1. Go to: www.cs.washington.edu/312

2. Take office hour survey at bottom of page.

312: FOUNDATIONS OF CS 2

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$\sim$ 8 homework 50%

May 6 $\rightarrow$ 1 midterm 20%

1 final 30%

Sets, Sequences, Functions

$\{1,2,3\} = \{3,2,1\}$

integers

$\mathbb{Z} = \{\ldots,-2,-1,0,1,2,\ldots\}$

natural

$\mathbb{N} = \{1,2,3,\ldots\}$

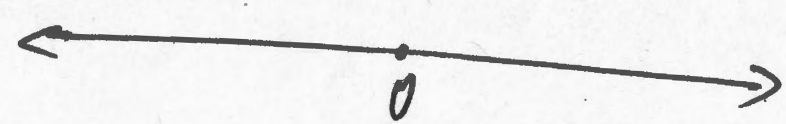
$[n] = \{1,2,3,\ldots,n\}$

rational #'s

$\mathbb{Q} = \{p/q : p,q \in \mathbb{Z}, q \neq 0\}$

real #'s

$\mathbb{R}$



S, T

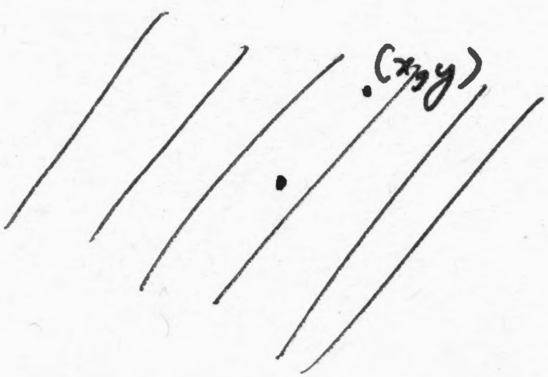
cartesian product

$$S \times T = \{ (x,y) : x \in S,\ y \in T \}$$

$$S^k = \underbrace{S \times S \times \cdots \times S}_{k \text{ times}}$$

$\mathbb{R}$ $\longleftrightarrow$

$\mathbb{R}^2$

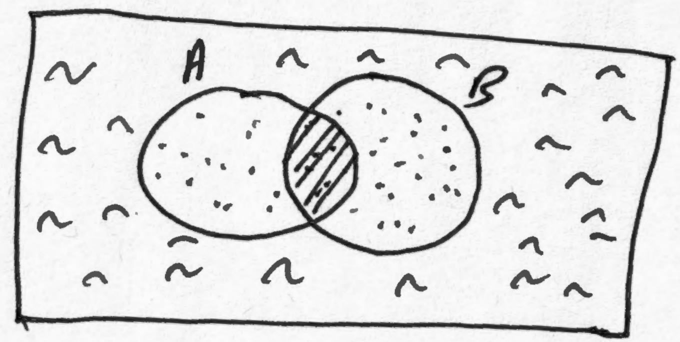


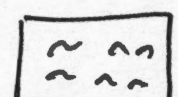
A, B

$$A \cup B = \{ c : c \in A \text{ or } c \in B \}$$

$$A \cap B = \{ c : c \in A \text{ and } c \in B \}$$

$$A^c = \{ x : x \notin A \}$$



 $\rightarrow$ $(A \cup B)^c = A^c \cap B^c$

S

$$2^S = \{ T : T \subseteq S \}$$

$S = \{1, 2\}$

$2^S = \{ \{\}, \{1\}, \{2\}, \{1, 2\} \}$

Sequences

$1, 3, 2, 4, 17, 8$

$\neq$

$1, 2, 3, 4, 17, 8$

S^k: sequences of length k with entries from S.

Function

$f : D \to R$

Associate a unique element $f(x) \in R$ to every element $x \in D$.

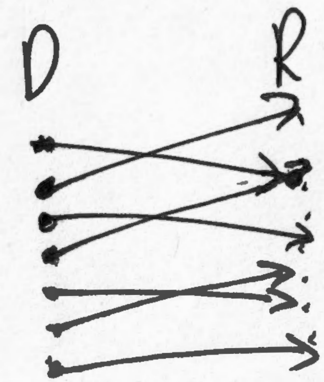
Every function is a set

Let

$F \subseteq D \times R$

$\forall x \in D$ there is a unique $y \in R$ s.t. $(x, y) \in F$

F defines $f(x) = y$.



Every set is a function

$A \subseteq [n]$

$f_A : [n] = \{0,1\}$

indicator function $f_A(x) = \begin{cases} 1 & x \in A \\ 0 & \text{otherwise} \end{cases}$

indicator vector $v \in \{0,1\}^n$

$v_x = 1$ iff $x \in A$

tuple, array, map, vector

$f : D \to R$

- injective or one-to-one
  If $x \neq x' \in D$
  then $f(x) \neq f(x')$
- surjective or onto
  $\forall y \in R$
  $\exists x$ s.t. $f(x) = y$.
- bijection or permutation
  f is both injective and surjective.

$$\overbrace{S_1, S_2, \ldots, S_k}^{\text{not all empty}} \subseteq [n]$$

are <u>closed under union</u>

if $\forall\ i, j$

$S_i \cup S_j$ is also in list.

<u>Conjecture</u> [Union Closed Conjecture]

If $S_1, \ldots, S_k$ are closed under union, then $\exists\ i \in [n]$ that is in $\geqslant k/2$ of sets.