# Interactive Exploration of Design Trade-offs
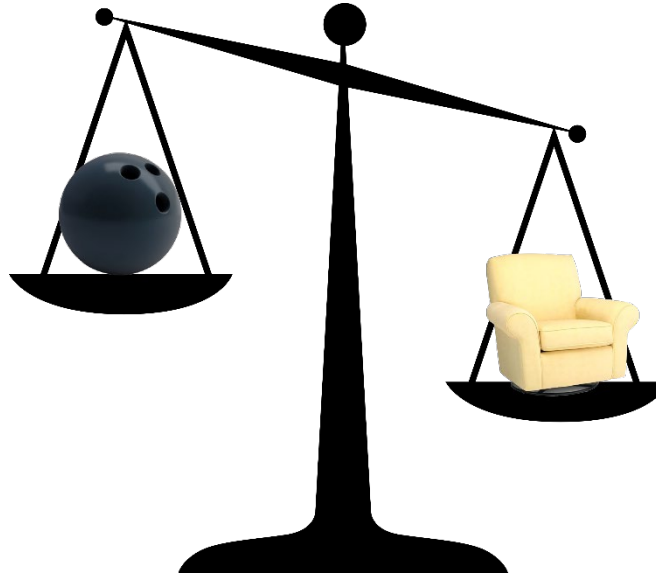
Adriana Schulz, Harrison Wang, Eitan Grinspun, Justin Solomon, and Wojciech Matusik

MIT CSAIL

# Performance-Driven Design



Performance metric:
**flexibility**



Performance metric:
**weight**



Performance metric:
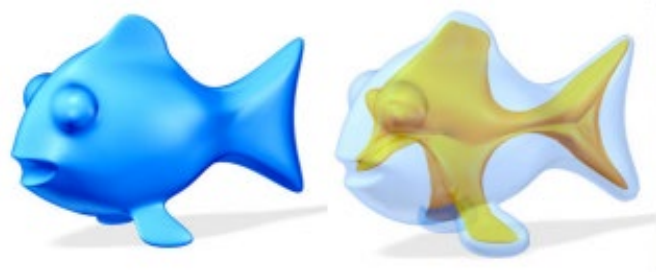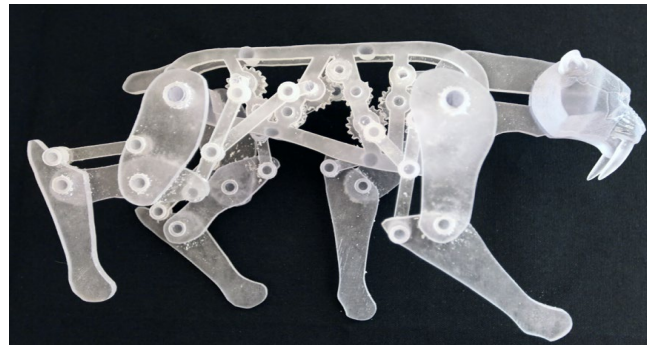**stability**

# Previous Work

[Prevost et al. 2014]

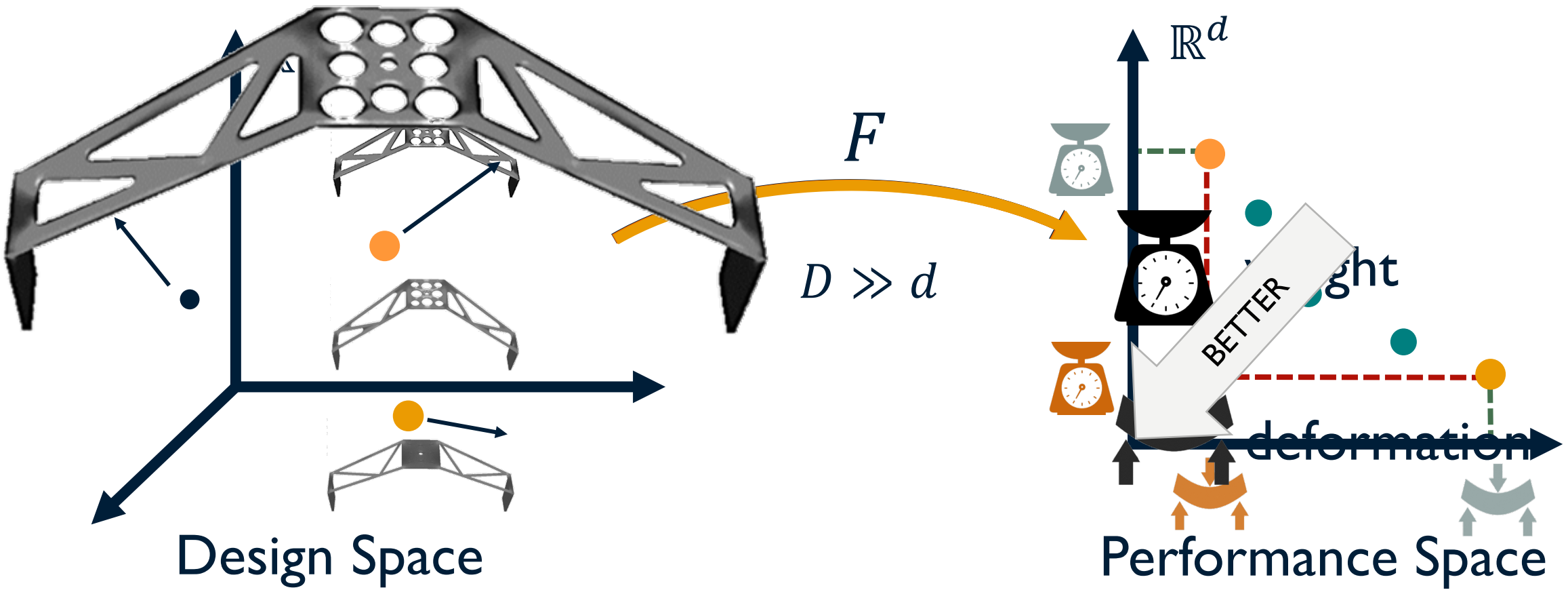[Bharaj et al. 2015]

[Li et al. 2016]

[Musialski et al. 2015]

[Coros et al. 2013]

[Umetani et al. 2014]

Single objective optimization!
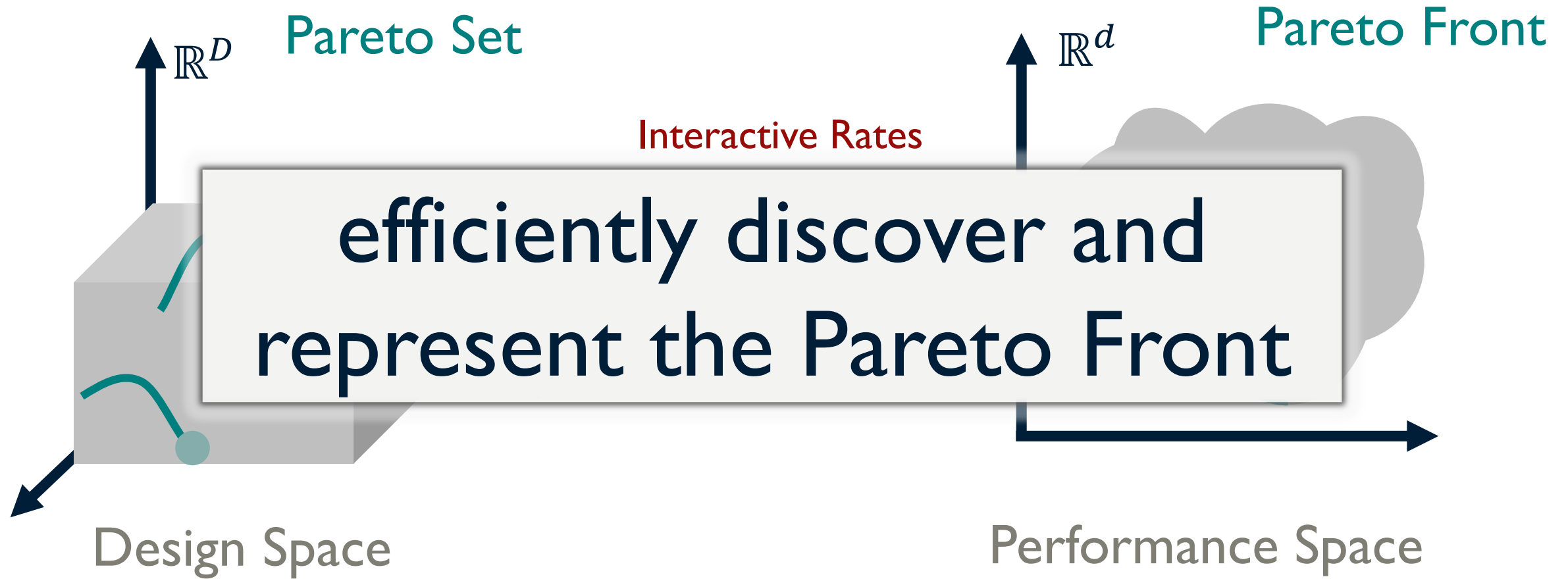
# Multiple Design Objectives



$F$

$D \gg d$

$\mathbb{R}^d$

BETTER

weight

deformation

Design Space

Performance Space

# Space of Optimal Solutions
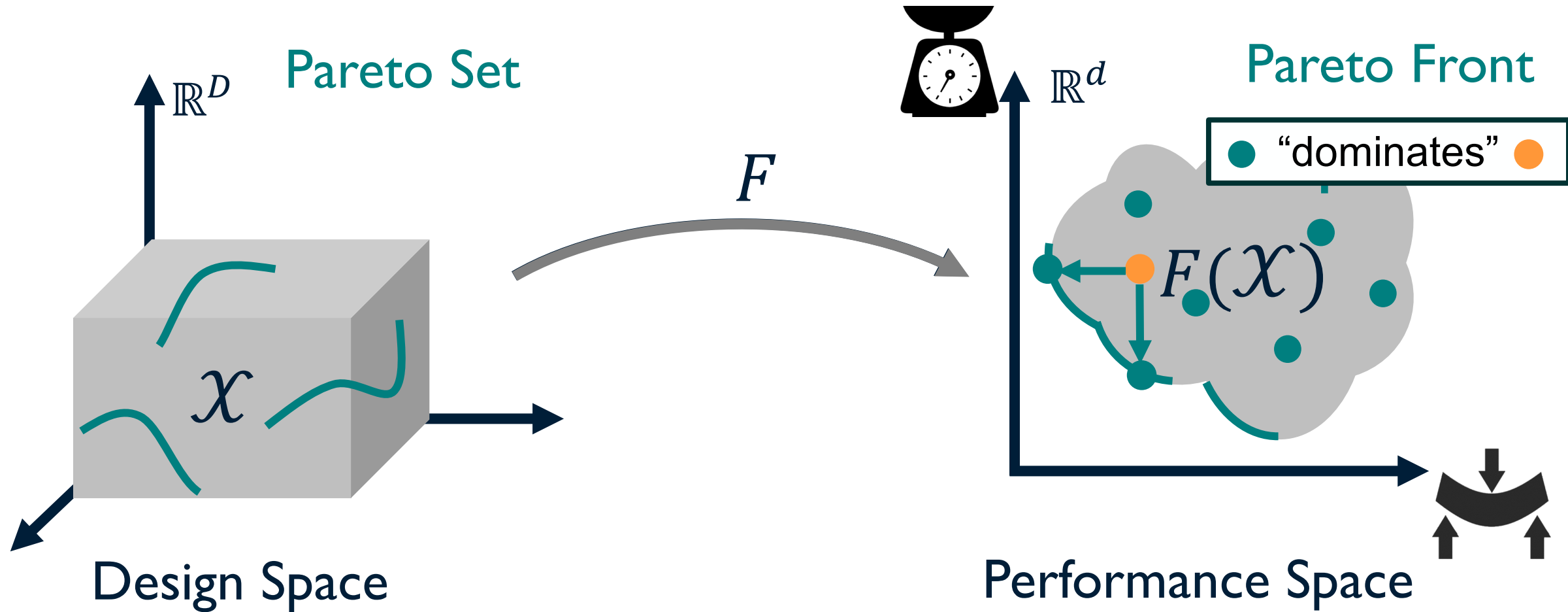


**Pareto Optimality:** Non-dominated Points

# Exploration of Design Trade-Offs

$\mathbb{R}^D$ Pareto Set

$\mathbb{R}^d$ Pareto Front

Interactive Rates

efficiently discover and represent the Pareto Front

Design Space

Performance Space

# Pareto Front Discovery: Challenges



$\mathbb{R}^D$ Pareto Set

Design Space

$\mathbb{R}^d$ Pareto Front
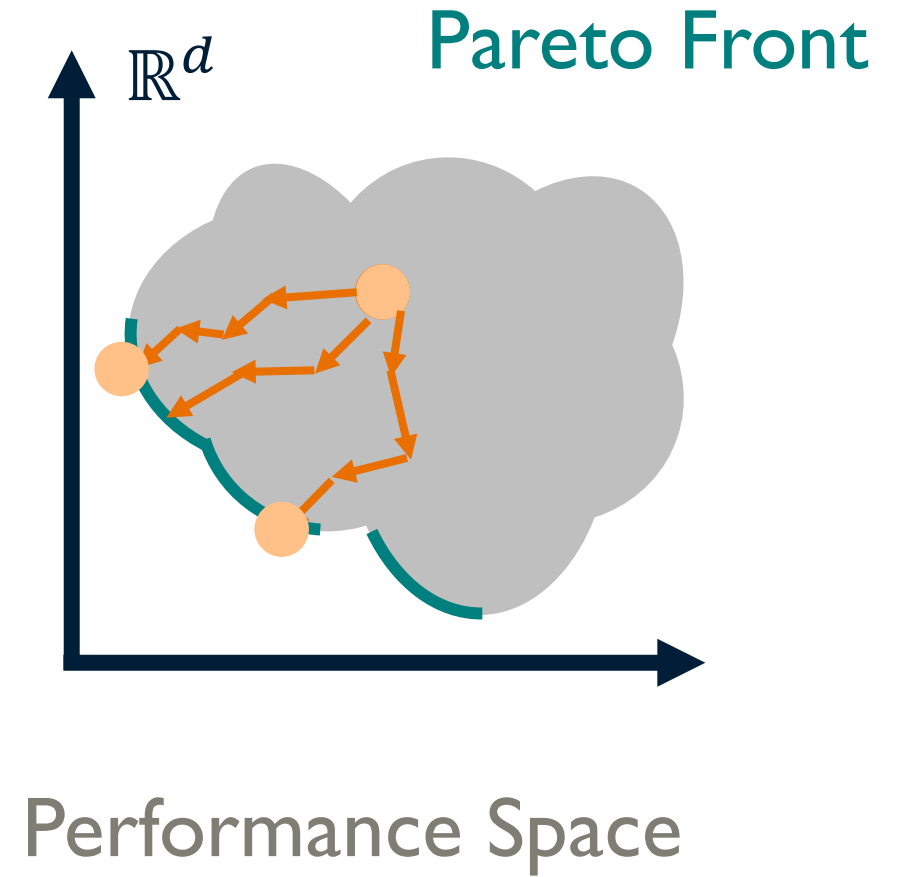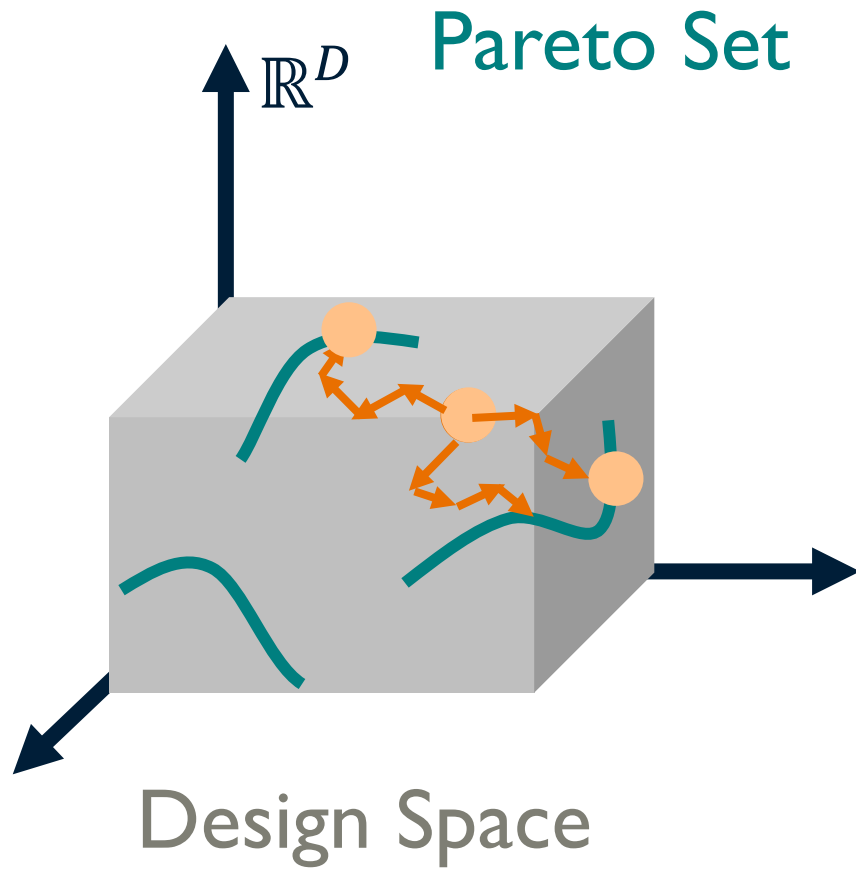
Performance Space

# Pareto Front Discovery: Previous Work



$\mathbb{R}^D$ Pareto Set

Design Space

$\mathbb{R}^d$ Pareto Front
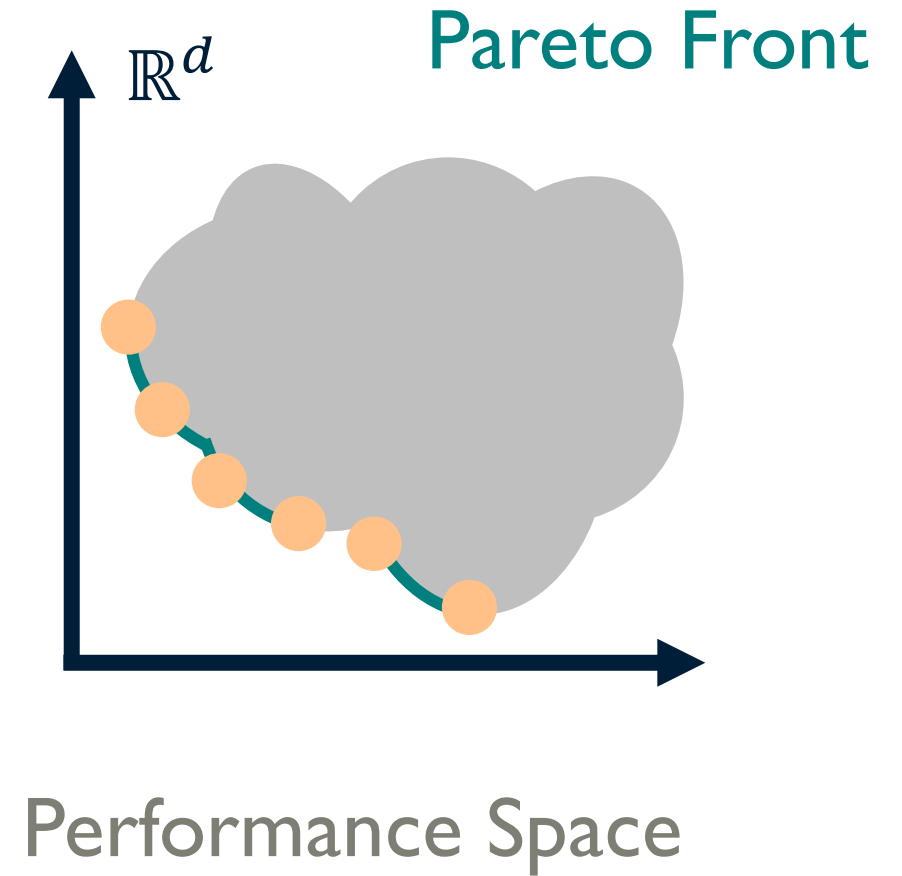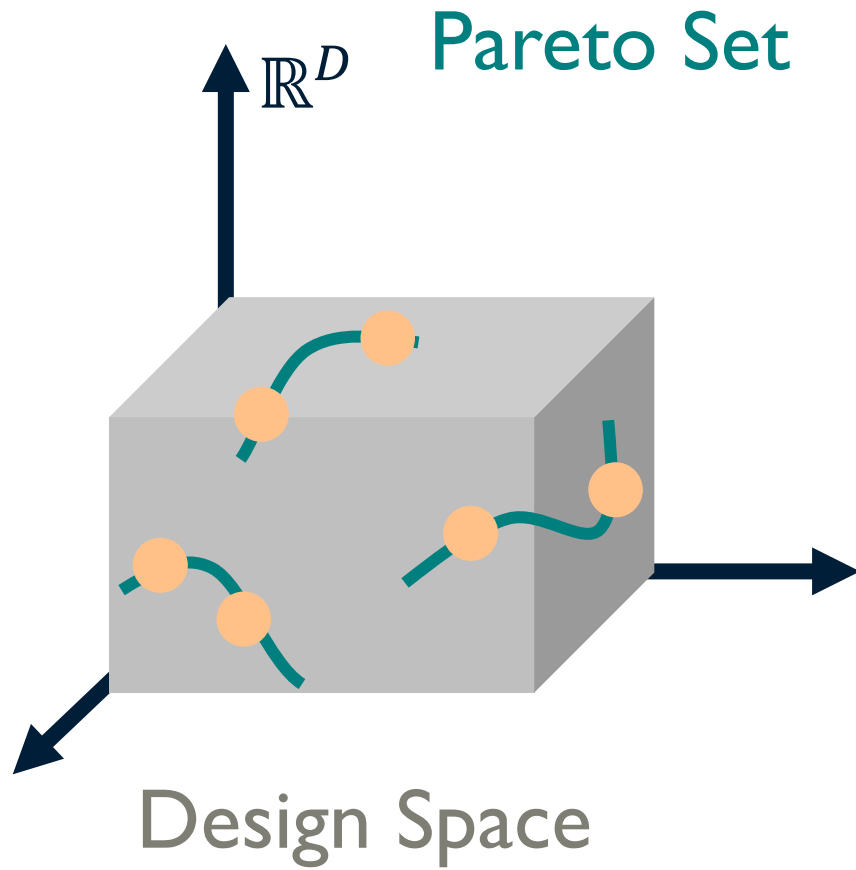
Performance Space

[Deb et al. 2017] (survey)

# Pareto Front Discovery: Our Approach

# Set of Manifolds

Pareto Set

$\mathbb{R}^D$

Pareto Front

$\mathbb{R}^d$

Design Space

Affine Subspaces
(Bounded, $d - 1$ dimensional)

Performance Space

Manifolds
(Bounded, $d - 1$ dimensional)

# Method
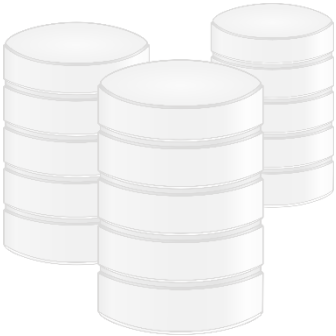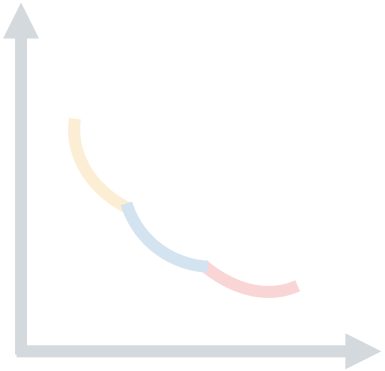


Performance Space

**Algorithm 1** Pareto set discovery given performance metrics $F$ and design constraints that define $\mathcal{X}$.

1: **procedure** PARETOFRONTDISCOVERY($\mathcal{X}$, $F$)
2:      $B$: performance buffer array
3:      $B(i) \leftarrow \emptyset, \forall i$
4:      **do**
5:          $\mathbf{x}_s^0, \ldots, \mathbf{x}_s^{N_s} \leftarrow$ stochasticSampling($B$, $F$, $\mathcal{X}$)
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10:             updateBuffer($B$, $F(M^i)$)
11:          **if** buffer not updated on past $N_t$ iterations **then**
12:             **break**
13:      **while** within computation budget
14:      **return** $B$

Performance Space

**Local Expansion**     Discovery Algorithm     Data-Structure     Sparse Approximation
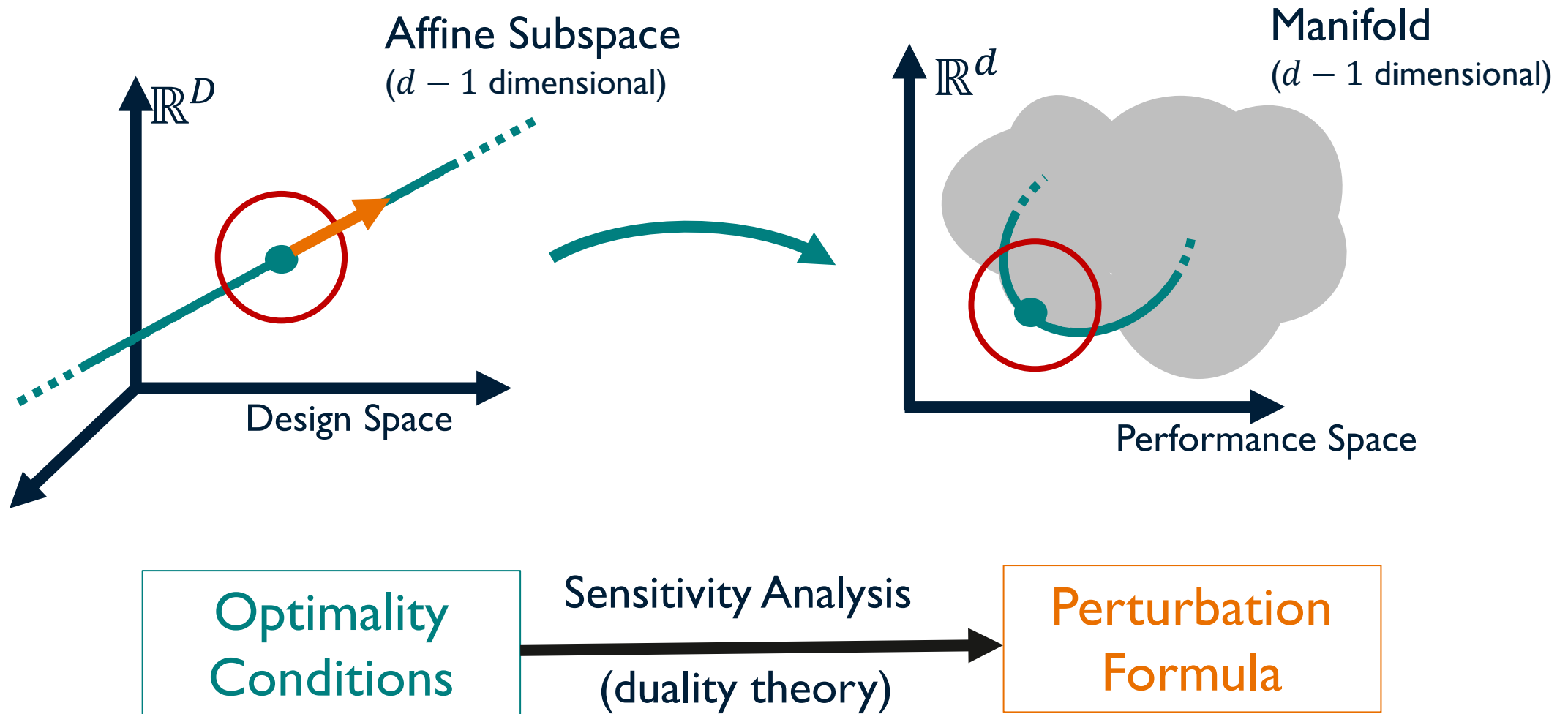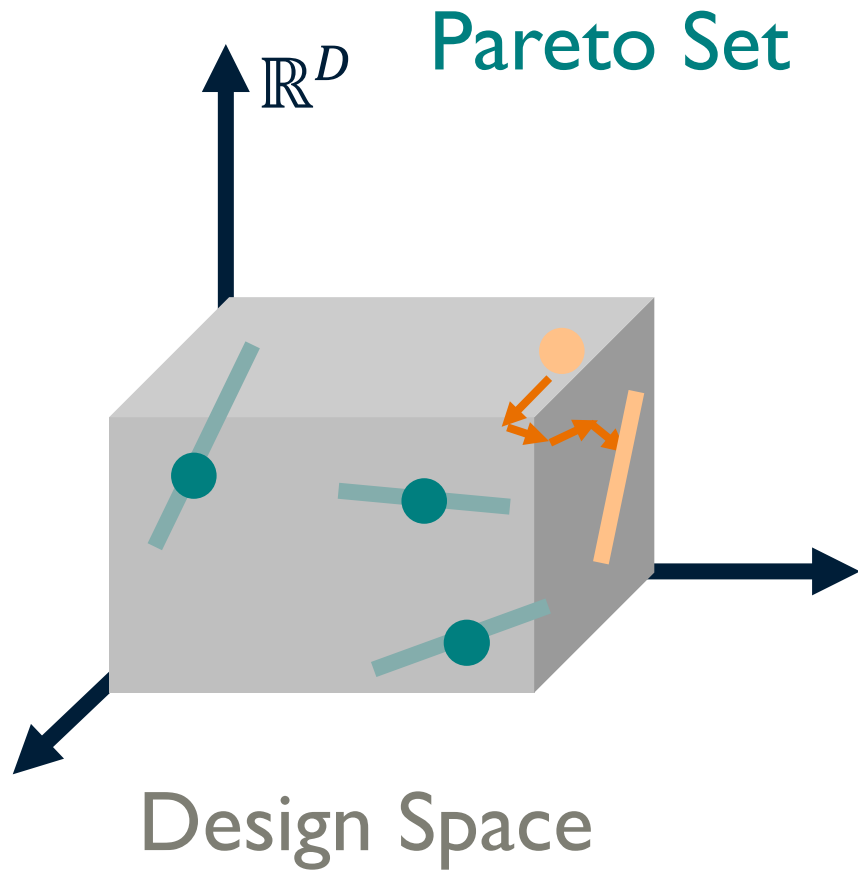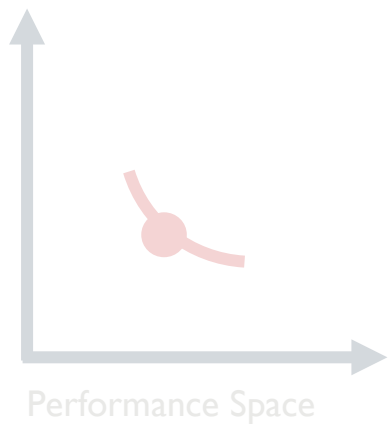
# First Order Approximation
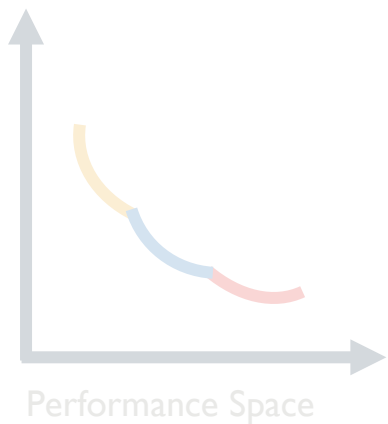
# Pareto Front Discovery

# Method



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Performance Space

Performance Space

Local Expansion

**Discovery Algorithm**

Data-Structure
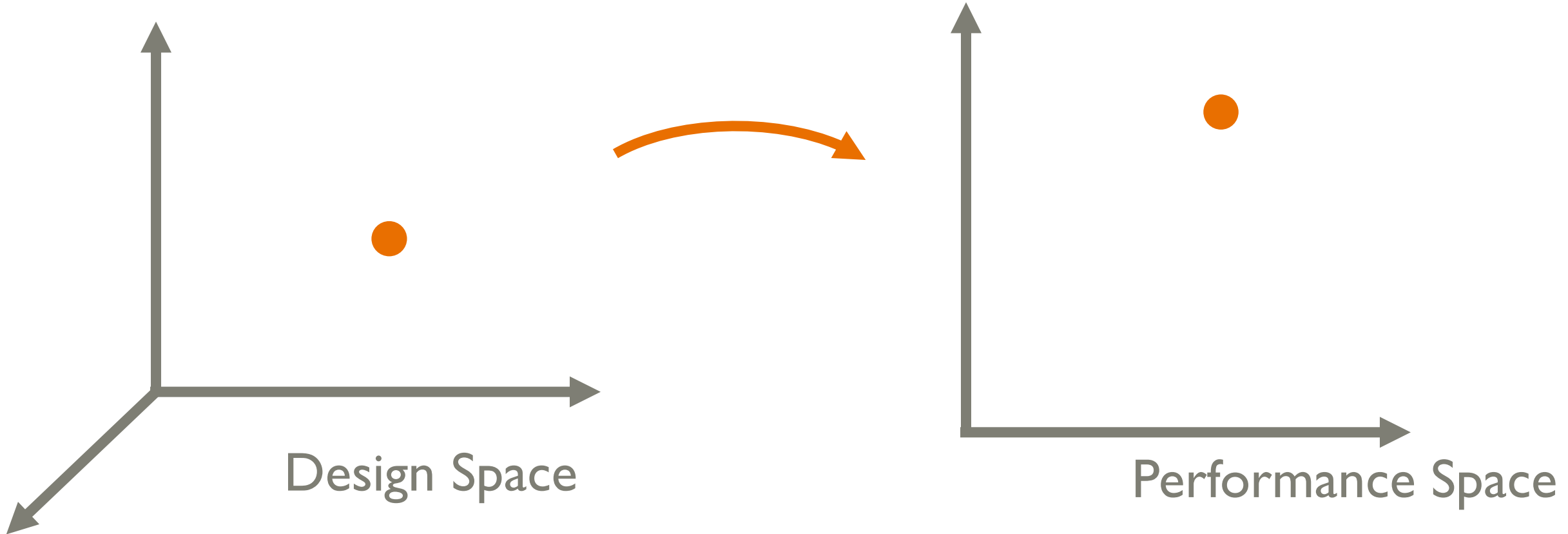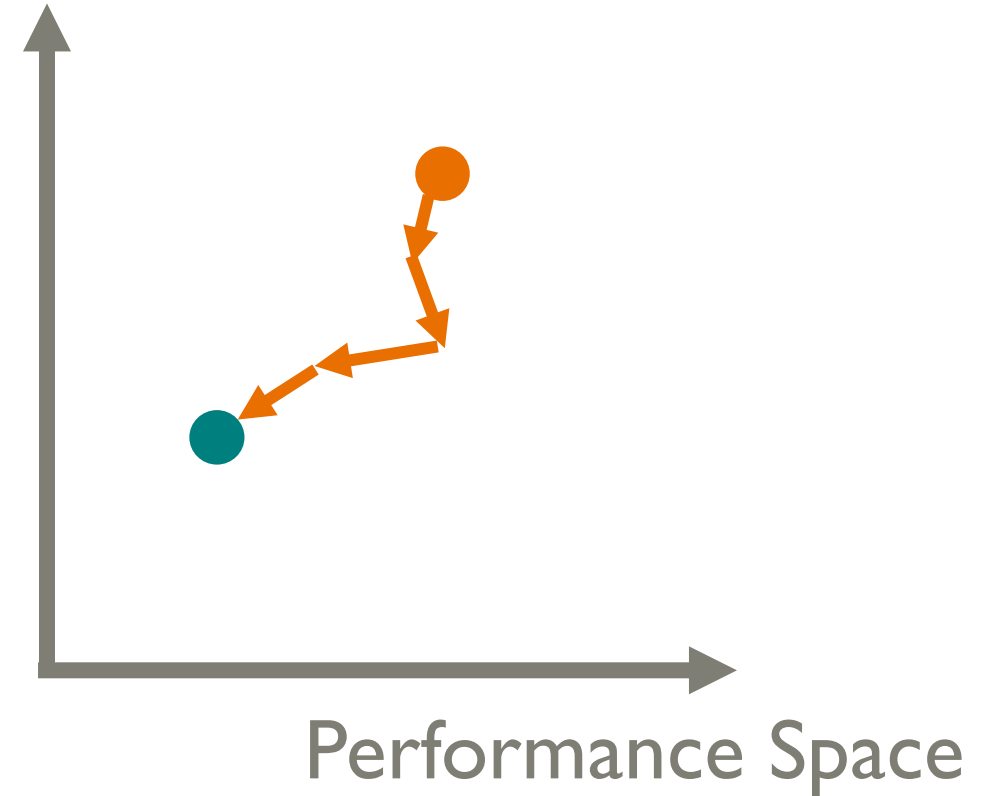
Sparse Approximation

# Algorithm Overview

**Step 1: Stochastic Sampling**
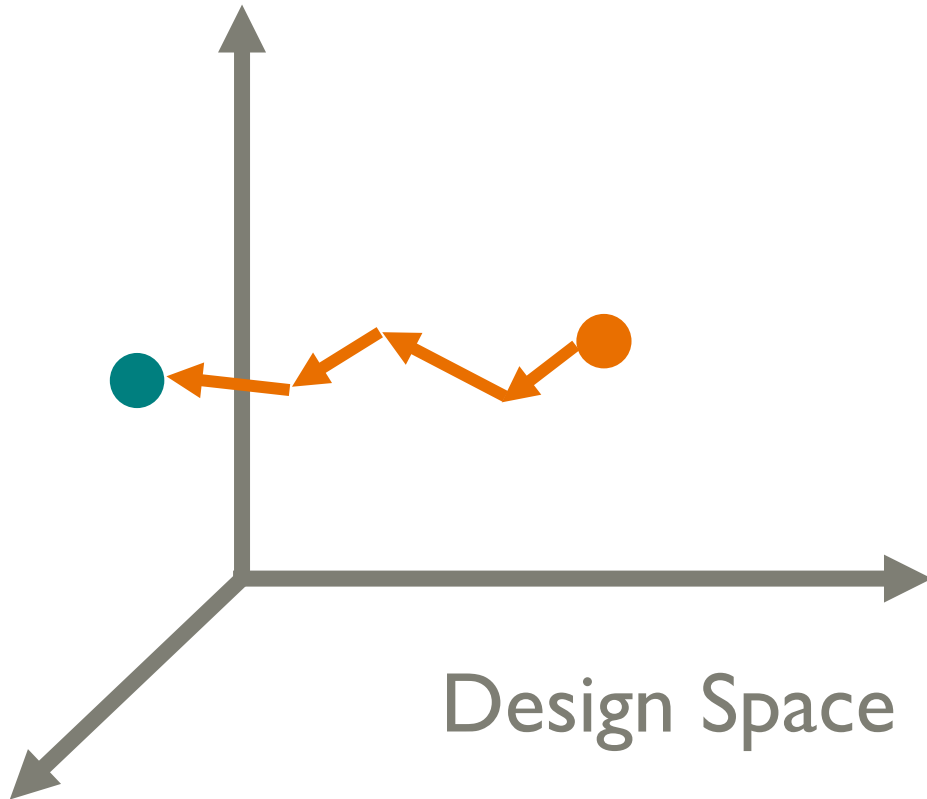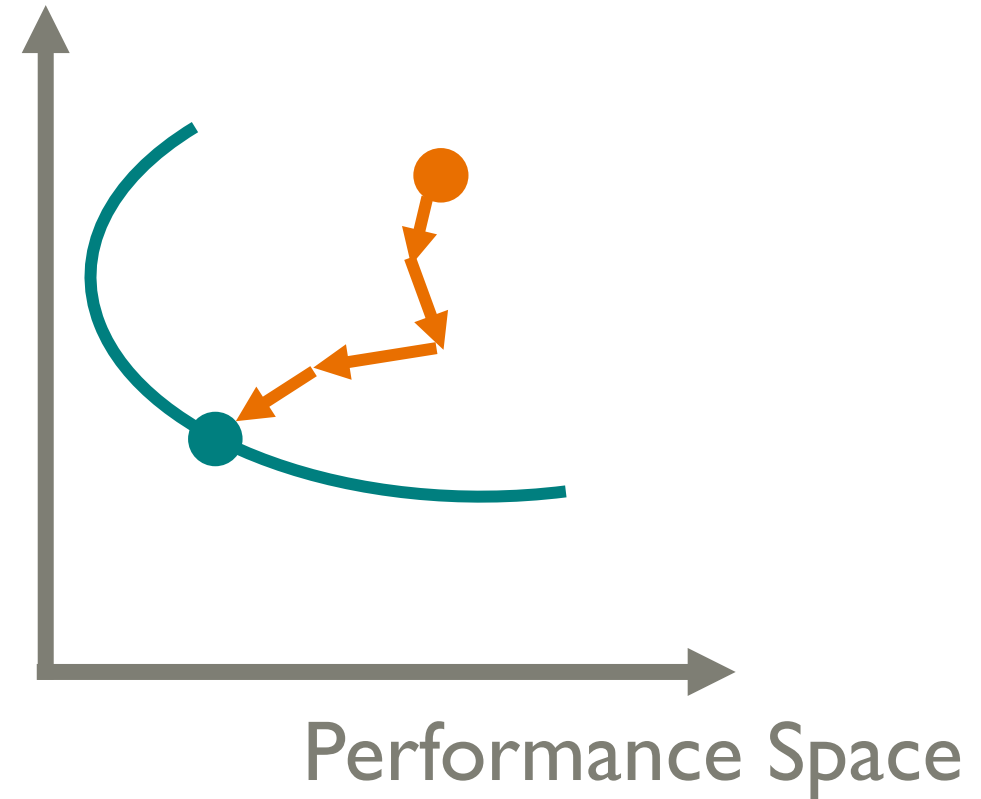Step 2: Local Optimization
Step 3: Local Expansion

Design Space

Performance Space

# Algorithm Overview

Design Space

Performance Space

# Algorithm Overview

Step 1: Stochastic Sampling
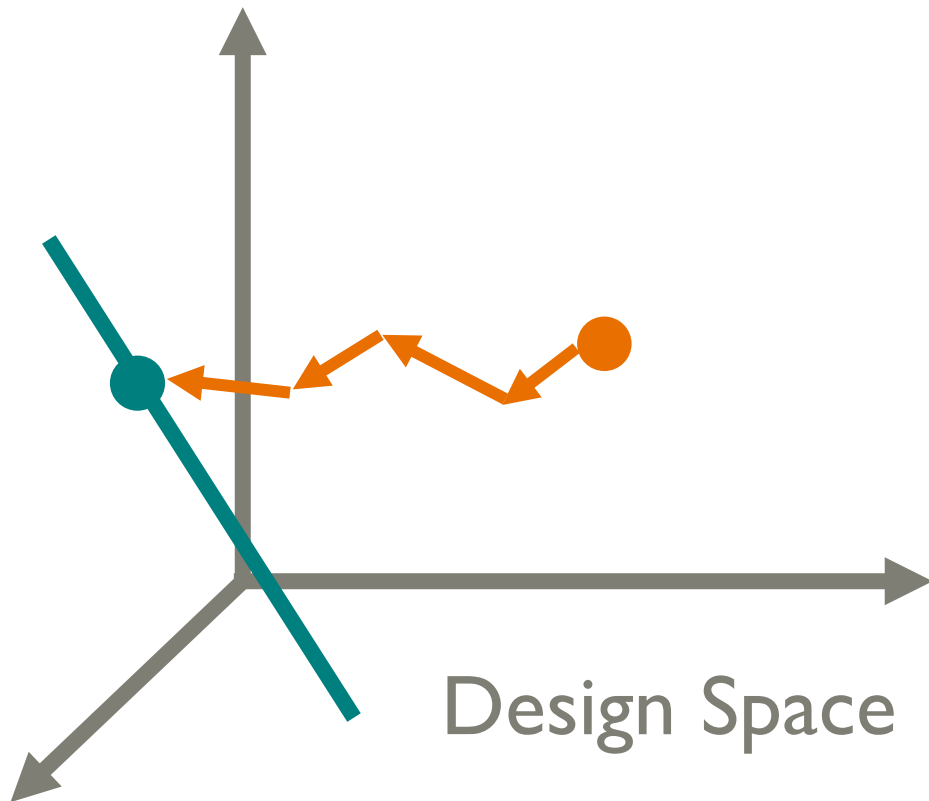Step 2: Local Optimization
**Step 3: Local Expansion**
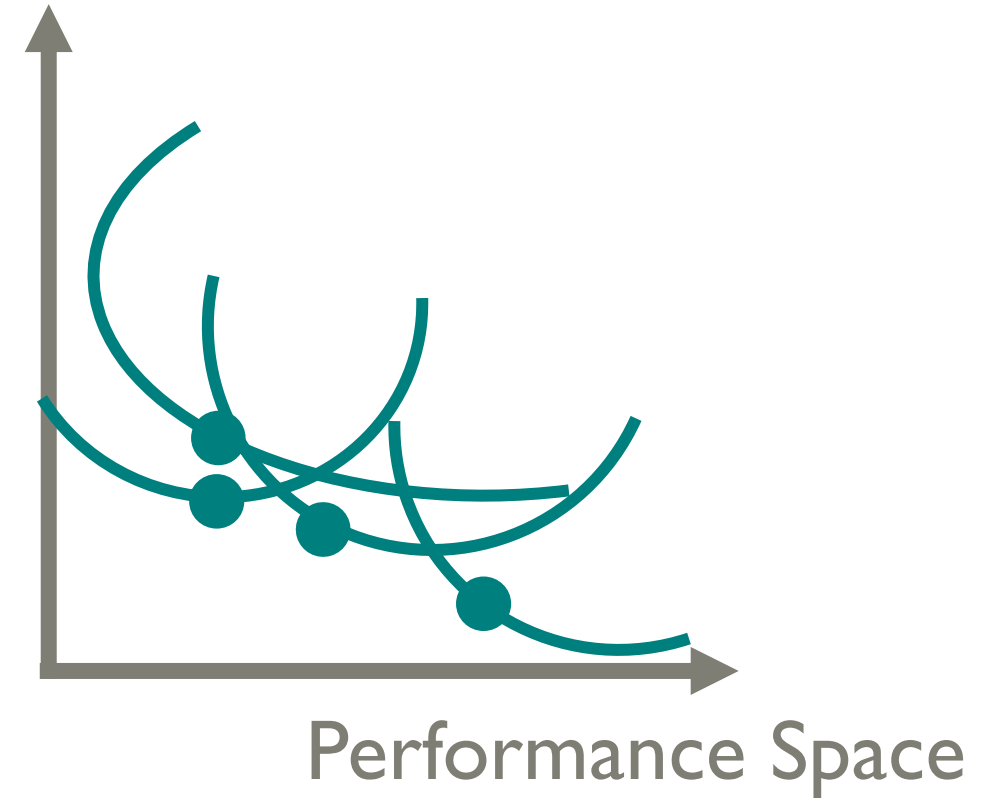


Design Space

Performance Space

# Algorithm Overview
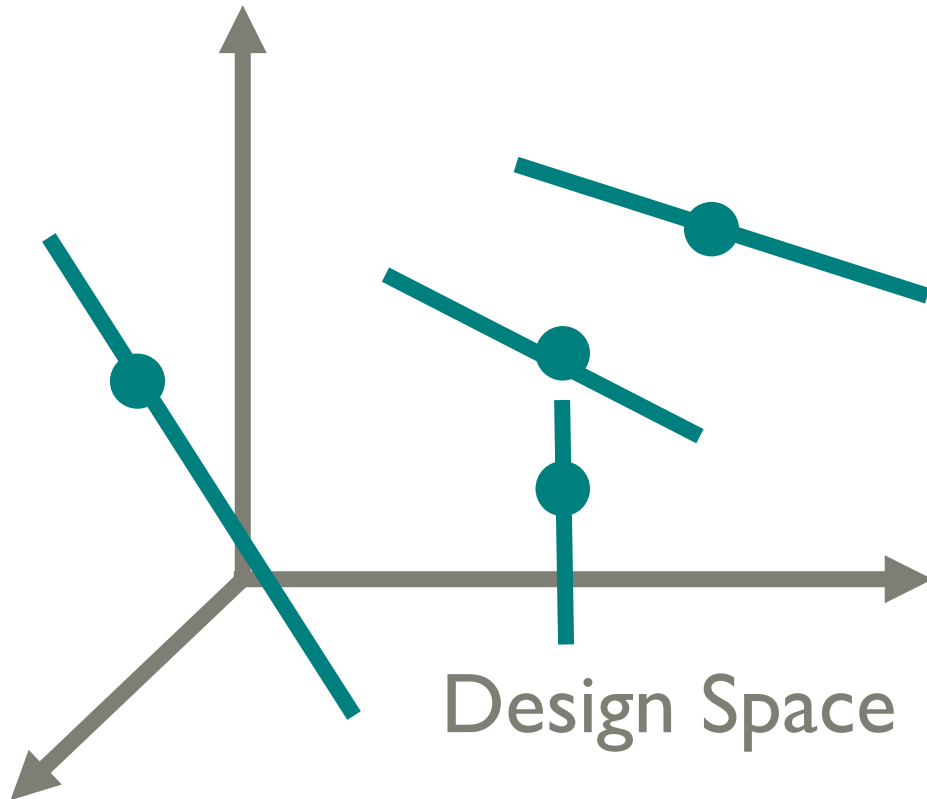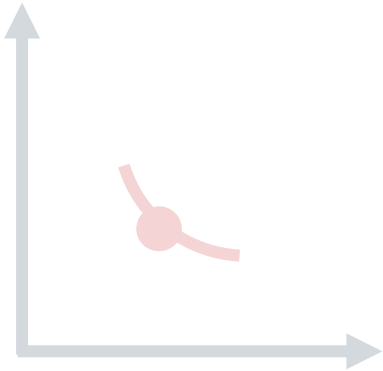
Step 1: Stochastic Sampling | Adaptive!
Step 2: Local Optimization | Direction Matters!
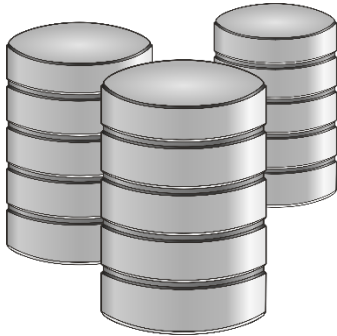Step 3: Local Expansion
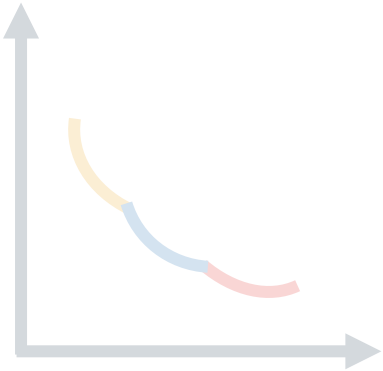
Design Space

Performance Space

# Method



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14:     **return** $B$

Performance Space

Performance Space

Local Expansion     Discovery Algorithm     Data-Structure     Sparse Approximation

# Data Structure



"Performance Buffer"

Pareto Front

Performance Space

$d_{\textcolor{orange}{\bullet}}$

$d_{\textcolor{teal}{\bullet}} < d_{\textcolor{orange}{\bullet}}$

$d_{\textcolor{teal}{\bullet}}$

Not Pareto:
$\textcolor{teal}{\bullet}$ "dominates" $\textcolor{orange}{\bullet}$

# Data Structure



Performance Buffer

Design Space
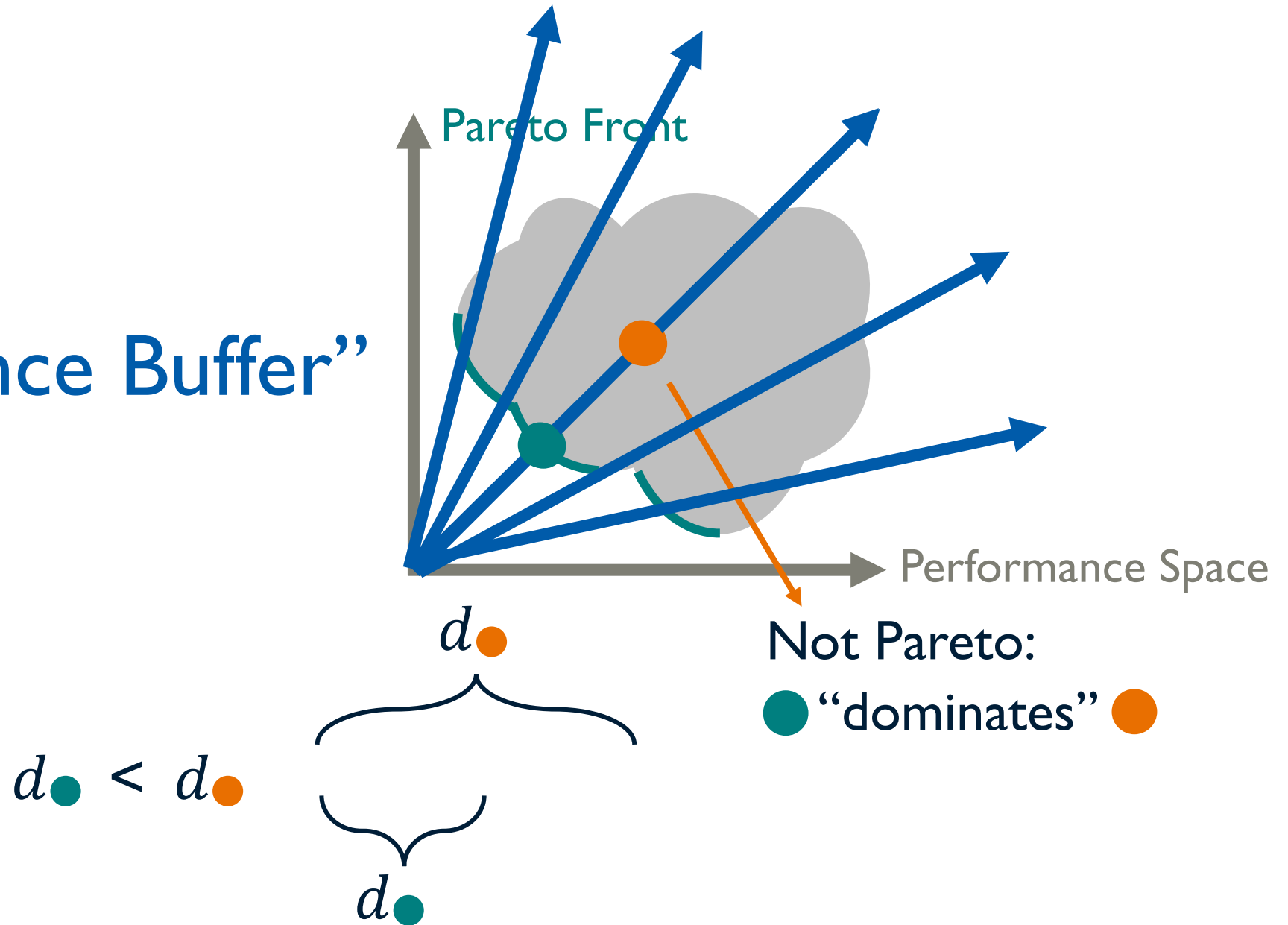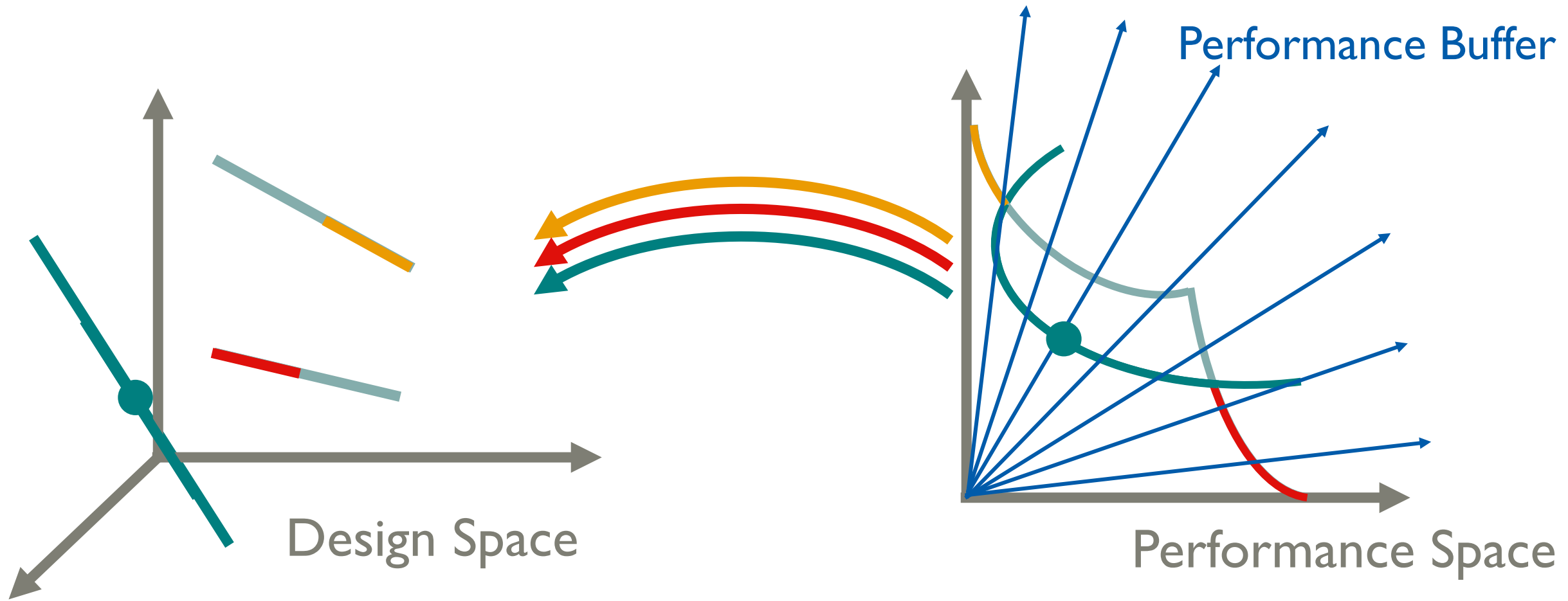
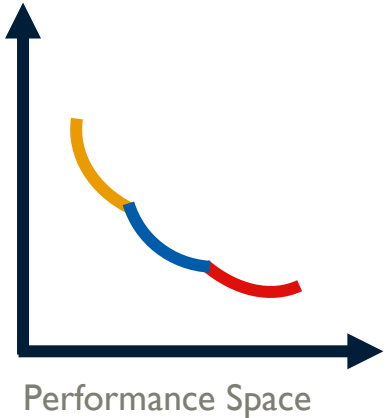Performance Space

# Method
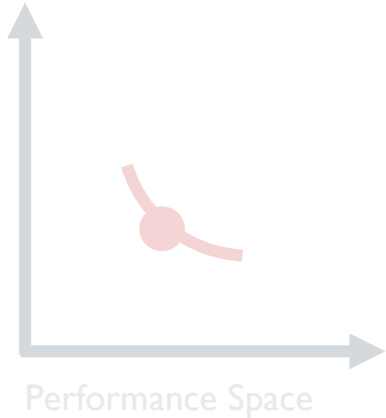


Local Expansion



**Algorithm 1** Pareto set discovery given performance metrics $F$ and design constraints that define $\mathcal{X}$.

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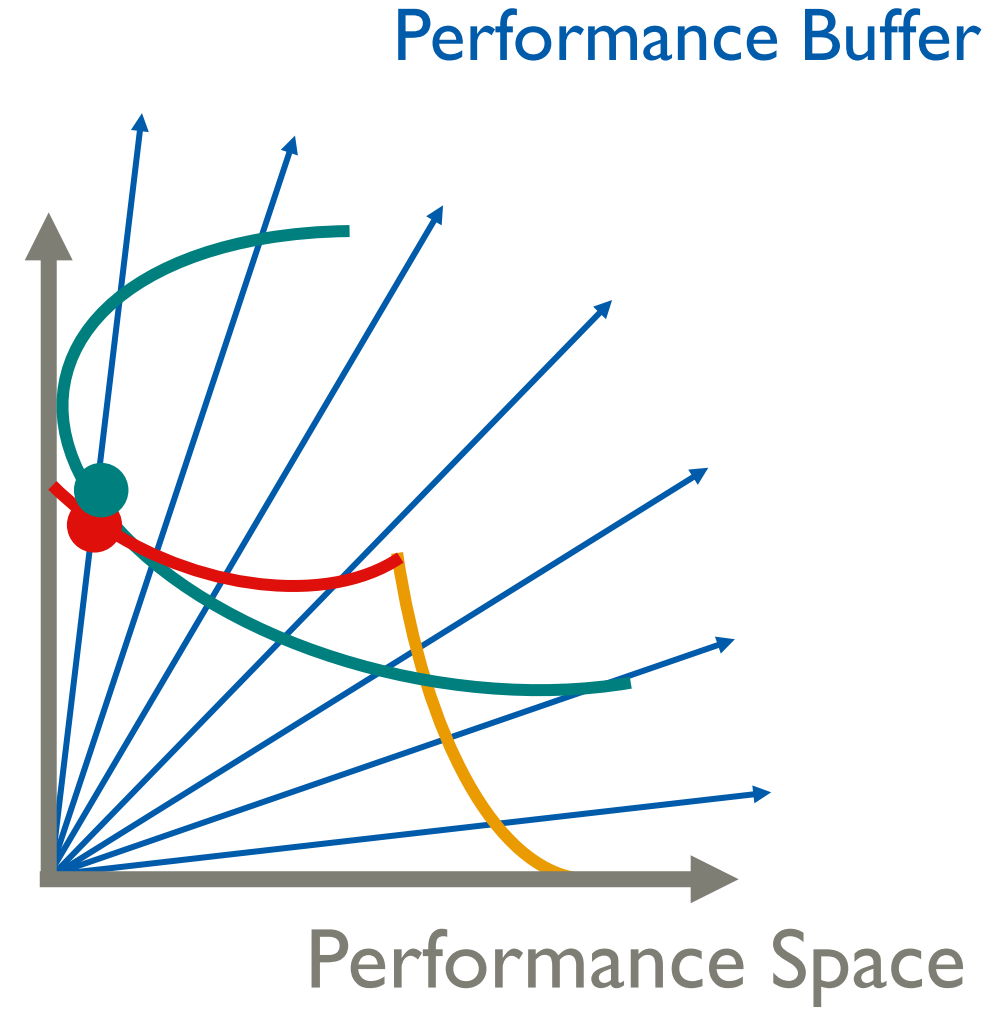14:     **return** $B$

Discovery Algorithm
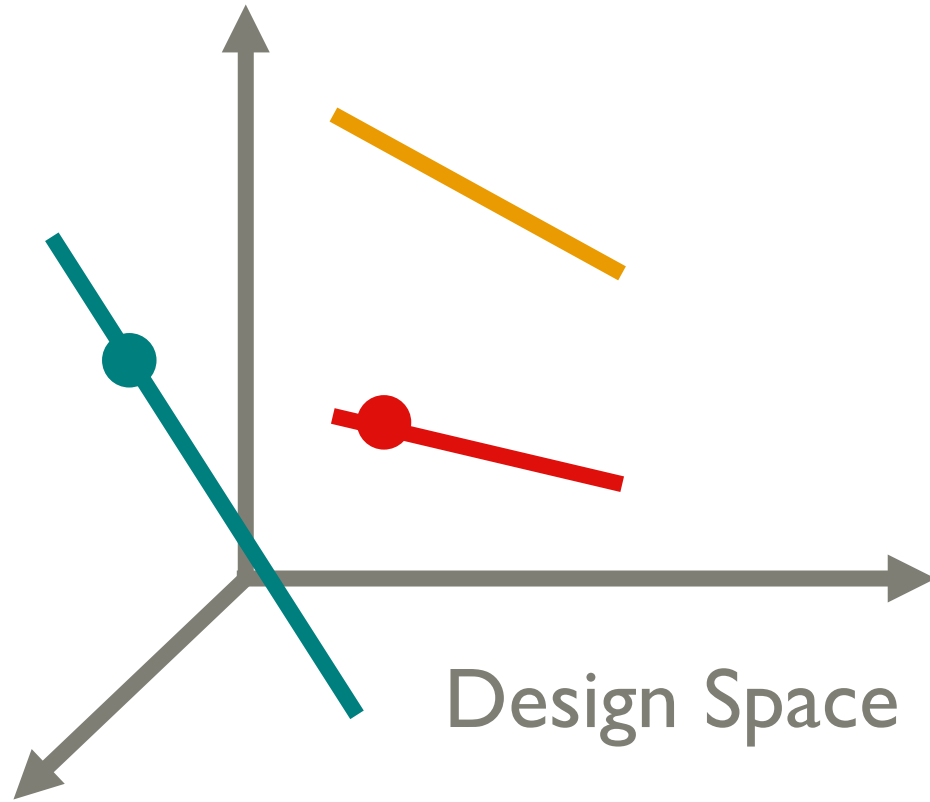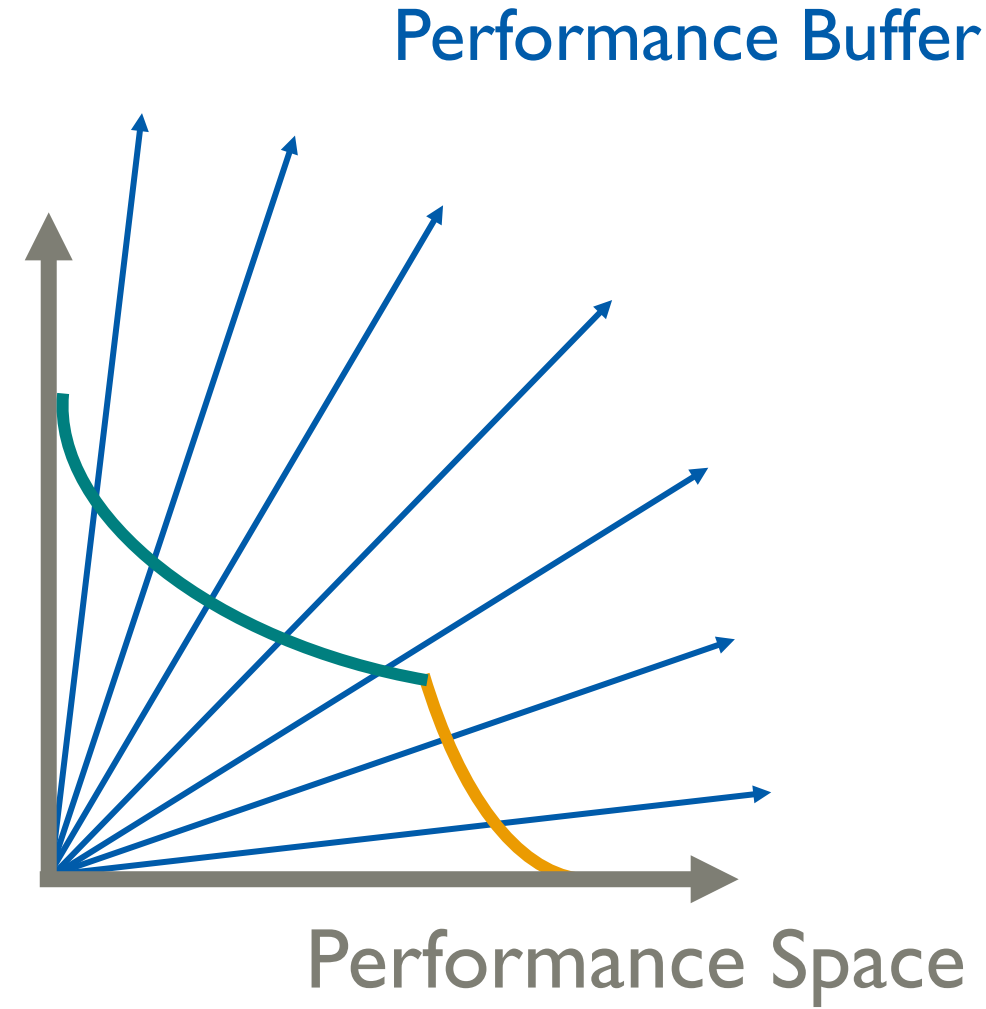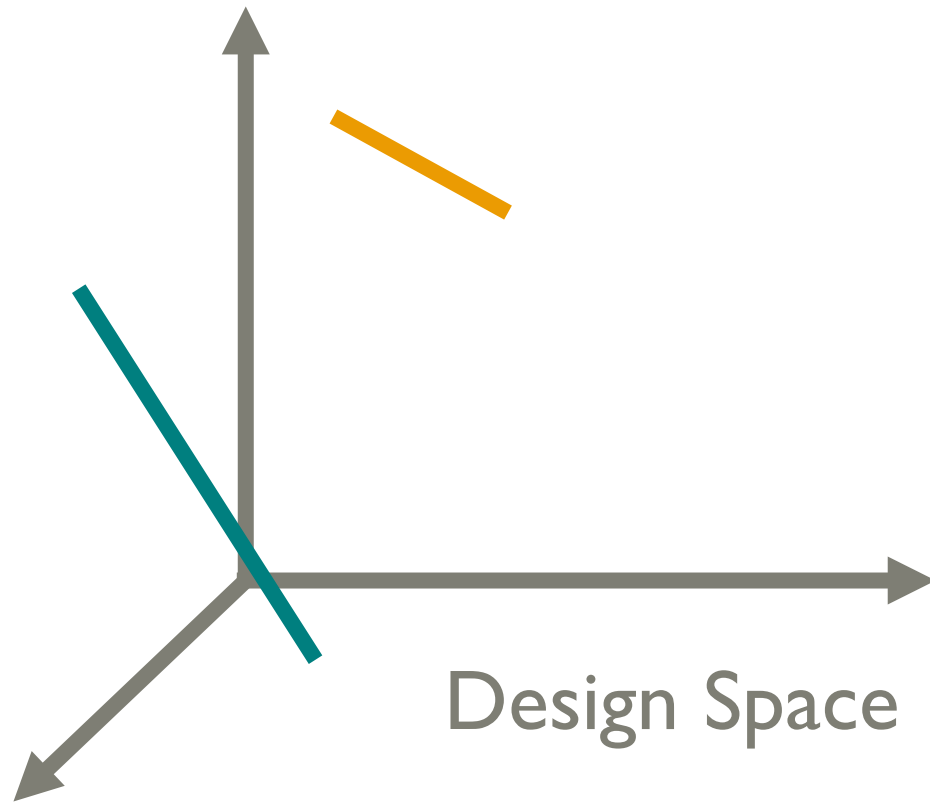


Data-Structure



Sparse Approximation

# Sparse Approximation



Performance Buffer
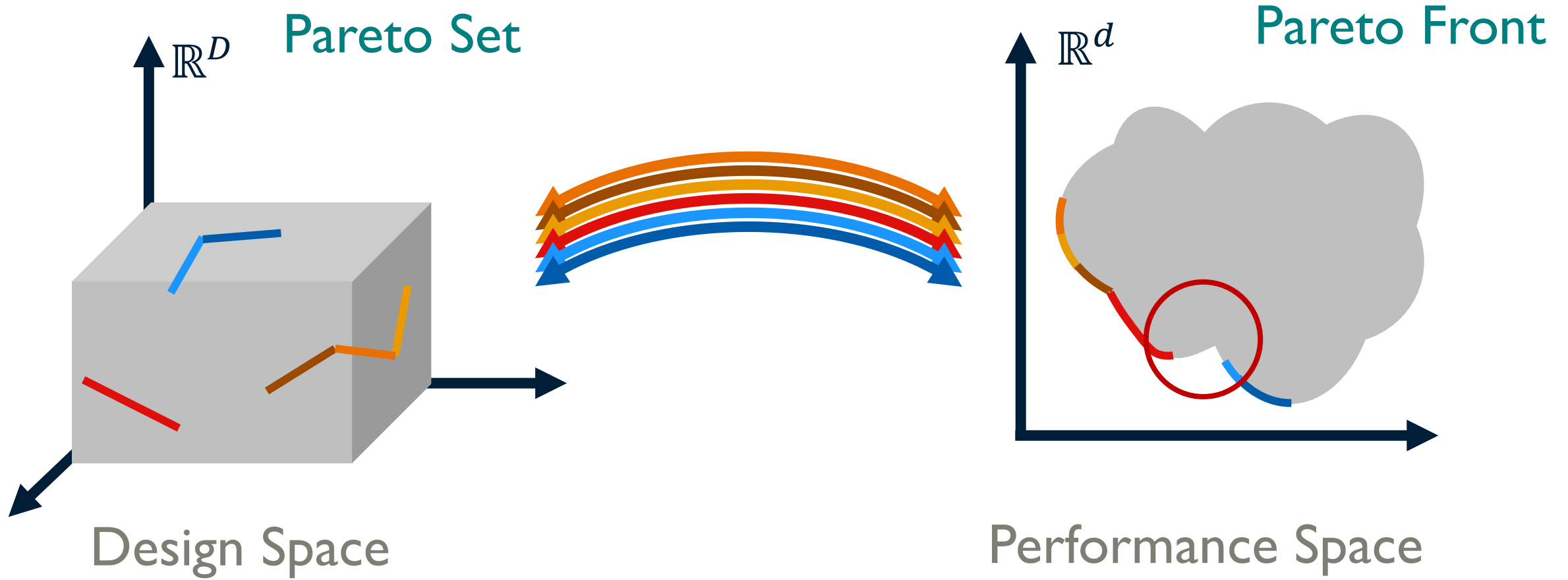
Design Space

Performance Space

# Sparse Approximation

Solution to Labeling problem:

Performance Buffer

Design Space

Performance Space

# Remove Non-Dominant Points



$\mathbb{R}^D$

Pareto Set

Design Space

$\mathbb{R}^d$

Pareto Front

Performance Space

# Remove Non-Dominant Points



$\mathbb{R}^D$

Pareto Set

Design Space

$\mathbb{R}^d$

Pareto Front
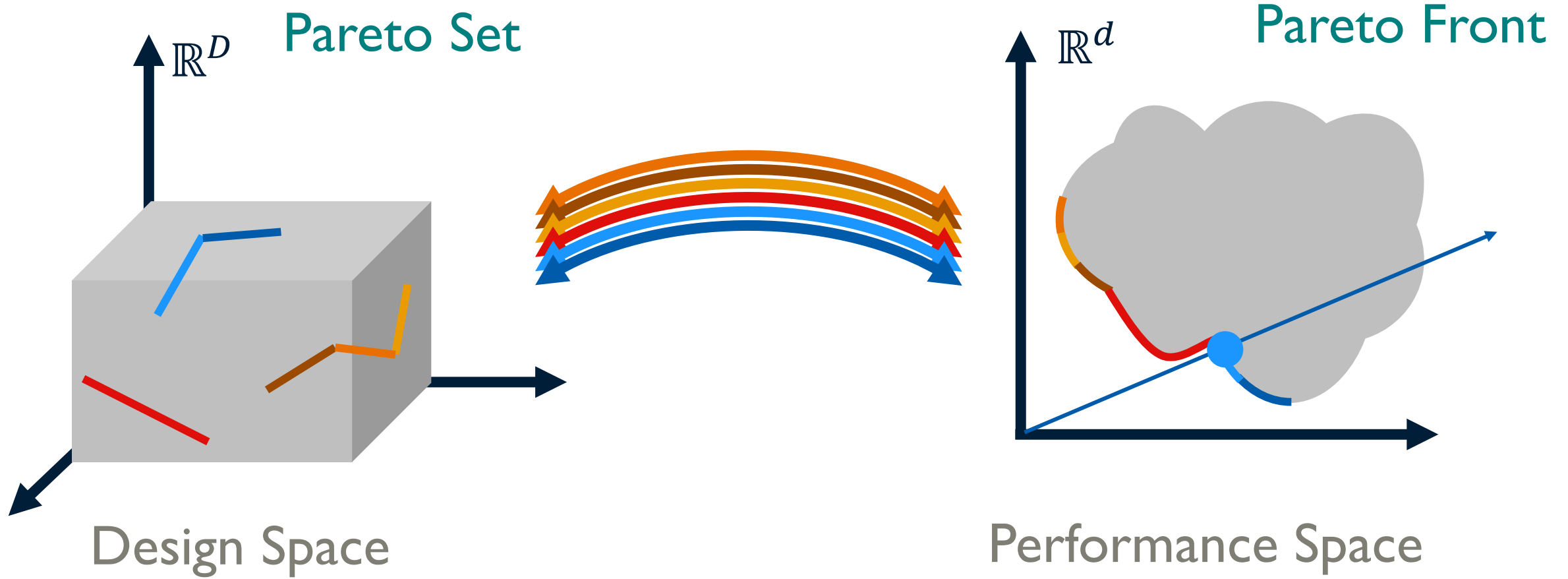
Performance Space

# Gaps in Performance Space
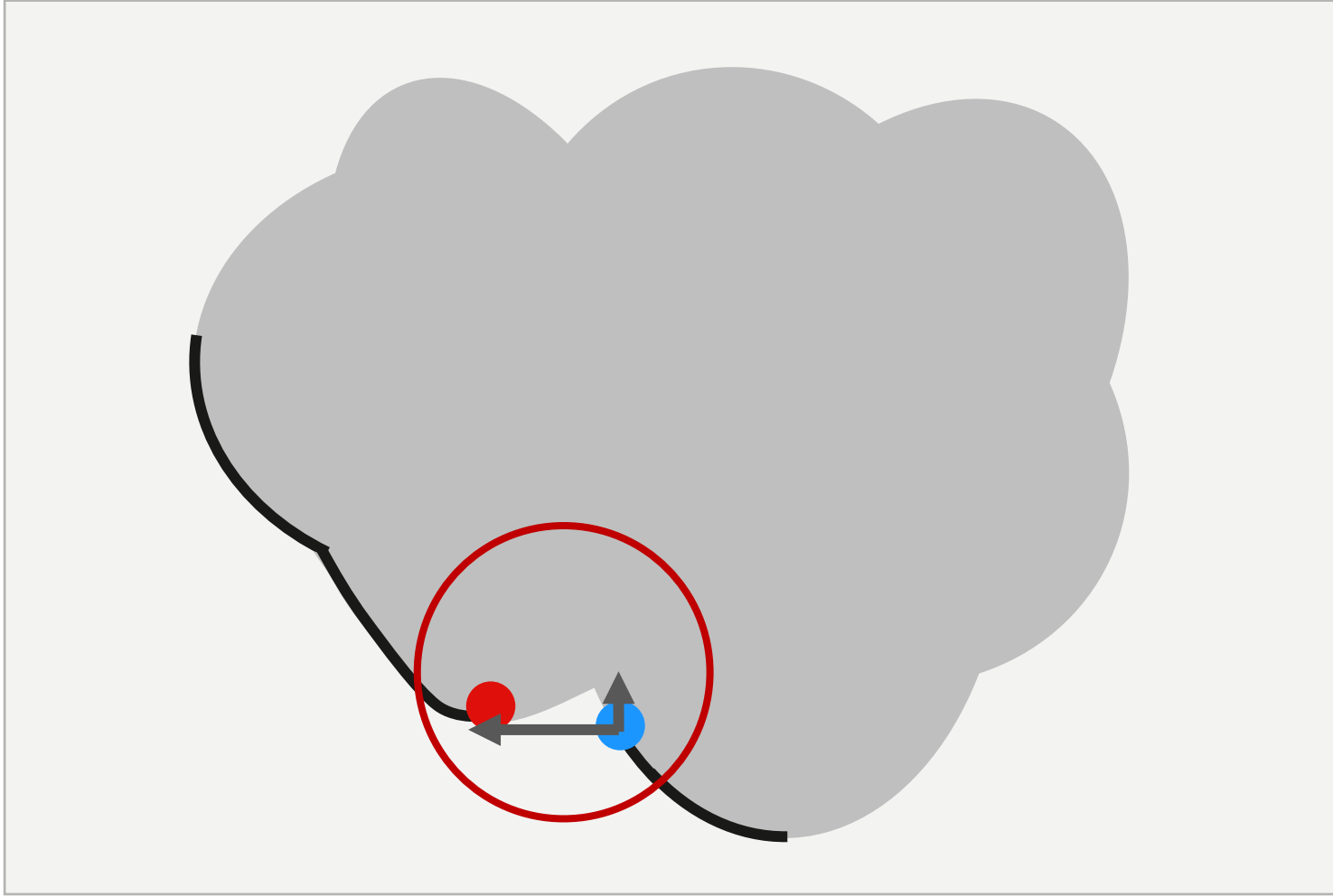
# Interactive Exploration Results

# Wrench (3 design variables, 3 performance metrics)



force/torque

mass

stress

Performance Space

mass

stress
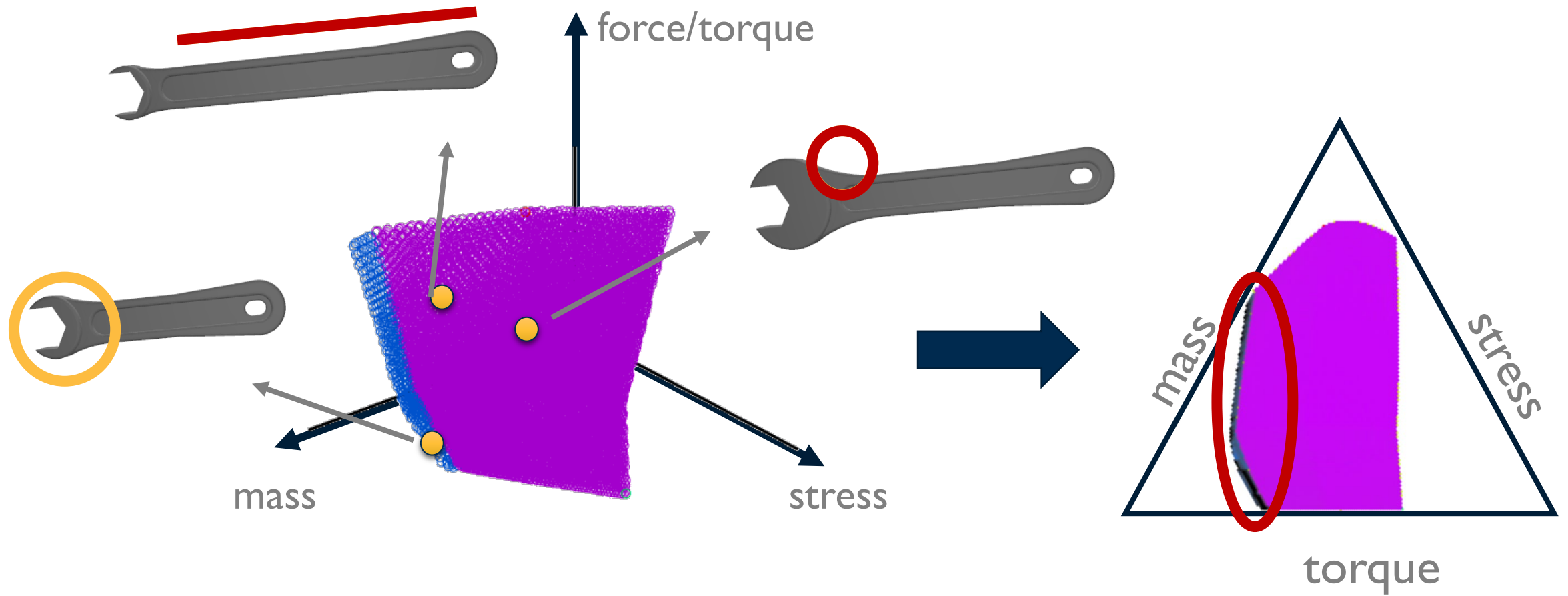
torque

# Wrench (3 design variables, 3 performance metrics)
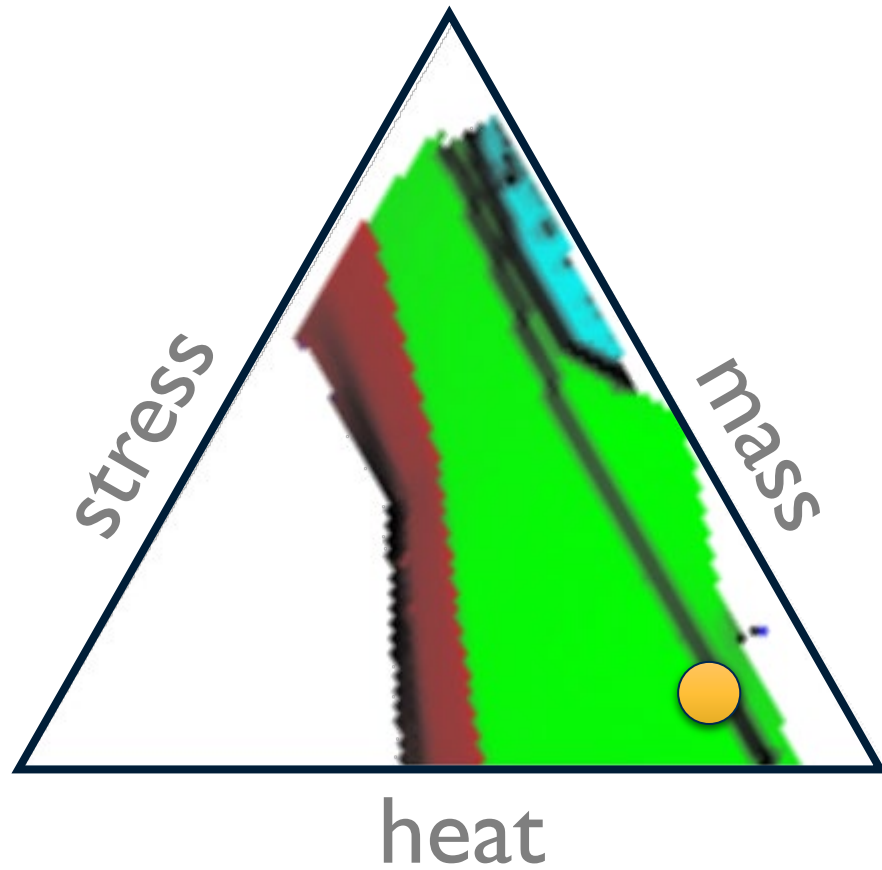
# Brake Hub (4 design variables, 3 performance metrics)
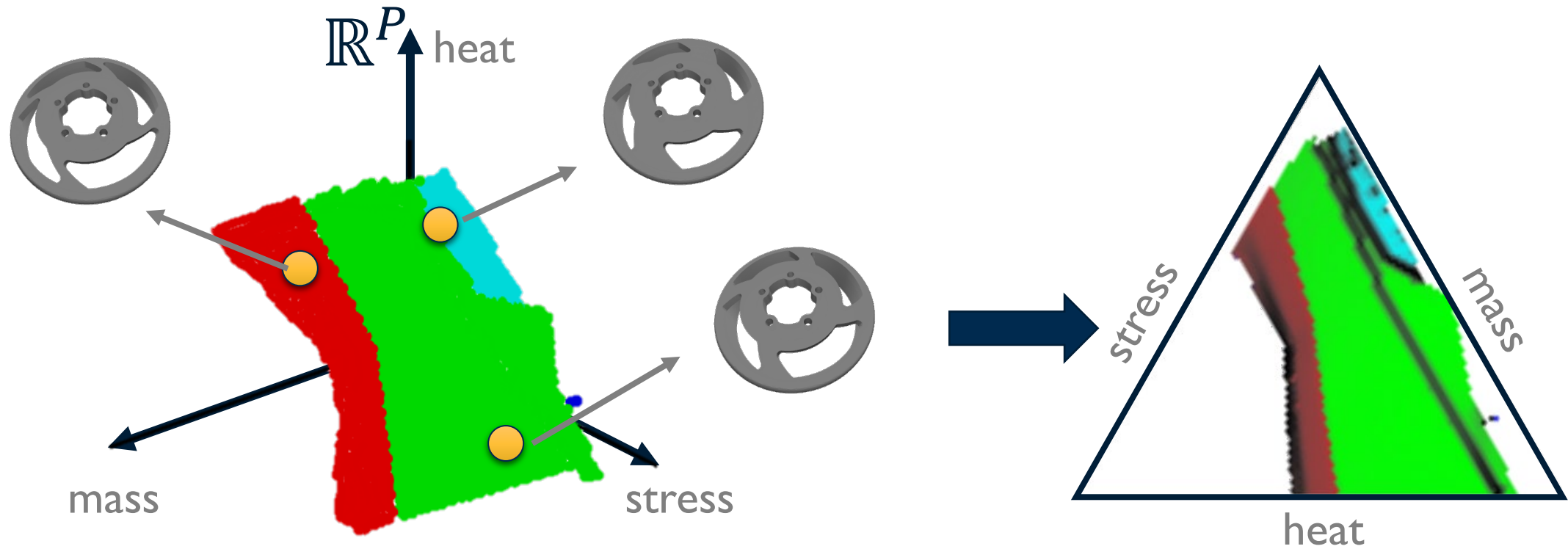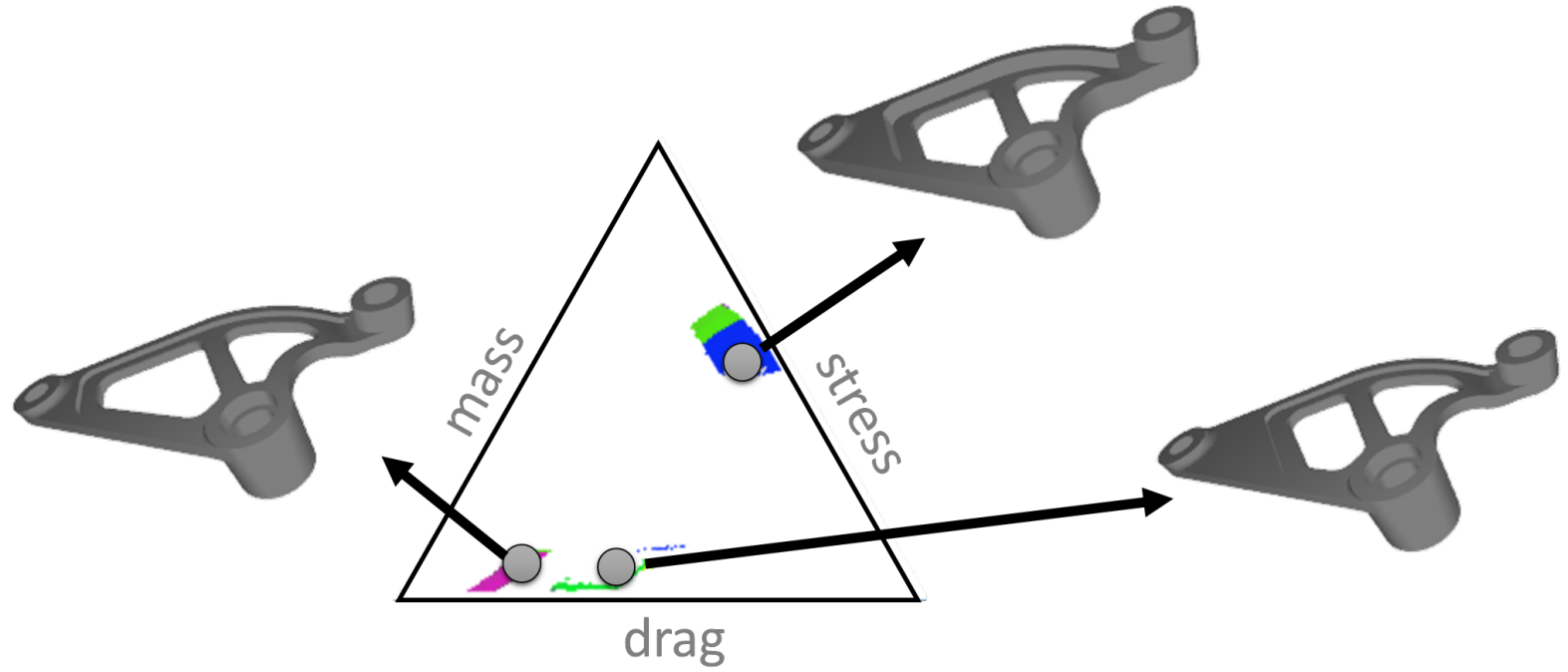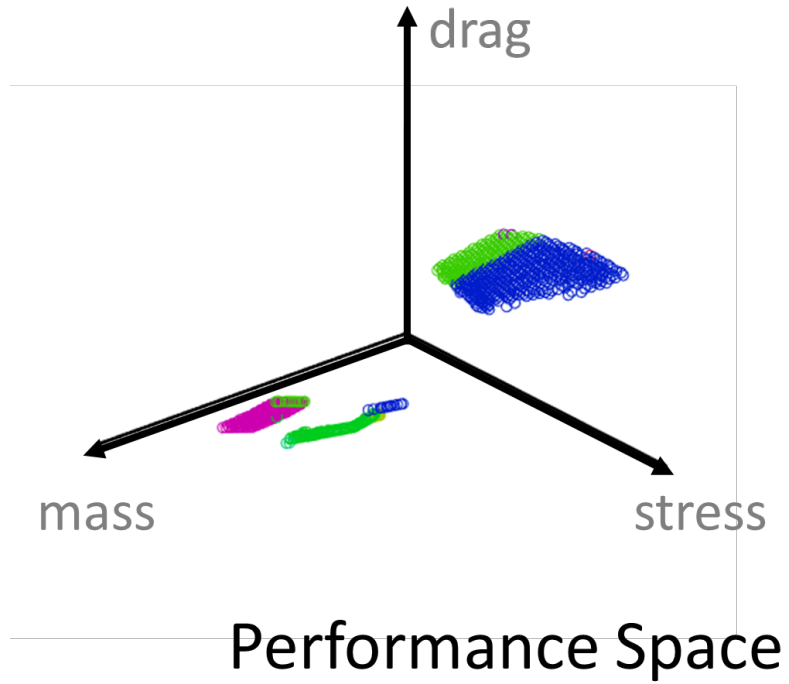


Performance Space

# Brake Hub (4 design variables, 3 performance metrics)
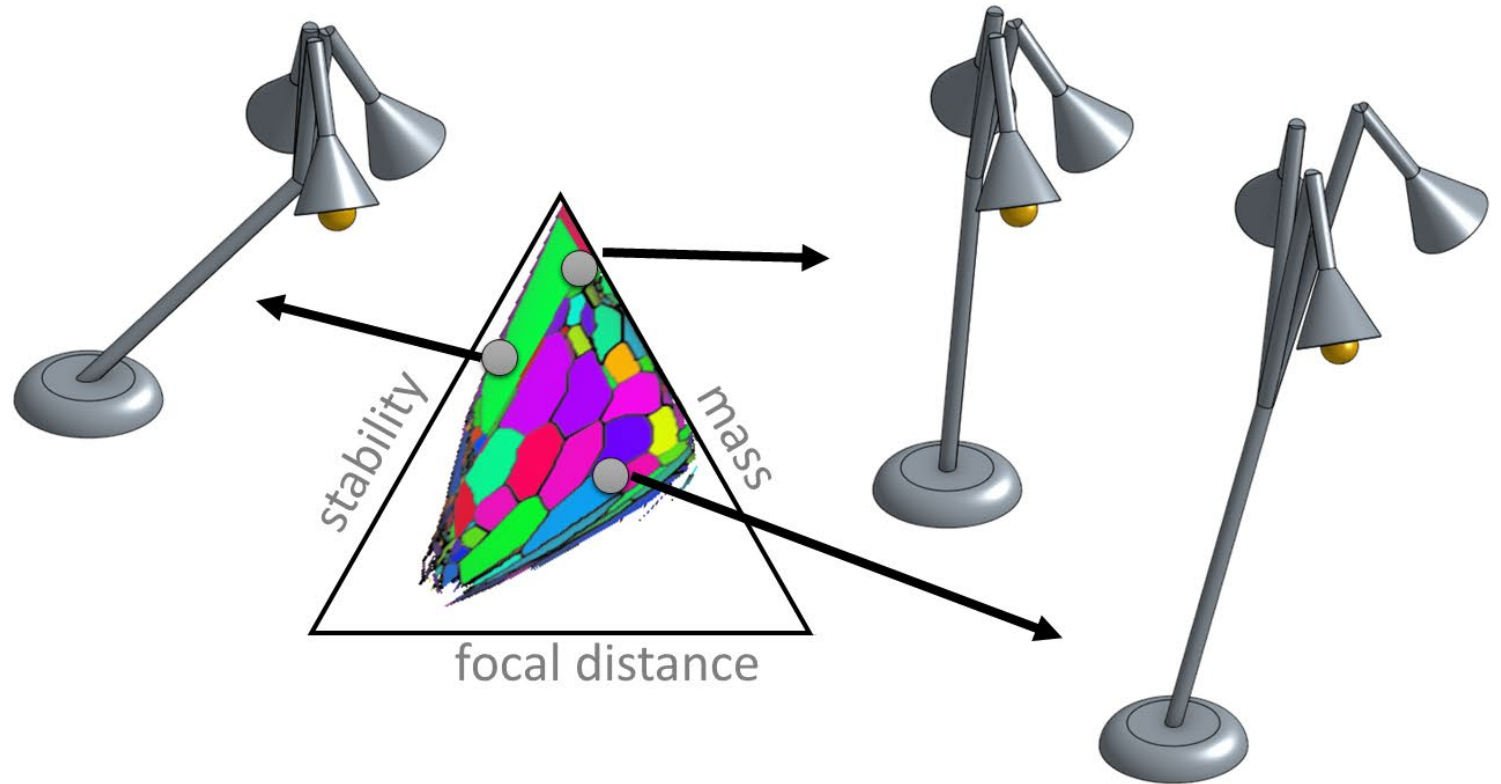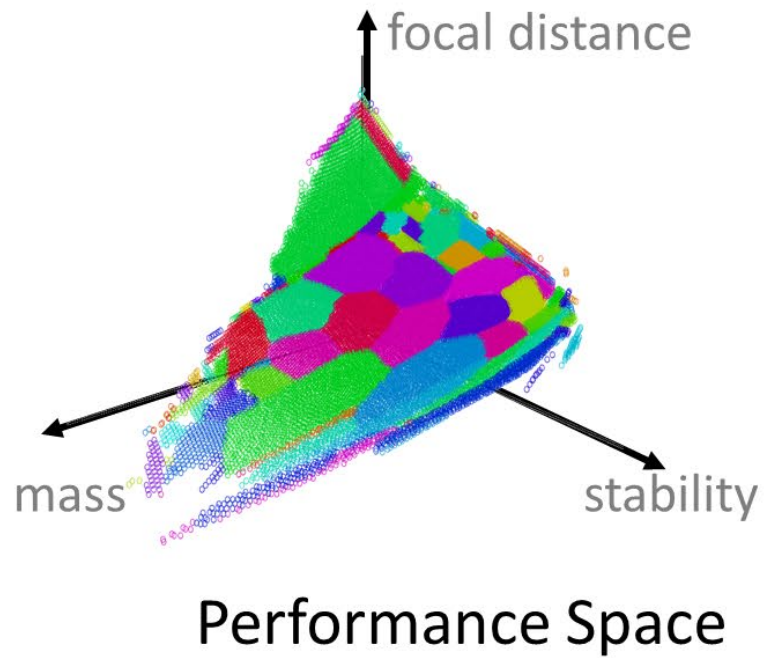
# Transition Example
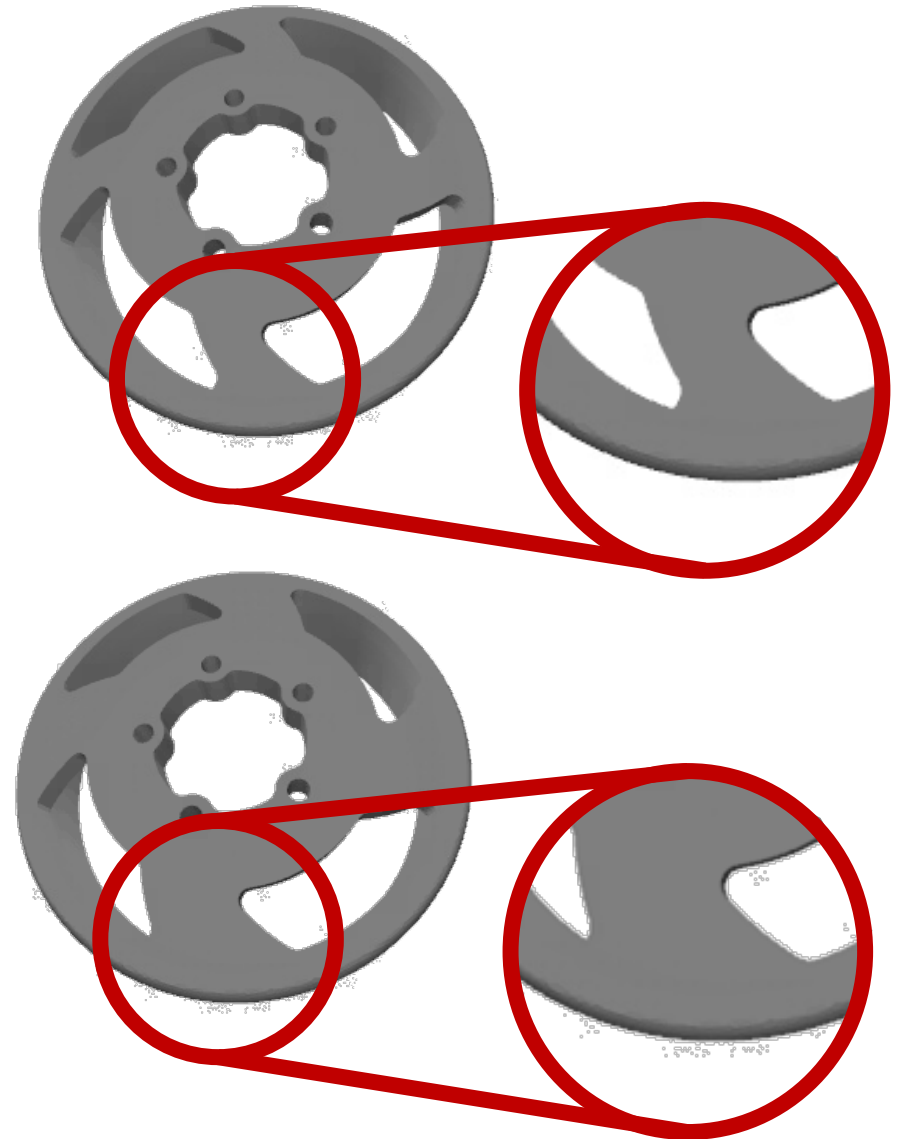
# Bike Frame (4 design variables, 3 performance metrics)
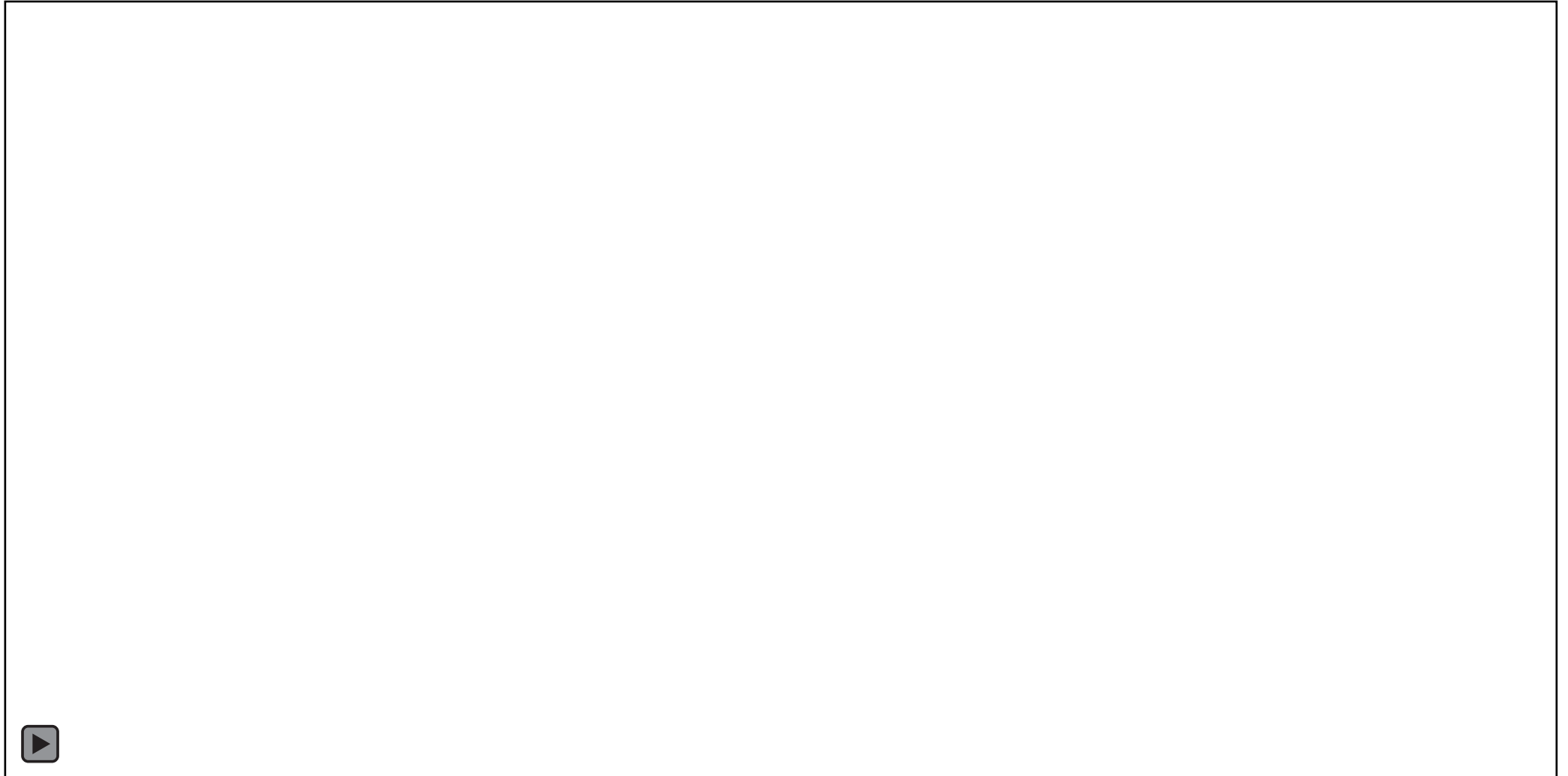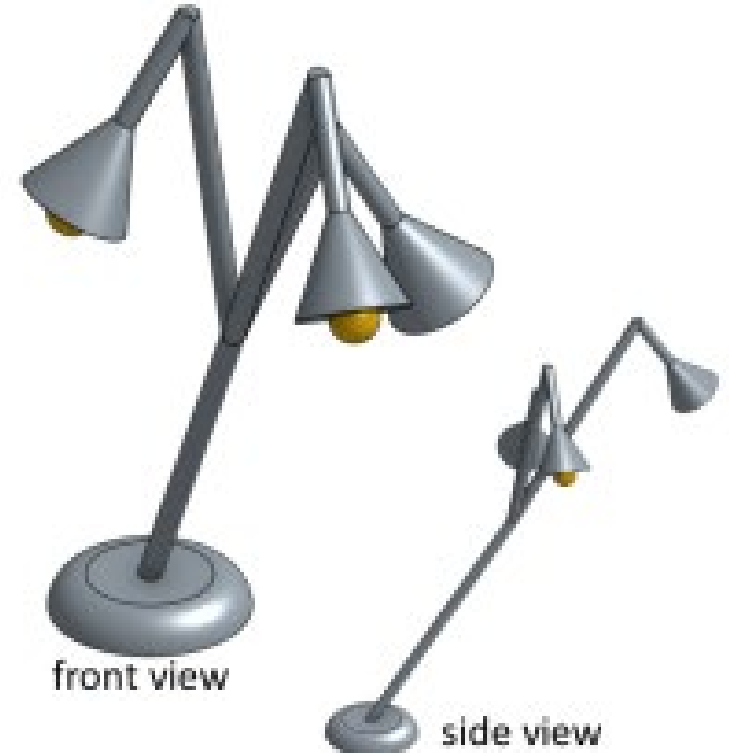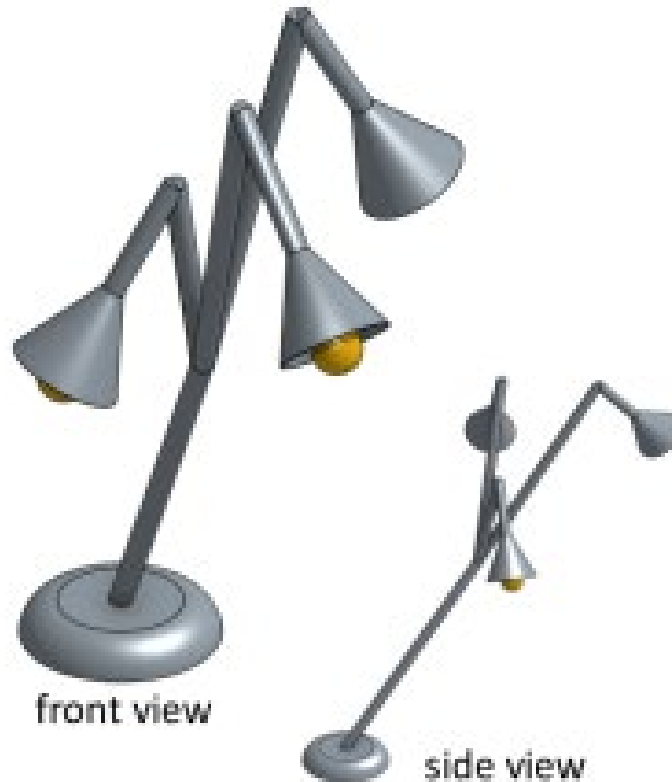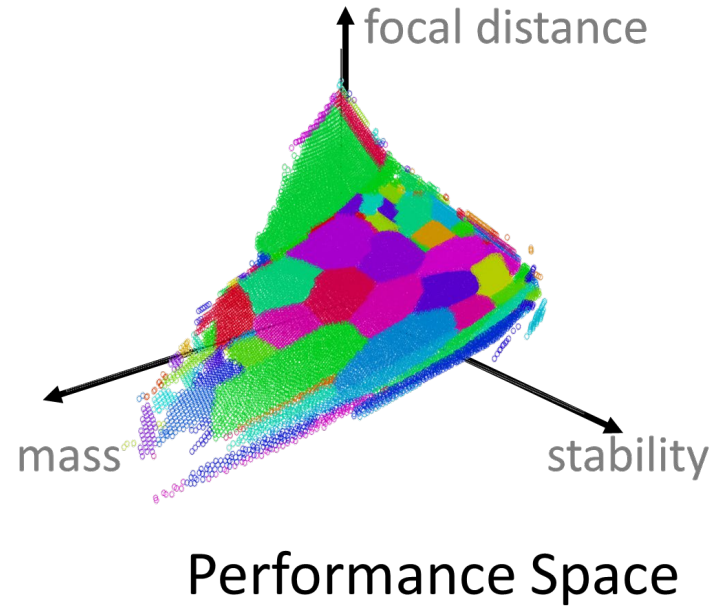


Performance Space

# Bike Frame

# Lamp (21 design parameters, 3 performance parameters)

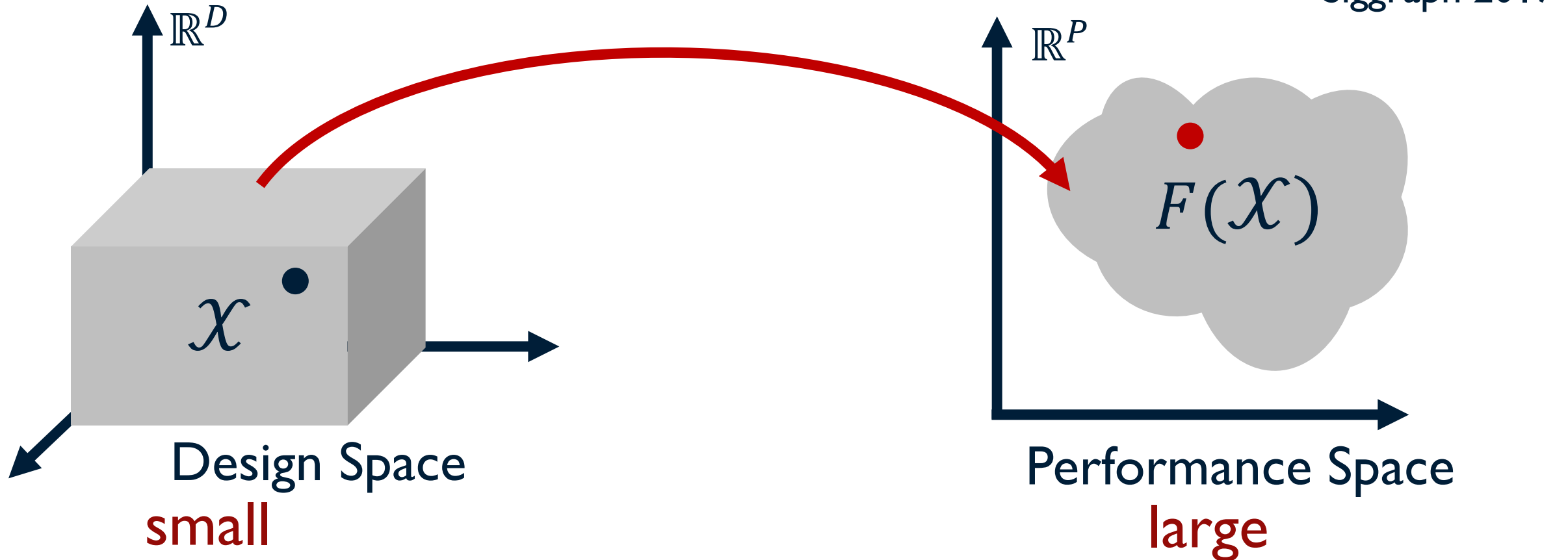# Lamp (21 design parameters, 3 performance parameters)

# Different Solutions with Same Performance



focal distance

mass

stability

Performance Space

front view

side view

front view

side view

# Future Work

- High-dimensional design and performance spaces

Siggraph 2017

$\mathbb{R}^D$

$\mathcal{X}$

Design Space
small

$\mathbb{R}^P$

$F(\mathcal{X})$

Performance Space
large

# Future Work

- High-dimensional design and performance spaces

Siggraph 2017



$\mathbb{R}^D$

$\mathcal{X}$

Design Space
large

$\mathbb{R}^P$

$F(\mathcal{X})$

Performance Space
small

# Future Work

- High-dimensional design and performance spaces

Siggraph 2017

$\mathbb{R}^D$

?

$\mathbb{R}^P$

$F(\mathcal{X})$

$\mathcal{X}$
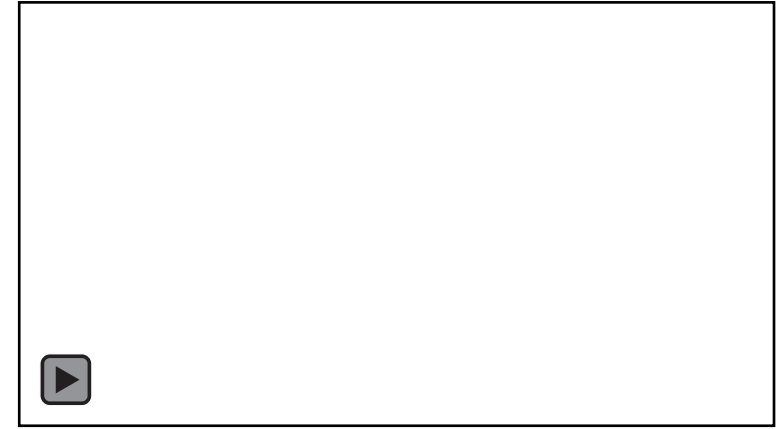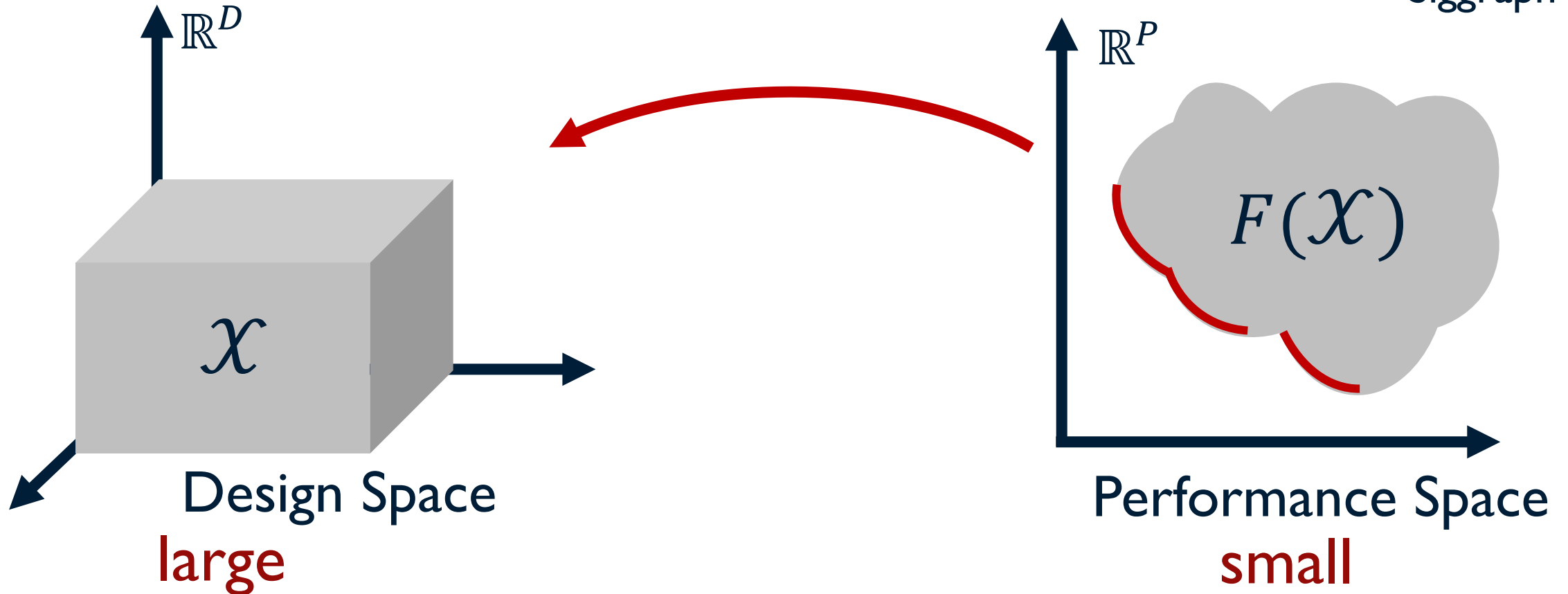
Design Space
large?
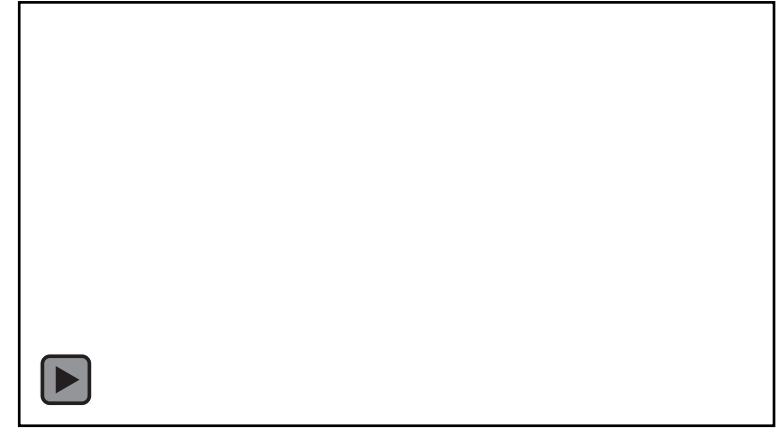
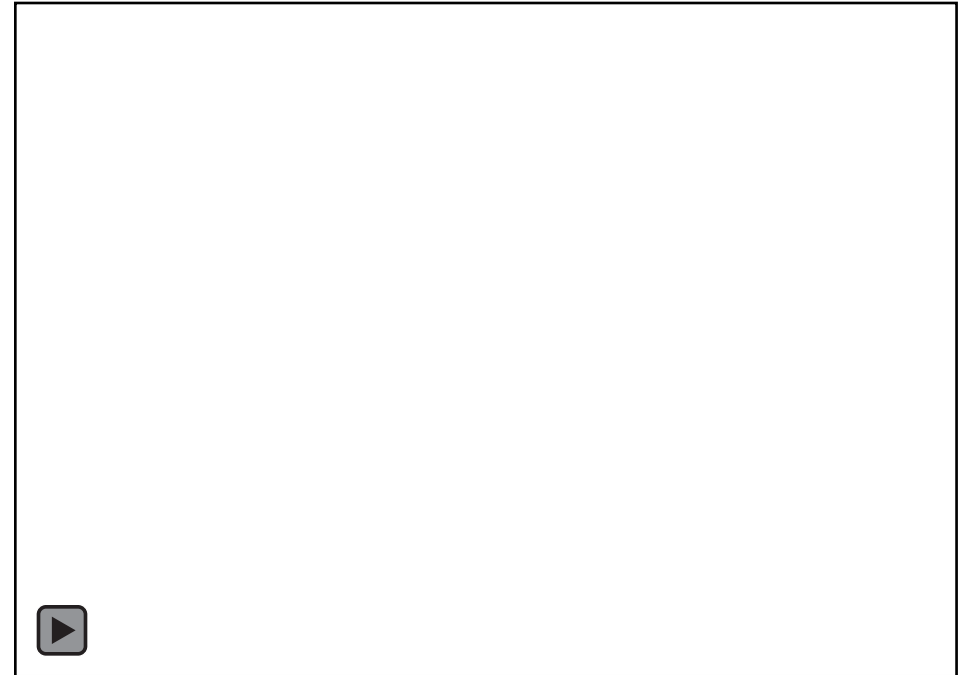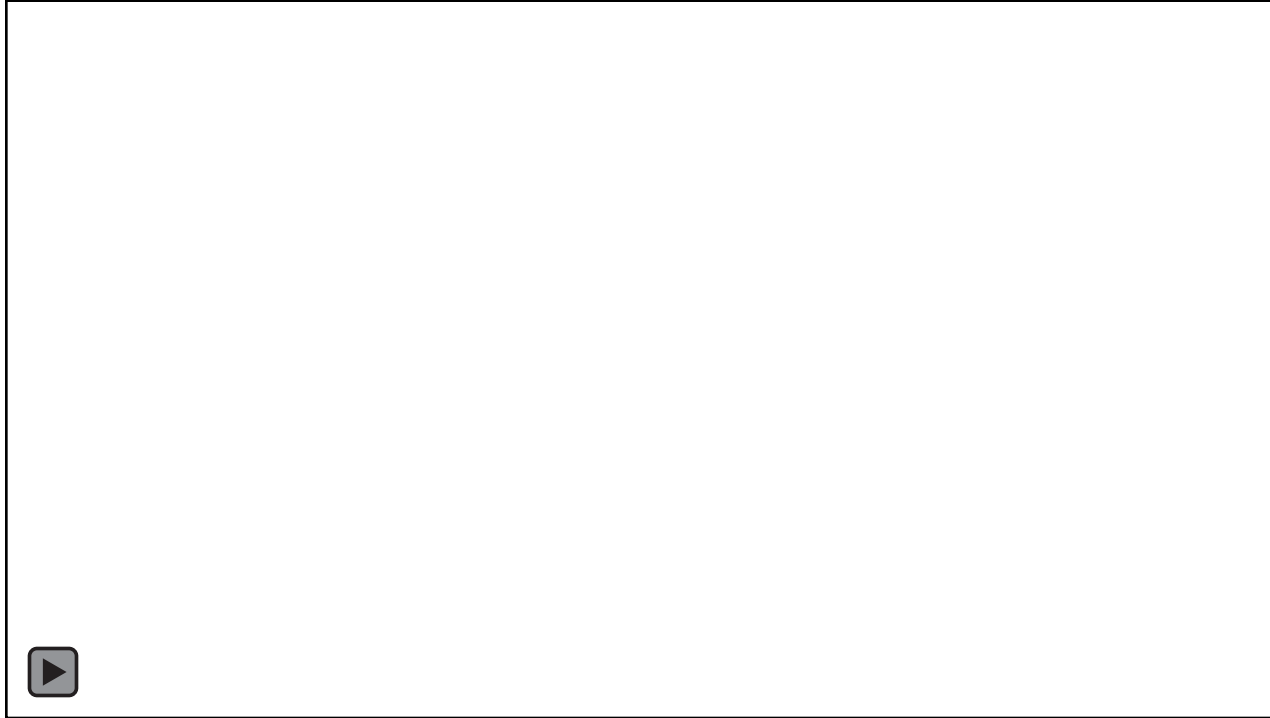Performance Space
large?
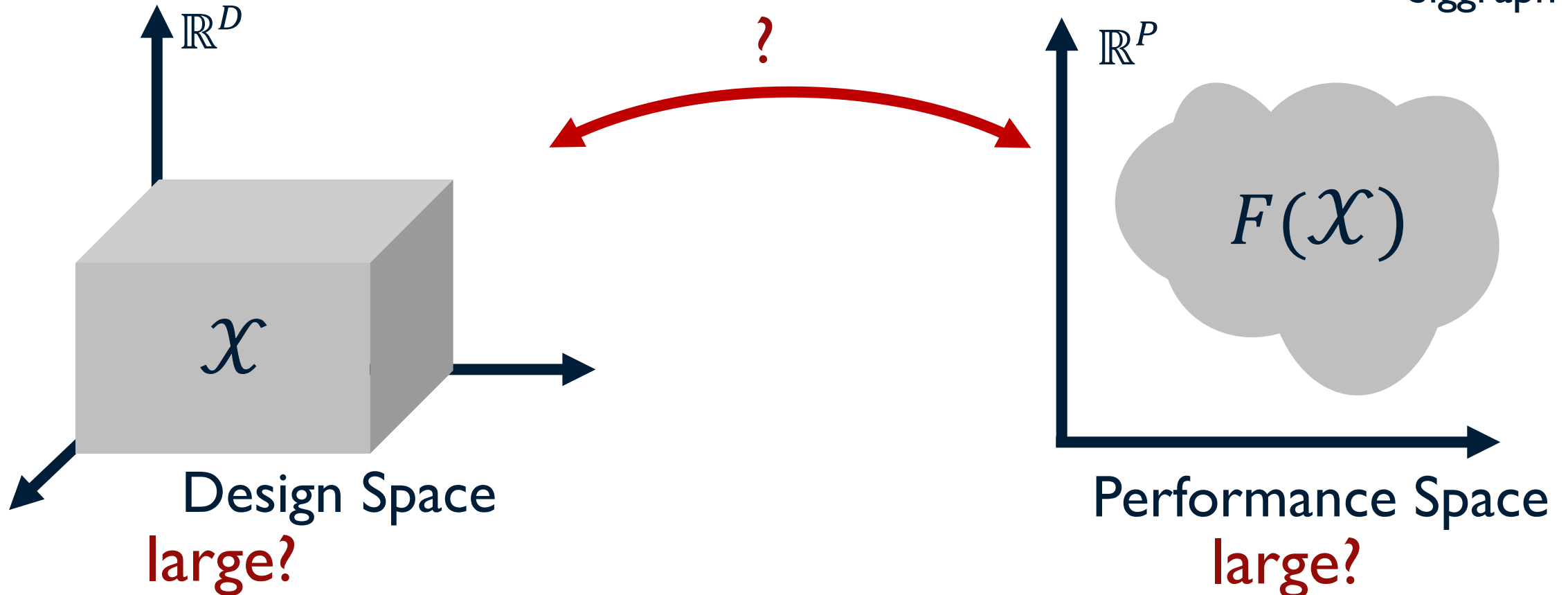
# Future Work

- High-dimensional design and performance spaces

- Mixture of continuous/discrete design variables

- Visualization tool

# Future Work

- High-dimensional design and Performance Spaces

- Mixture of continuous/discrete design variables

- Visualization tool

# Thank you!

Looking for students and postdocs!

adriana@cs.washington.edu