NAME: ____________________________

CSE 421
Introduction to Algorithms
Sample Midterm Exam Autumn 2021

Anup Rao

1 November 2021

DIRECTIONS:

- Answer the problems on the exam paper.
- Justify all answers with proofs, unless the facts you need have been proved in class or in the book.
- If you need extra space use the back of a page
- You have 50 minutes to complete the exam.
- Please do not turn the exam over until you are instructed to do so.
- Good Luck!

1	/45
2	/20
3	/25
Total	/90
Extra	/10

1. (45 points, 5 each) For each of the following problems answer **True** or **False** and BRIEFLY JUSTIFY you answer.

   (a) $2^{(\log n)^2} = O(n^{1.5})$.

   (b) Suppose we are given a connected undirected graph with distinct edge weights. We greedily delete the heaviest edge that does not disconnect the graph, until we cannot delete any more edges. Then the result is guaranteed to be a minimum spanning tree of the original graph.

   (c) If the running time of an algorithm satisfies the recurrence $T(n) = 3T(n/6) + cn$, for some positive constant c, and $T(1) = 1$, then $T(n) = O(n)$.

   (d) You are given $2^{\sqrt{\ell}}$ sets and promised that some k of them cover the set $\{1, 2, \ldots, \ell\}$. Then the set cover algorithm from class would find $O(k \log \ell)$ of the sets whose union is $\{1, 2, \ldots, \ell\}$.

(e) There is an algorithm for computing the optimal tour in the Traveling Salesperson problem that runs in time $2^{O(n)}$.

(f) You are given a directed graph whose edges have non-negative weights. There is an algorithm for finding the length of the shortest path between every pair of vertices in time $O(n^3)$.

(g) In class, we have seen a polynomial time algorithm for computing the optimal vertex cover of a graph.

(h) The Fast-Fourier transform algorithm is super-easy to understand.

(i) There is an algorithm for multiplying two $n \times n$ matrices in time $O(n^{2.999})$.

2. (20 points) You are given a directed graph that does not have any cycles. Give a polynomial time algorithm that outputs the vertices in an order satisfying that if (a, b) is an edge, then a is output before b. HINT: Find a greedy algorithm for this problem.

3. (25 points) A perfect matching of an undirected graph on $2n$ vertices is a set of n edges such that each vertex is part of exactly one edge. Give a polynomial time algorithm that takes a tree on $2n$ vertices as input and finds a perfect matching in the tree, if there is one.

4. (Extra Credit: 10 points) Is it possible that it is asymptotically faster to square an n-bit number than it is to multiply two n-bit numbers? Justify your answer.