CSE421: Design and Analysis of Algorithms	November 26, 2021
Homework 7	
Anup Rao	Due: December 3, 2021

Read the fine print[1]. Each problem is worth 10 points:

1. Write down the dual of the following linear program:

$$\begin{aligned} &\text{maximize } a - b + c \\ &\text{subject to} \\ &\quad 5a + 2b \leq 3 \\ &\quad c - a \leq -2 \\ &\quad b + c \leq 0 \\ &\quad a, b, c \geq 0 \end{aligned}$$

2. You are given the following points in the plane: $(1,3), (2,5), (3,7), (5,11), (7,14), (8,15), (10,19)$. You want to find a line $y = ax + b$ that approximately passes through these points (no line is a perfect fit). Write a linear program (you do not need to solve it) to find the line that minimizes the maximum absolute error,

$$\max_{1 \leq i \leq 7} |y_i - ax_i - b|$$

3. You are running a truck business and need to fill a truck that can carry a total weight of 100 tons and volume 30 cubic meters. You can put three types of materials into the truck.

   (a) Item 1 has density 2 tons per cubic meter, maximum available amount is 40 cubic meters and the revenue associated with it is 1000 dollars per cubic meter.

   (b) Item 2 has density 5 tons per cubic meter, maximum available amount is 20 cubic meters and the revenue associated with it is 2000 dollars per cubic meter.

   (c) Item 3 has density 7 tons per cubic meter, maximum available amount is 15 cubic meters and the revenue associated with it is 1500 dollars per cubic meter.

   Write a linear program to calculate how much of each amount the truck should carry to maximize profits (no need to solve it).

[1] In solving the problem sets, you are allowed to collaborate with fellow students taking the class, but **each submission can have at most one author**. If you do collaborate in any way, you must acknowledge, for each problem, the people you worked with on that problem. The problems have been carefully chosen for their pedagogical value, and hence might be similar to those given in past offerings of this course at UW, or similar to other courses at other schools. Using any pre-existing solutions from these sources, for from the web, constitutes a violation of the academic integrity you are expected to exemplify, and is strictly prohibited. Most of the problems only require one or two key ideas for their solution. It will help you a lot to spell out these main ideas so that you can get most of the credit for a problem even if you err on the finer details. Please justify all answers. Some other guidelines for writing good solutions are here: `http://www.cs.washington.edu/education/courses/cse421/08wi/guidelines.pdf`.

4. We all love vertex covers. The reason is that they are subsets of the vertices that touch every edge of an undirected graph. Give an algorithm to find a vertex cover of smallest size in a bipartite graph. Hints:

   (a) Construct a flow network from the input bipartite graph just as in the maximum matching algorithm.

   (b) Show that a "nice" min-cut in this flow network gives a vertex cover. Namely, if the graph has bipartitions A, B and X, Y are the corresponding components of the "nice" min-cut, show that $(A - X) \cup (B - Y)$ must be a vertex cover of smallest size.

   (c) Write down the algorithm and prove that it works.