CSE421: Design and Analysis of Algorithms November 12, 2021

Homework 6

Anup Rao Due: November 19, 2021

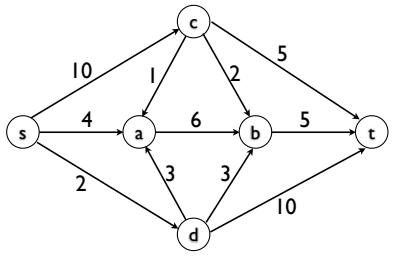
Read the fine print[1]. Each problem is worth 10 points:

1. Given two strings $x_1, \ldots, x_m$ and $y_1, \ldots, y_n$, we want to calculate the length of the longest common substring, namely the largest k for which there are i, j such that $x_i x_{i+1} \ldots x_{i+k-1} = y_j y_{j+1} \ldots y_{j+k-1}$. Show how to do this in time $O(mn)$.

2. You are a large corporation that wants to open a chain of stores along a highway. There are n possible locations, which are at mileposts $m_1, \ldots, m_n$ on the highway. At each location m_i, you may open one store, which will give you an expected profit of p_i. However, if you open stores at m_i, m_j, then these stores must be at least k miles apart (i.e. $|m_i - m_j| \geq k$). Give an efficient algorithm to find the optimal locations to open stores on input $k, p_1, \ldots, p_n$ and $m_1, \ldots, m_n$. (For full credit it is enough to calculate the maximum expected profit from the best solution).

3. Suppose we are given a flow network, where instead of capacities on edges, each internal vertex has a capacity on the total flow that is allowed to pass through it. So for each vertex v, there is a non-negative integer c_v, and the flow must satisfy $f^{in}(v) \leq c_v$. Each edge can carry an arbitrary amount of flow. Give a polynomial time algorithm to find the maximum flow in such a network. (Hint: try to convert the problem into a flow network of the type we are used to.)

4. Draw out a maximum $s - t$ flow for the graph below, and the corresponding residual graph G_f. What is the minimum cut that corresponds to this max flow?



[1] In solving the problem sets, you are allowed to collaborate with fellow students taking the class, but **each submission can have at most one author**. If you do collaborate in any way, you must acknowledge, for each problem, the people you worked with on that problem. The problems have been carefully chosen for their pedagogical value, and hence might be similar to those given in past offerings of this course at UW, or similar to other courses at other schools. Using any pre-existing solutions from these sources, for from the web, constitutes a violation of the academic integrity you are expected to exemplify, and is strictly prohibited. Most of the problems only require one or two key ideas for their solution. It will help you a lot to spell out these main ideas so that you can get most of the credit for a problem even if you err on the finer details. Please justify all answers. Some other guidelines for writing good solutions are here: `http://www.cs.washington.edu/education/courses/cse421/08wi/guidelines.pdf`.