Read the fine print[1]. Each problem is worth 10 points:

1. Show that if P=NP, then there is a polynomial time algorithm that given a 3SAT formula, finds a satisfying assignment to the formula, if there is one.

2. Show that if P=NP, then there is a polynomial time algorithm for factoring. Here you are given an n-bit number N, and you need to find a factor a that divides N, with $a \neq 1$, and $a \neq N$, if such an a exists.

3. Given two decision problems X and Y, let $X \wedge Y$ denote the decision problem, where on input the answer is yes if and only if the answer to both X and Y is yes. Similarly define $X \vee Y$. If X, Y are both in NP, does this imply that $X \wedge Y$ is in NP? Does it imply that $X \vee Y$ is in NP?

4. Consider a special version of the 3SAT problem, where every clause has exactly 3 literals, and each variable appears at most 3 times. Show that there is a polynomial time algorithm to find a satisfying assignment for such a formula. HINT: Consider the bipartite graph with clauses on the left, and variables on the right. Connect a clause to a variable if the variable appears in the clause. Prove that this graph has a matching whose size is the same as the number of clauses. Then give an algorithm to find the matching and use the matching to build a satisfying assignment.