CSE421: Design and Analysis of Algorithms	November 6, 2020
Homework 5	
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Read the fine print[1]. Each problem is worth 10 points:

1. Given a sequence of integers $x_1, \dots, x_n$ (possibly including negative integers) and an interval of coordinates $I = [i, j]$, write x_I to denote the sum $\sum_{i \leq k \leq j} x_k$. Give a linear time algorithm to find the interval that maximizes x_I, given the numbers $x_1, \dots, x_n$ as input.

2. Given a sequence of characters $c_1, \dots, c_n$, we say that a subsequence is a *palindrome* if it reads the same forwards and backwards. For example, "a,b,a,c,a,b,a" is a palindrome. Give an $O(n^2)$ time algorithm to find the longest palindrome subsequence in the input sequence $c_1, \dots, c_n$. For example, in the sequence $c, l, m, a, l, f, d, c, a, f, m$, the longest palindrome subsequence is m, a, d, a, m. HINT: For $i < j$, let $p(i, j)$ denote the length of the longest palindrome in $x_i, \dots, x_j$. Express $p(i, j)$ in terms of $p(i+1, j), p(i, j-1), p(i+1, j-1)$. Evaluate the values $p(i, j)$ in order of increasing $|i - j|$.

3. You are given a rectangular piece of cloth with dimensions $X \times Y$, where X and Y are positive integers, and a list of n products that can be made using the cloth. For each product i you know that a rectangle of cloth of dimensions $a_i \times b_i$ is needed and that the selling price of the product is c_i Assume the a_i, b_i and c_i are all positive integers. You have a machine that can cut any rectangular piece of cloth into two pieces either horizontally or vertically. Design an algorithm that runs in time that is polynomial in X, Y, n and determines the best return on the $X \times Y$ piece of cloth, that is, a strategy for cutting the cloth so that the products made from the resulting pieces give the maximum sum of selling prices. You are free to make as many copies of a given product as you wish, or none, if desired.

4. Let G be an input graph to the max flow problem. Let A be a minimum $s - t$ cut in the graph. Suppose we add 1 to the capacity of every edge in the graph. Is it necessarily true that A is still a minimum cut? If so, prove it, if not give a counterexample.

[1] In solving the problem sets, you are allowed to collaborate with fellow students taking the class, but **each submission can have at most one author**. If you do collaborate in any way, you must acknowledge, for each problem, the people you worked with on that problem. The problems have been carefully chosen for their pedagogical value, and hence might be similar to those given in past offerings of this course at UW, or similar to other courses at other schools. Using any pre-existing solutions from these sources, for from the web, constitutes a violation of the academic integrity you are expected to exemplify, and is strictly prohibited. Most of the problems only require one or two key ideas for their solution. It will help you a lot to spell out these main ideas so that you can get most of the credit for a problem even if you err on the finer details. Please justify all answers. Some other guidelines for writing good solutions are here: `http://www.cs.washington.edu/education/courses/cse421/08wi/guidelines.pdf`.