CSE431: Complexity Theory October 17, 2018

Homework 2

Anup Rao Due: October 26, 2018

Read the fine print[1]. Each problem is worth 10 points:

1. Prove that the following functions are in **NP**:

   (a)
   $$4\mathbf{COL}(G) = \begin{cases} 1 & \text{if graph } G \text{ has a coloring with 4 colors,} \\ 0 & \text{otherwise.} \end{cases}$$
   Recall that a coloring of G with c colors is an assignment of a number in $[c]$ to each vertex such that no adjacent vertices get the same number.

   (b)
   $$\mathbf{CONN}(G, s, t) = \begin{cases} 1 & \text{if graph } G \text{ has a path from vertex } s \text{ to vertex } t, \\ 0 & \text{otherwise.} \end{cases}$$

2. Consider the following function $h : \{0,1\}^* \to \{0,1\}$.
   $$h(\alpha) = \begin{cases} 1 & \text{if there is some } x \text{ such that } M_\alpha(x) \text{ halts with output 1,} \\ 0 & \text{else.} \end{cases}$$
   Someone claims to have a program that can compute h. Prove that their program must have a bug by showing that no turing machine can compute $h(\alpha)$ for every α.

3. If $f, g \in \mathbf{NP}$, discuss whether each of the following functions can be deduced to be in **NP** or not:
   - $h_1(x) = f(x) \wedge g(x)$.
   - $h_2(x) = f(x) \vee g(x)$.

4. Prove that $\mathsf{L} \neq \mathsf{DTIME}(n^{\log n})$.

5. Let HALT be the halting function we defined in class (i.e. $\mathsf{HALT}(\alpha, x) = 1$ iff $M_\alpha(x)$ halts). Show that HALT is **NP**-hard. Is it **NP**-complete?

[1] In solving the problem sets, you are allowed to collaborate with fellow students taking the class, but **each submission can have at most one author**. If you do collaborate in any way, you must acknowledge, for each problem, the people you worked with on that problem. The problems have been carefully chosen for their pedagogical value, and hence might be similar to those given in past offerings of this course at UW, or similar to other courses at other schools. Using any pre-existing solutions from these sources, for from the web, constitutes a violation of the academic integrity you are expected to exemplify, and is strictly prohibited. Most of the problems only require one or two key ideas for their solution. It will help you a lot to spell out these main ideas so that you can get most of the credit for a problem even if you err on the finer details. Please justify all answers. Some other guidelines for writing good solutions are here: `http://www.cs.washington.edu/education/courses/cse421/08wi/guidelines.pdf`.