CSE421: Design and Analysis of Algorithms	November 20, 2020

Homework 7

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Due: December 5, 2020

Read the fine print[1]. Each problem is worth 10 points:

1. Let G be an input graph to the max flow problem. Let (A, B) be a minimum capacity $s-t$ cut in the graph. Suppose we add 1 to the capacity of every edge in the graph. Is it necessarily true that A is still a minimum cut? If so, prove it, if not give a counterexample.

2. An edge of a flow network is called *critical* if decreasing the capacity of this edge results in a decrease in the maximum flow. Give a polynomial time algorithm that finds a critical edge in a network.

3. Suppose you are given a bipartite graph G with n vertices on each side, and a matching M with $n-1$ edges that is contained in the graph. Give an $O(n^2)$ time algorithm to check whether or not the graph has a matching of size n.

[1] In solving the problem sets, you are allowed to collaborate with fellow students taking the class, but **each submission can have at most one author**. If you do collaborate in any way, you must acknowledge, for each problem, the people you worked with on that problem. The problems have been carefully chosen for their pedagogical value, and hence might be similar to those given in past offerings of this course at UW, or similar to other courses at other schools. Using any pre-existing solutions from these sources, for from the web, constitutes a violation of the academic integrity you are expected to exemplify, and is strictly prohibited. Most of the problems only require one or two key ideas for their solution. It will help you a lot to spell out these main ideas so that you can get most of the credit for a problem even if you err on the finer details. Please justify all answers. Some other guidelines for writing good solutions are here: `http://www.cs.washington.edu/education/courses/cse421/08wi/guidelines.pdf`.