CSE421: Design and Analysis of Algorithms	April 8, 2023

Homework 2

Anup Rao Due: April 16, 2023

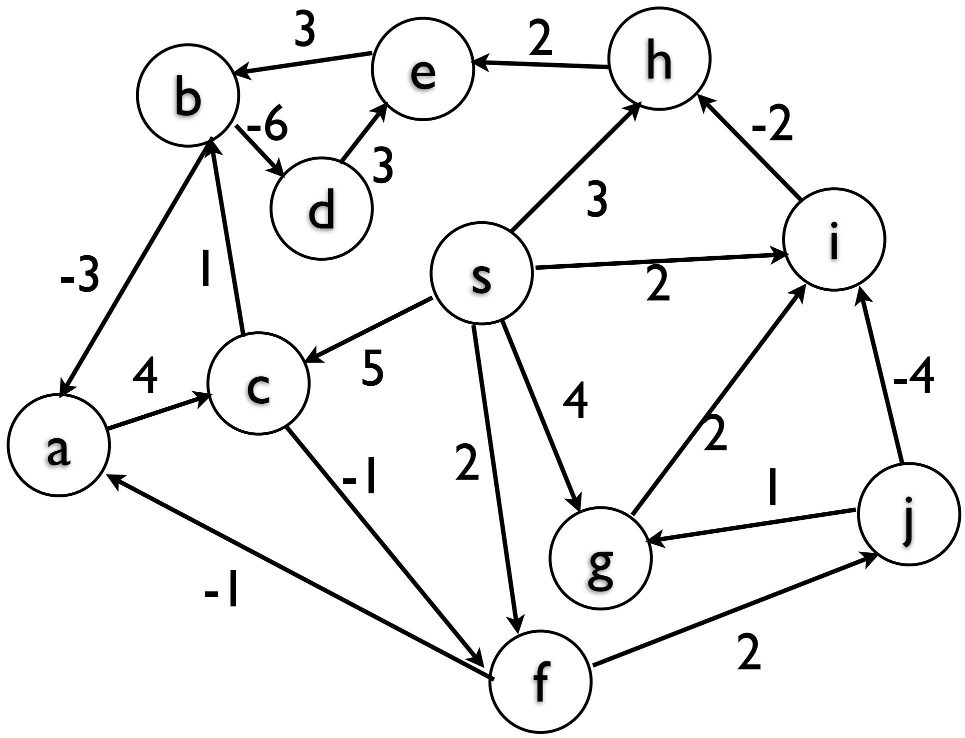
Read the fine print[1]. Each problem is worth 10 points:

1. Prove that in every undirected graph, there must be two vertices that have the same degree. HINT: Argue that If the degrees are all distinct then the vertices must have degrees $0, 1, 2, \dots, n-1$, and this cannot happen.

2. A walk of length k in a graph is a sequence of vertices $v_0, v_1, \dots, v_k$ such that v_i is a neighbor of v_{i+1} for $i = 0, 1, 2, \dots, k-1$. Suppose the product of two $n \times n$ matrices can be computed in time $O(n^\omega)$ for a constant $\omega \geq 2$. Give an algorithm that counts the number of walks of length k in a graph with n vertices in time $O(n^\omega \log k)$. HINT: If A is the adjacency matrix, prove that the $(i,j)'th$ entry of A^k is exactly the number of walks of length k that start at i and end at j. Then, for example, to compute A^8, compute $((A^2)^2)^2$.

3. Compute the shortest path tree for the following graph to find all shortest path distances from s:

[1] In solving the problem sets, you are allowed to collaborate with fellow students taking the class, but **each submission can have at most one author**. If you do collaborate in any way, you must acknowledge, for each problem, the people you worked with on that problem. The problems have been carefully chosen for their pedagogical value, and hence might be similar to those given in past offerings of this course at UW, or similar to other courses at other schools. Using any pre-existing solutions from these sources, for from the web, constitutes a violation of the academic integrity you are expected to exemplify, and is strictly prohibited. Most of the problems only require one or two key ideas for their solution. It will help you a lot to spell out these main ideas so that you can get most of the credit for a problem even if you err on the finer details. Please justify all answers. Some other guidelines for writing good solutions are here: `http://www.cs.washington.edu/education/courses/cse421/08wi/guidelines.pdf`.



You only need to show the shortest path tree for full credit.

4. Prove that in any tree with n vertices, the number of nodes with 3 or more neighbors is at most $2(n-1)/3$. Use the fact that every tree on n vertices has exactly $n-1$ edges, and apply the identity $\sum_v deg(v) = 2m$.