Algorithm: Sel(numbers,k)

Given: numbers $x_1,\ldots,x_n$, k, output k'th smallest number.

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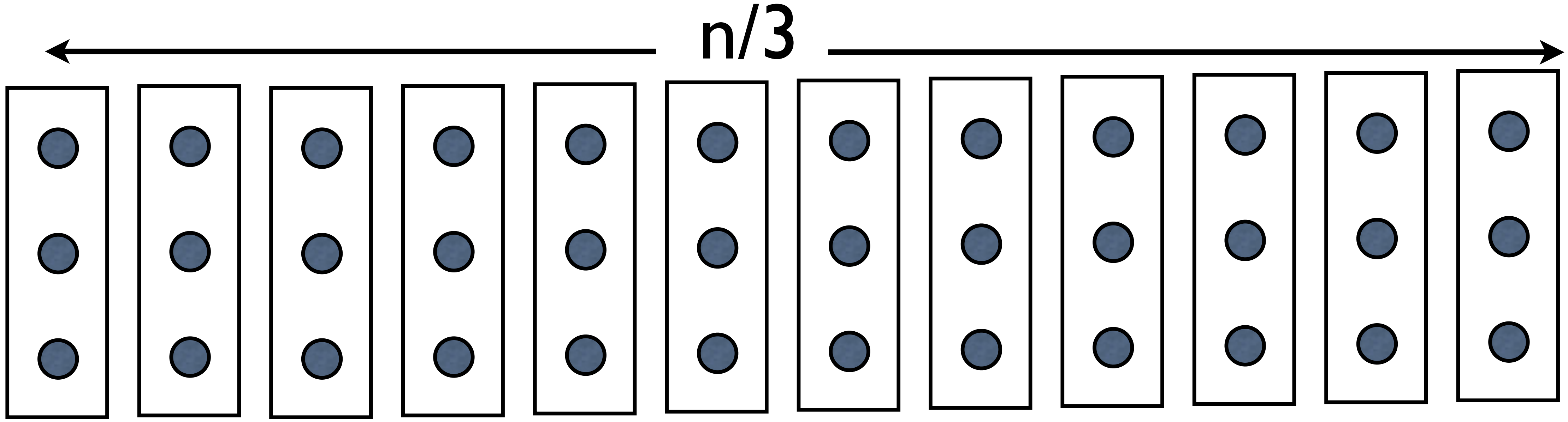
Sort the numbers! O(n log n) time.

Can we do better?

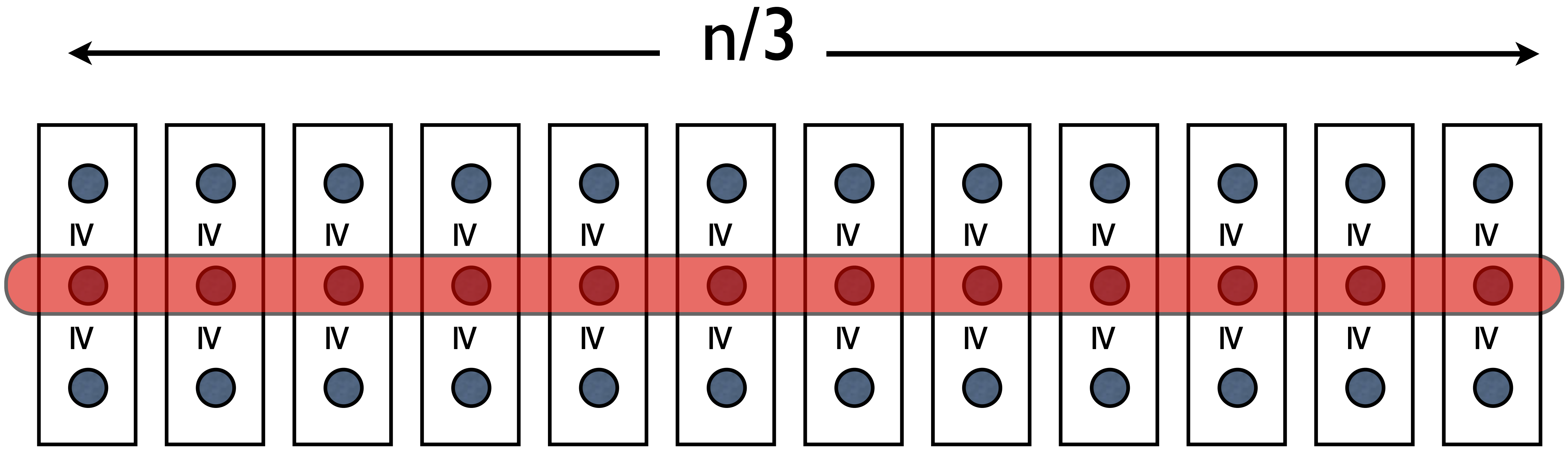
Given: $x_1, \ldots, x_n$, k, output k'th smallest number.

Algorithm: Sel(numbers,k)



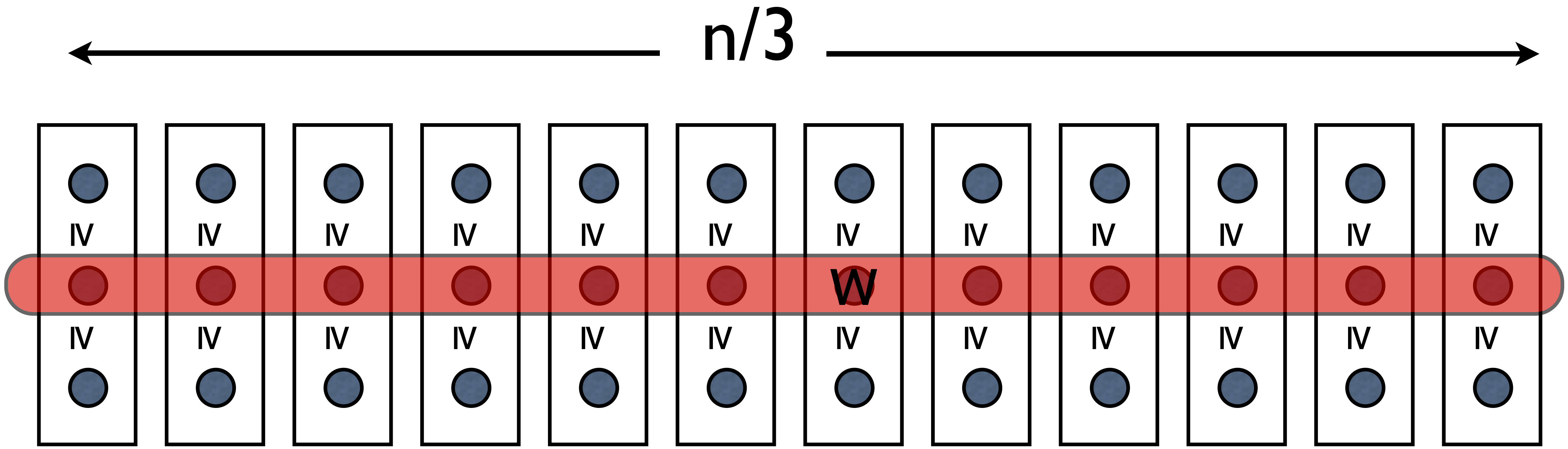
1. Partition numbers into sets of size 3

Algorithm: Sel(numbers,k)



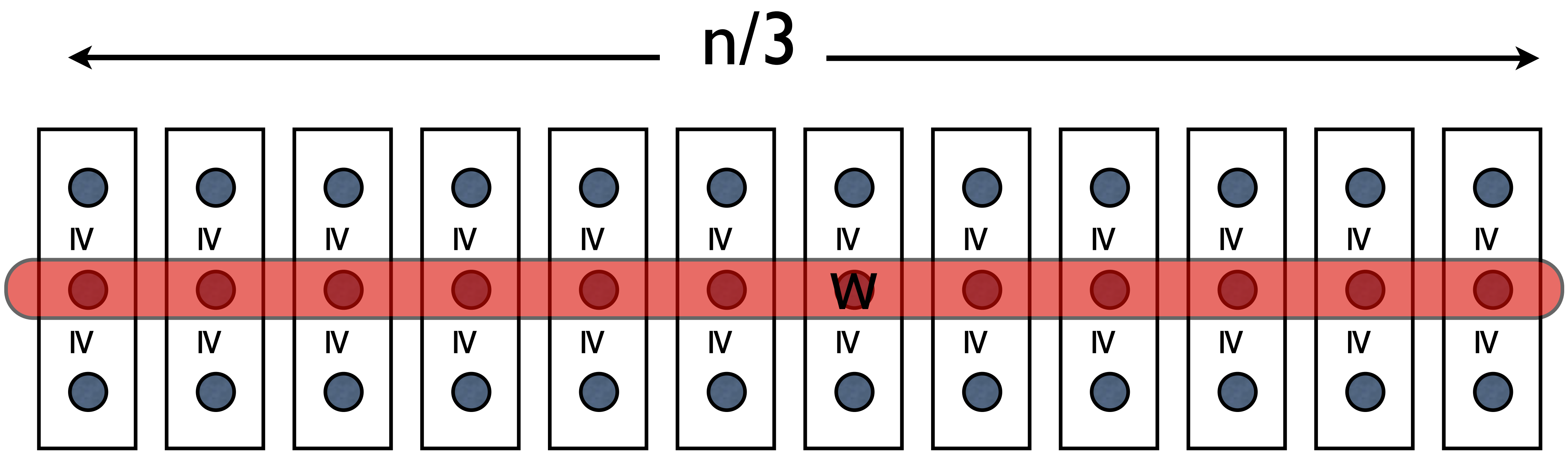
1. Partition numbers into sets of size 3
2. Sort each set

Algorithm: Sel(numbers,k)



1. Partition numbers into sets of size 3
2. Sort each set
3. w = Sel( ,n/6)

Algorithm: Sel(numbers,k)



1. Partition numbers into sets of size 3
2. Sort each set
3. w = Sel( ,n/6)

$S_L(w) = \{x_i \mid x_i < w\}$

$S_E(w) = \{x_i \mid x_i = w\}$

$S_G(w) = \{x_i \mid x_i > w\}$

} Can be computed in linear time

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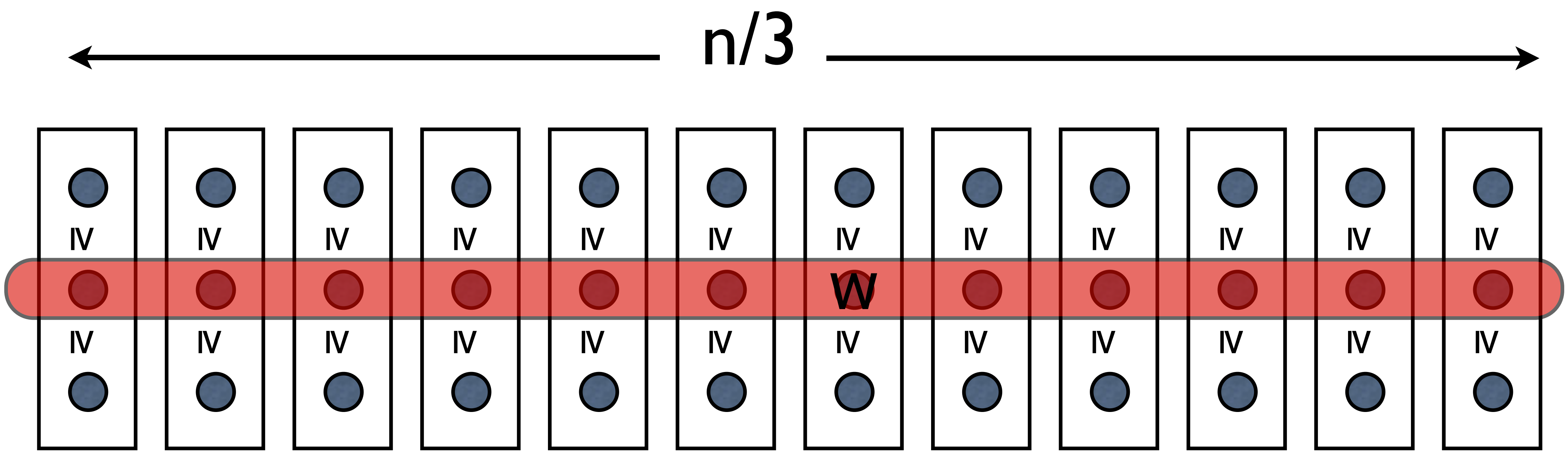

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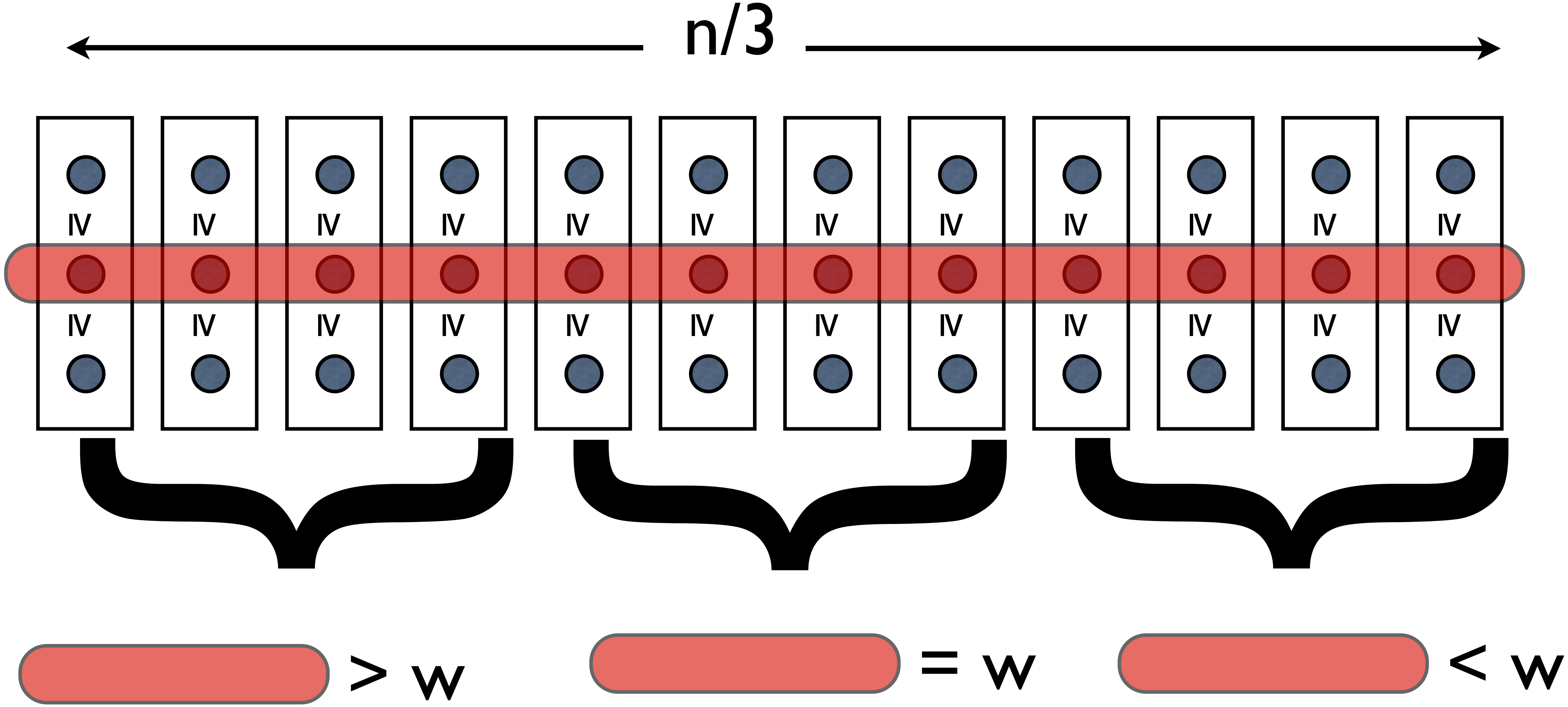
} Can be computed in linear time

4. if $k \leq |S_L(w)|$, output $Sel(S_L(w), k)$
   else if $k \leq |S_L(w)| + |S_E(w)|$, output w
   else output $Sel(S_G(w), k - |S_L(w)| - |S_E(w)|)$

Algorithm: Sel(numbers,k)



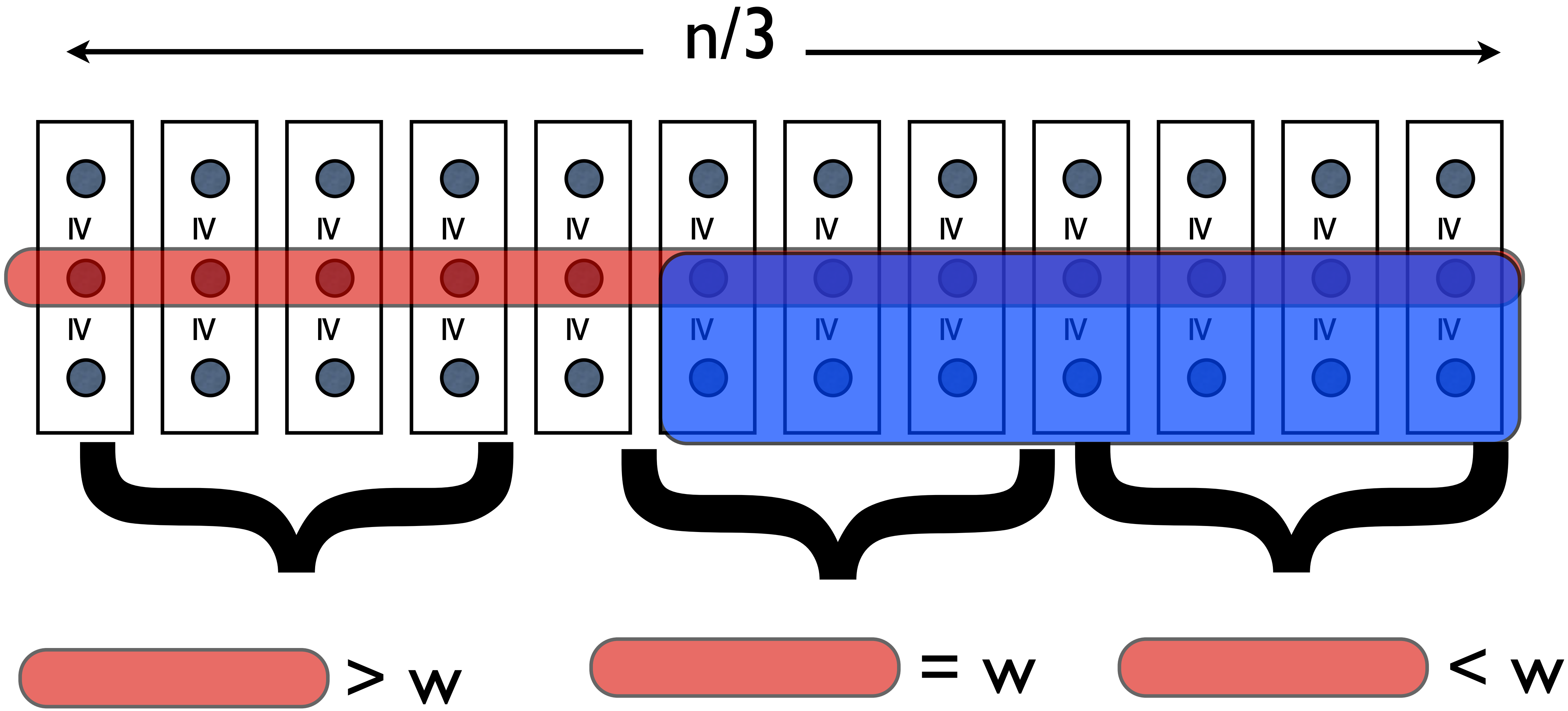
w = Sel(,n/6)

if $k \leq |S_L(w)|$, output Sel($S_L(w)$, k)

else if $k \leq |S_L(w)| + |S_E(w)|$, output w

else output Sel($S_G(w)$, $k - |S_L(w)|-|S_G(w)|$)

Algorithm: Sel(numbers,k)



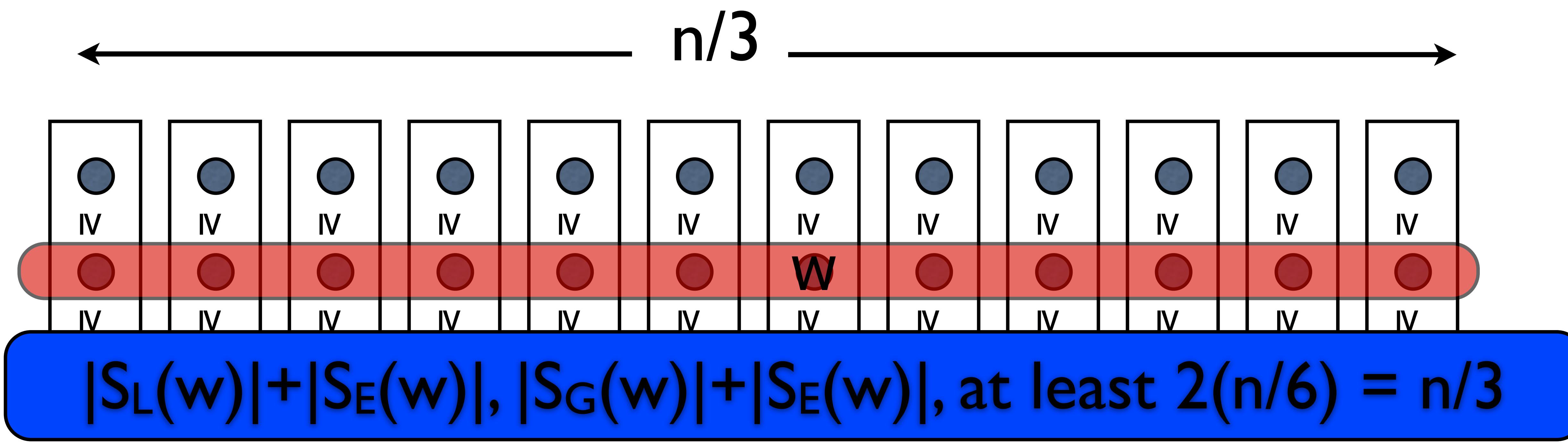
$|S_L(w)|+|S_E(w)|$, at least 2(n/6) = n/3

w = Sel(,n/6)

if $k \leq |S_L(w)|$, output $Sel(S_L(w), k)$

else if $k \leq |S_L(w)| + |SE(w)|$, output w

else output $Sel(S_G(w), k - |S_L(w)|-|S_G(w)|)$



Algorithm: Sel(numbers,k)

1. Partition numbers into sets of size 3
2. Sort each set
3. w = median of

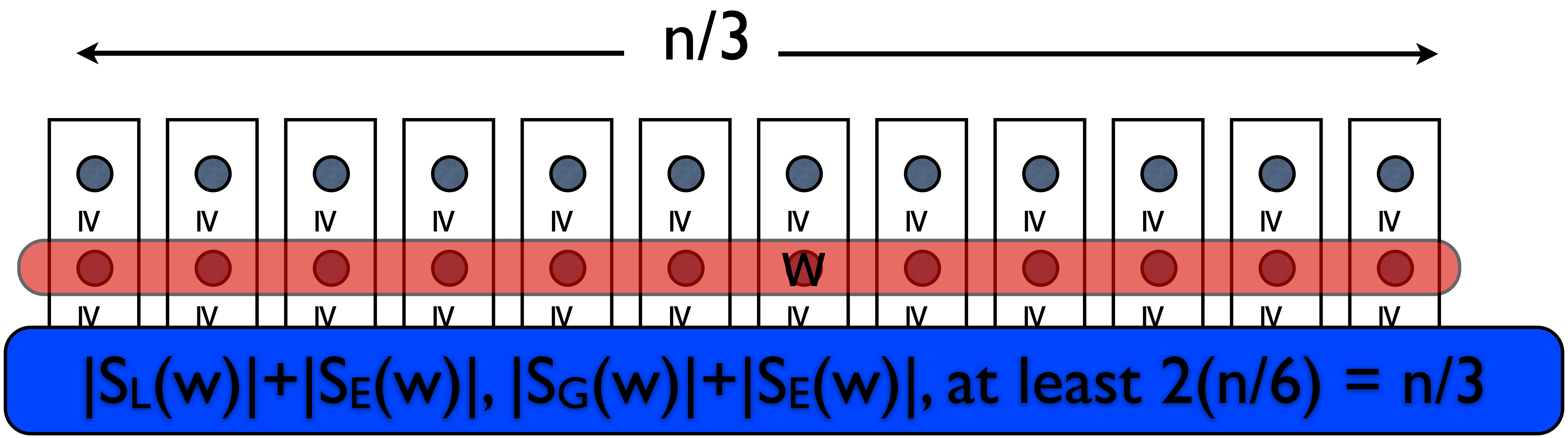
$$T(n) = T(n/3) + T(2n/3) + O(n)$$

$S_L(w) = \{x_i \mid x_i < w\}$
$S_E(w) = \{x_i \mid x_i = w\}$
$S_G(w) = \{x_i \mid x_i > w\}$

} Can be computed in linear time

4. if $k \leq |S_L(w)|$, output $Sel(S_L(w), k)$
   else if $k \leq |S_L(w)| + |S_E(w)|$, output w
   else output $Sel(S_G(w), k - |S_L(w)| - |S_E(w)|)$

Algorithm: Sel(numbers,k)



1. Partition numbers into sets of size 3
2. Sort each se
3. w = median

$S_L(w) = \{x_i$

$S_E(w) = \{x_i$

$S_G(w) = \{x_i \mid x_i > w\}$

$T(n) = T(n/3) + T(2n/3) + O(n)$

so

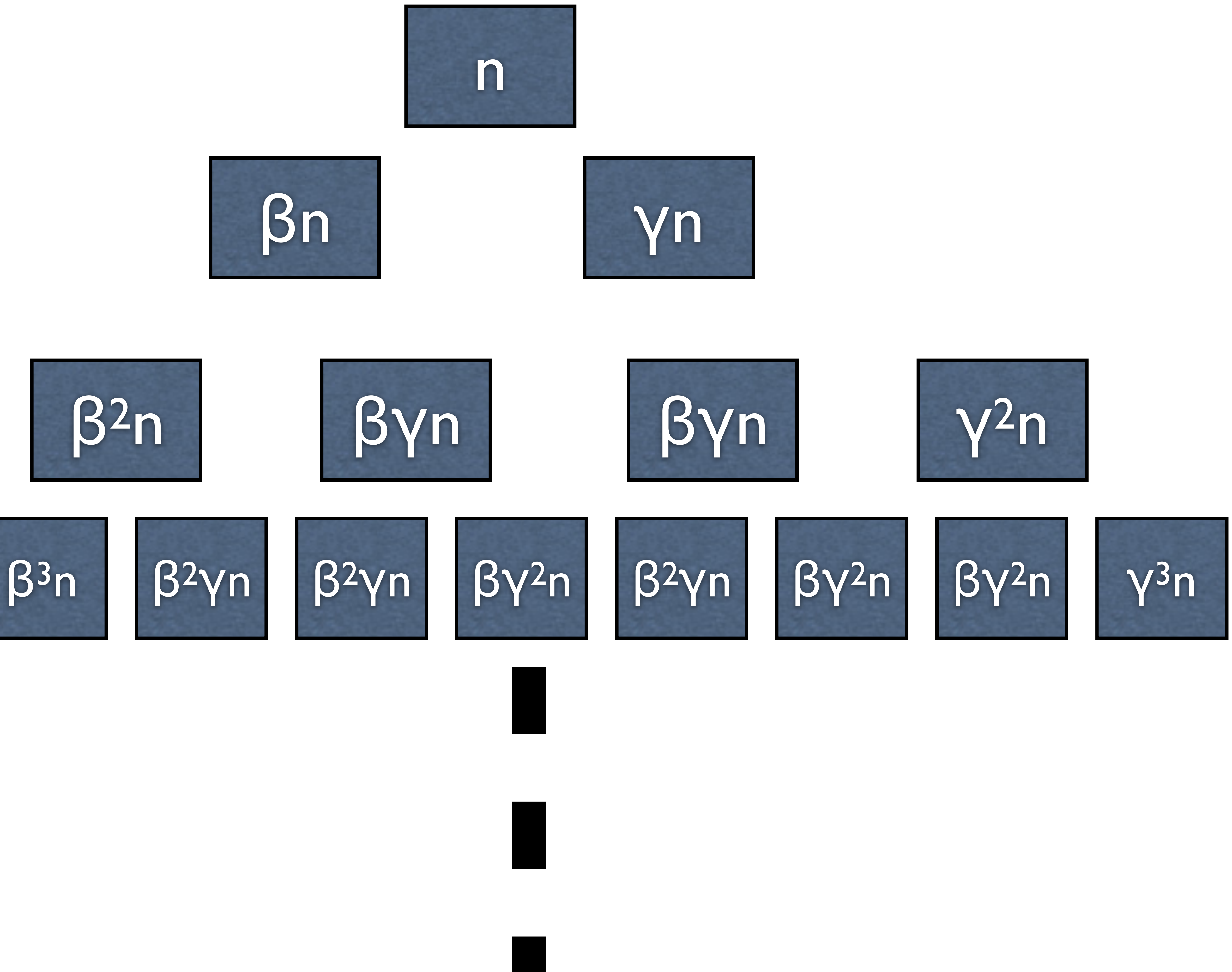
$T(n) = O(n\log n)$

(what's the point???)

4. if $k \le |S_L(w)|$, output $Sel(S_L(w), k)$
   else if $k \le |S_L(w)| + |S_E(w)|$, output w
   else output $Sel(S_G(w), k - |S_L(w)| - |S_E(w)|)$

Recurrences

$$T(n) = T(\gamma n) + T(\beta n) + n$$



$$n$$

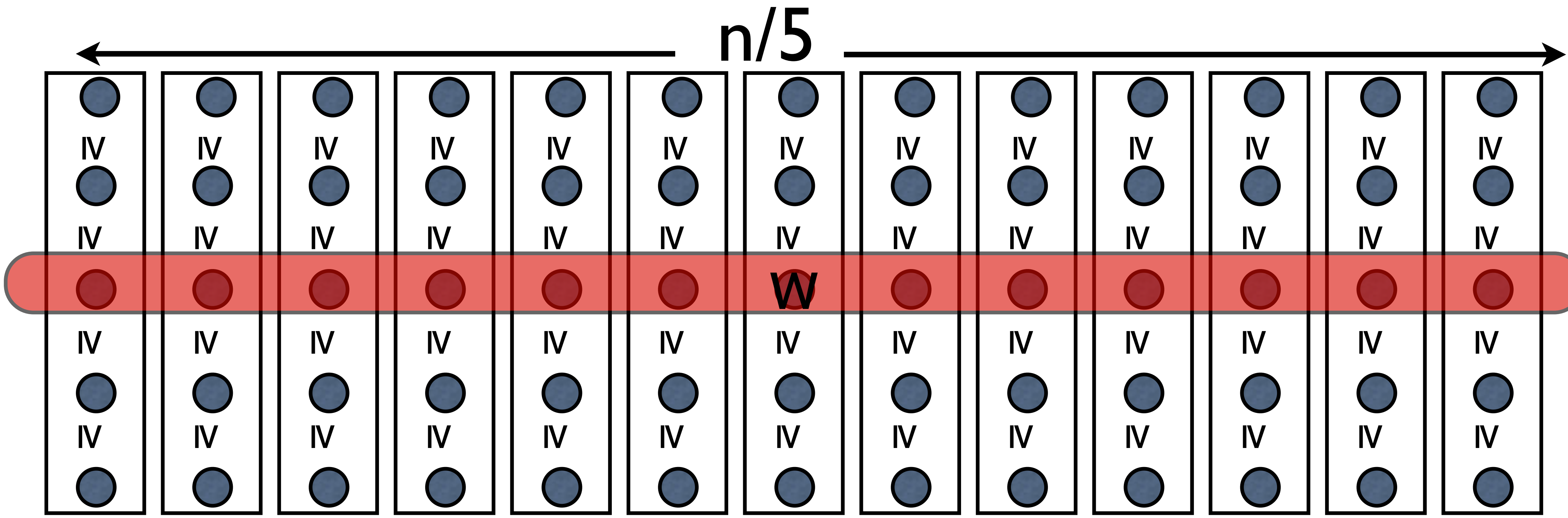
$$(\beta+\gamma)n$$

$$(\beta+\gamma)^2 n$$

$$(\beta+\gamma)^3 n$$

$$n \sum_i (\beta+\gamma)^i$$

Algorithm: Sel(numbers,k)



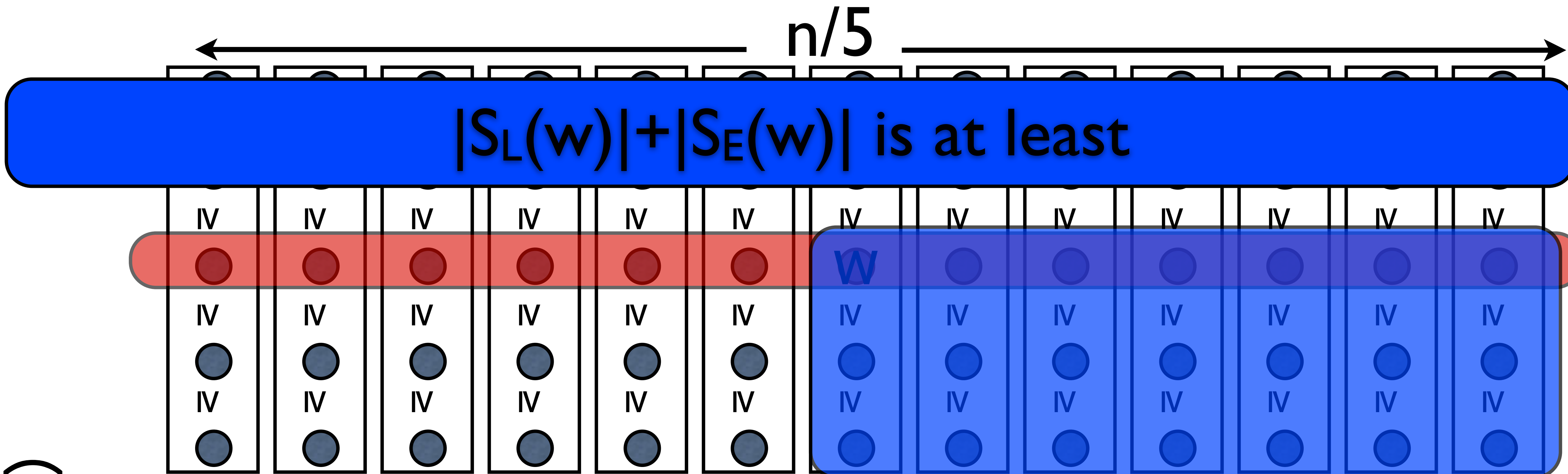
1. Partition numbers into sets of size 5
2. Sort each set
3. w = median of 

$S_L(w) = \{x_i \mid x_i < w\}$
$S_E(w) = \{x_i \mid x_i = w\}$
$S_G(w) = \{x_i \mid x_i > w\}$

} Can be computed in linear time

4. if $k \leq |S_L(w)|$, output $Sel(S_L(w), k)$
   else if $k \leq |S_L(w)| + |S_E(w)|$, output w
   else output $Sel(S_G(w), k - |S_L(w)| - |S_E(w)|)$

Algorithm: Sel(numbers,k)



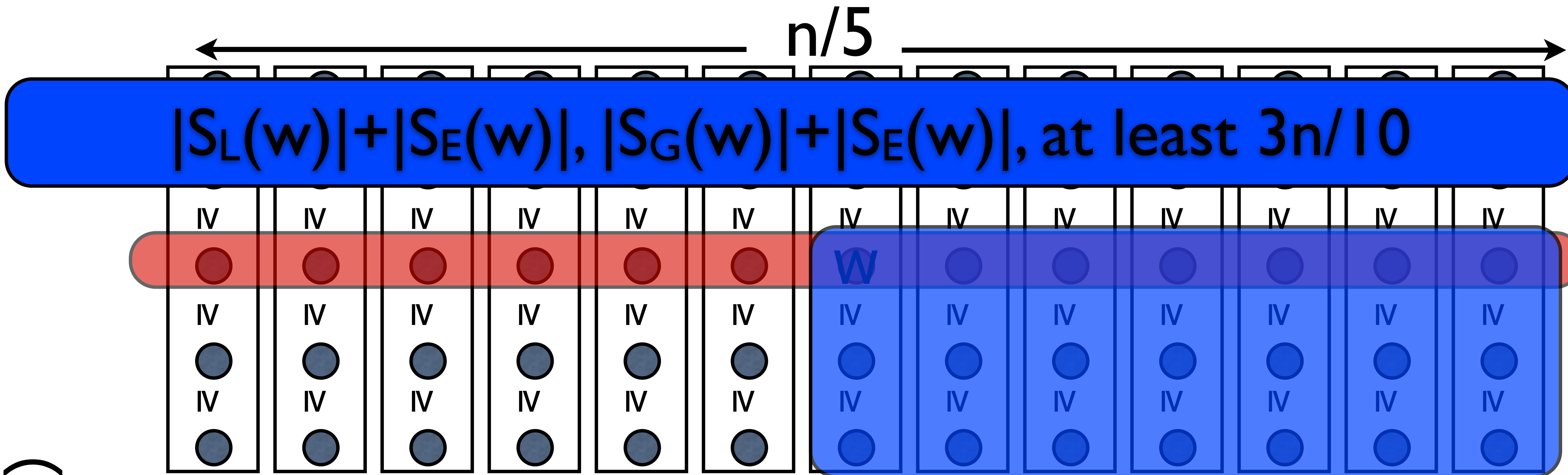
1. Partition numbers into sets of size 5
2. Sort each set
3. w = median of

$S_L(w) = \{x_i \mid x_i < w\}$
$S_E(w) = \{x_i \mid x_i = w\}$
$S_G(w) = \{x_i \mid x_i > w\}$

} Can be computed in linear time

4. if $k \leq |S_L(w)|$, output $Sel(S_L(w), k)$
   else if $k \leq |S_L(w)| + |S_E(w)|$, output w
   else output $Sel(S_G(w), k - |S_L(w)| - |S_E(w)|)$

Algorithm: Sel(numbers,k)



1. Partition numbers into sets of size 5
2. Sort each set
3. w = median of

$S_L(w) = \{x_i \mid x_i < w\}$
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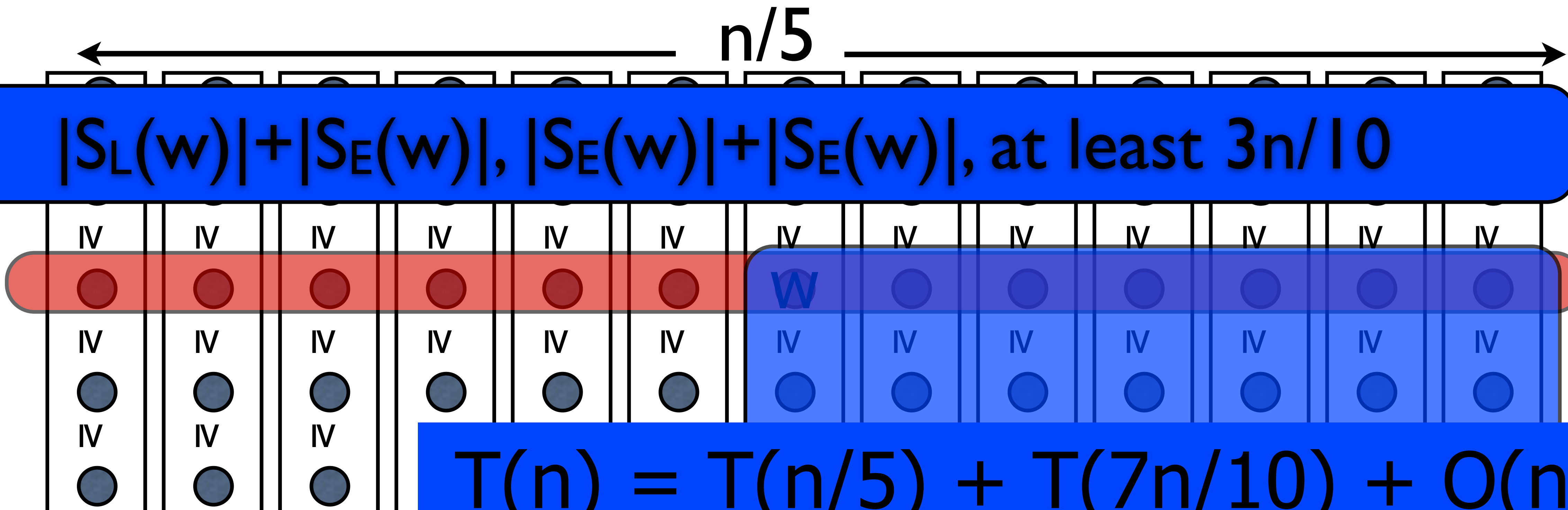
} Can be computed in linear time

4. if $k \leq |S_L(w)|$, output Sel($S_L(w)$, k)
   else if $k \leq |S_L(w)| + |S_E(w)|$, output w
   else output Sel($S_G(w)$, k - $|S_L(w)|$-$|S_E(w)|$)

Algorithm: Sel(numbers,k)



$|S_L(w)|+|S_E(w)|$, $|S_E(w)|+|S_E(w)|$, at least 3n/10

1. Partition nu

2. Sort each s

3. w = mediar

$$T(n) = T(n/5) + T(7n/10) + O(n)$$

so

$$T(n) = O(n)$$

$$S_L(w) = \{x_i \mid x_i < w\}$$
$$S_E(w) = \{x_i \mid x_i = w\}$$
$$S_G(w) = \{x_i \mid x_i > w\}$$

} Can be computed in linear time

4. if $k \leq |S_L(w)|$, output Sel($S_L(w)$, k)
   else if $k \leq |S_L(w)| + |S_E(w)|$, output w
   else output Sel($S_G(w)$, k - $|S_L(w)|$-$|S_E(w)|$)