quad-plane controller



Tail-sitter controller





Our Approach: NN Controller



Reinforcement Learning



Reinforcement Learning









Real Flight Tests



Our Approach: NN Controller



Real Flight Tests



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Computational Design Stack



Computational Design Stack



Additive Manufacturing Processes

- Thermoplastic Extrusion
 - Fused deposition modeling (FDM)
- UV Curable Resins/thermosets
 - Stereolithography (SLA) & DLP Printing
 - Photopolymer Inkjet Printing
- Powders
 - Selective laser sintering (SLS)
 - Binder jetting/3D Printing
- Sheets
 - Laminated object manufacturing (LOM



Figure 2. Various 3D printing techniques. a) Selective laser sintering (SLS), b) Fused deposition modeling (FDM, also termed "thermoplastic extrusion"), c) Photopolymer inkjet printing, d) Binder ketting, also trademarked as 3DP, e) Laminated object manufacturing (LOM), f) Stereo-Hibbrgraph (SL), Images countrey of CustomPartNetzon.

Computational Design and Fabrication Pipeline



Hardware

Machine Code



Performance

3D Printing Software Pipeline





• For a discrete z value, compute an intersection of a plane with a model



Computational Design Stack



Hardware

Machine Code



Performance

Design Space

- Each design can be mathematically represented as a point in \mathbb{R}^{D}



Design Space

Parametric Design and CAD



Procedural Modeling



Source: Converting 3D Furniture Models to Fabricable Parts and Connectors, Lau et al., Siggraph 2011
Deformation Methods



Computational Design Stack



Hardware

Machine Code



Performance

Design Driven By Performance



Simulation

- Mechanical
 - dynamic
 - static
- Acoustic
- Thermal
- Electromagnetic
- etc.



Performance Space Representations: Gamut



Inverse: From Performance Space to Design Space

• Inverse problem is much more difficult



From Performance Space to Design Space



Optimization in Reduced Space



Rest shape

Topology Optimization



Multi-Objective Optimization: Pareto Front



8 candies



4 candies

3 candies

7 candies



3 carrots 9 candies

Multi-Objective Optimization: Pareto Front



Pareto Front



Speeding up Simulation



Expert Systems for Computational Design



Data-driven Systems for Computational Design



Computational Design Stack



Hardware

Machine Code



Performance