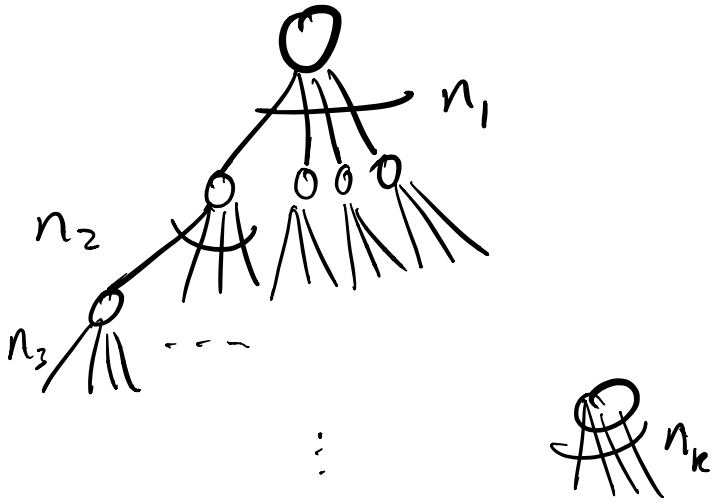
Counting

Product Rule

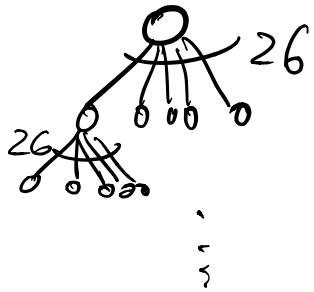


leaves [o o o o o o o o o o o o o o o o o o o o

$= n_1 \cdot n_2 \cdot \cdots \cdot n_k$

How many 5 letter words?

$26^5 = 26 \cdot 26 \cdots 26$



words of length 5 [o o o o o o o o o o

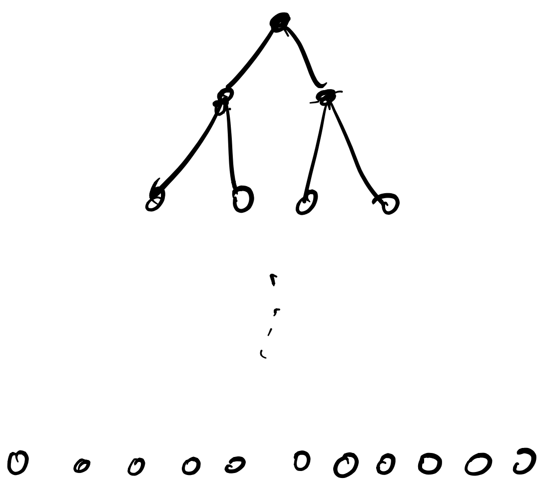
- S, $|S^k| = |S|^k$

S = letters of alphabet

S^5 = words of length 5.

Suppose S is a finite size n

- How many subsets does S have?



$2 \cdot 2 \cdot 2 \cdots 2 = 2^n$

$v \in \{0,1\}^n$ indicator vector of subset

2 choices for v_1

2 choices for v_2

⋮

2 choices for v_n

] 2^n choices

6 TA's, 4 sections

How many ways to assign TAs to sections so that no section gets 0 TAs?

A = set of sections taught by Yifan

B = " " " Yael

⋮

F = " " " Su

2^4 choices for A
2^4 " for B
⋮
2^4 " " F

} $(2^4)^6 = 2^{24}$ choices.

A: set of TA's teaching 1st section

B: " " 2nd "

C: " " 3rd "

D: " " 4th "

2^6 possible subsets of TA's

$2^6 - 1$ of them are non-empty

choices for A = $2^6 - 1$

" B = $2^6 - 1$

" C = $2^6 - 1$

] # choices $(2^6 - 1)^4$ ✓

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5 books and 3 children

Alice, Bob, Charlie

1
2
3
4
5

A: set of books given to Alice

B: ———

C: ———

$2^5 \times 2^5 \times 2^5 = 2^{15}$ ✗

Book 1 : 3 choices

⋮ : 3 choices

choices in total = $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

$= 3^5$ ✓

FACTORIAL

How many ways to rearrange letters in GRAPEFRUIT — 10 letters

XGAPEFRUIT

XAGPEFRUIT

⋮

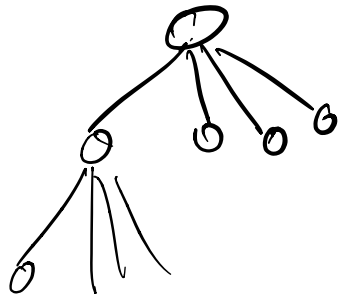
? ?

10 choices · 9 choices ⋯ # choices

$= 10 \cdot 9 \cdot 8 \cdots \cdot 1$

$= 10!$

$$n! = n \cdot (n-1) \cdots 1$$



0 0 0 0 0 0 0 0 0

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1 2 3 4 ... 10

G R A P E F X U I T

2 3 1 4 5 ... 10

R A G P E F X U I T

# of bijections from $[10] \to [10]$

How many 5 letter words in English where letters are distinct?

? ? ? ? ?

$$26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 = \frac{26!}{21!}$$

26 choices

$$P(n,k) = \frac{n!}{(n-k)!}$$

= # ways to make a sequence of length k from a set of size n with no repetitions

OVERCOUNTING

26 letter words, all letters distinct $= 26!$

For every such word, take first 5 letters

Each 5 letter word is counted $21!$ times

$\Rightarrow$ # 5 letter words

$$= \frac{26!}{21!}$$

How many ways to arrange GRAPEFRUIT?

$\frac{10!}{2!}$

GR$_1$APEFR$_2$UIT

#ways $10!$

R$_1$R$_2$APEGFUIT

R$_2$R$_1$APEGFUIT

RRAPEGFUIT