

Homework 8

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Due: December 10, 2021

Read the fine print¹. Each problem is worth 10 points:

1. Show that if $P=NP$, then there is a polynomial time algorithm that given a 3SAT formula, finds a satisfying assignment to the formula, if there is one.
2. Show that if $P=NP$, then there is a polynomial time algorithm for factoring. Here you are given an n -bit number N , and you need to find a factor a that divides N , with $a \neq 1$, and $a \neq N$, if such an a exists.
3. Given two decision problems X and Y , let $X \wedge Y$ denote the decision problem, where on input the answer is yes if and only if the answer to both X and Y is yes. Similarly define $X \vee Y$. If X, Y are both in NP , does this imply that $X \wedge Y$ is in NP ? Does it imply that $X \vee Y$ is in NP ?
4. Consider a special version of the 3SAT problem, where every clause has exactly 3 literals, and each variable appears at most 3 times. Show that there is a polynomial time algorithm to find a satisfying assignment for such a formula. HINT: Consider the bipartite graph with clauses on the left, and variables on the right. Connect a clause to a variable if the variable appears in the clause. Prove that this graph has a matching whose size is the same as the number of clauses. Then give an algorithm to find the matching and use the matching to build a satisfying assignment.

¹In solving the problem sets, you are allowed to collaborate with fellow students taking the class, but **each submission can have at most one author**. If you do collaborate in any way, you must acknowledge, for each problem, the people you worked with on that problem. The problems have been carefully chosen for their pedagogical value, and hence might be similar to those given in past offerings of this course at UW, or similar to other courses at other schools. Using any pre-existing solutions from these sources, for from the web, constitutes a violation of the academic integrity you are expected to exemplify, and is strictly prohibited. Most of the problems only require one or two key ideas for their solution. It will help you a lot to spell out these main ideas so that you can get most of the credit for a problem even if you err on the finer details. Please justify all answers. Some other guidelines for writing good solutions are here: <http://www.cs.washington.edu/education/courses/cse421/08wi/guidelines.pdf>.