

Homework 4

Anup Rao

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Read the fine print¹. An algorithm is said to run in polynomial time if it runs in time $O(n^d)$ for some constant d on inputs of size n . Each problem is worth 10 points:

1. Prove or disprove the following: Given any undirected graph G with weighted edges, and a minimum spanning tree T for that G , there exists some sorting of the edge weights $w(e_1) \leq w(e_2) \leq \dots \leq w(e_m)$, such that running Kruskal's algorithm with that sorting produces the tree T .
2. You are given a graph G with n vertices and m edges, and a minimum spanning tree T of the graph. Suppose one of the edge weights $w(e)$ of the graph is updated.
 - (a) Give an algorithm that runs in time $O(m)$ to test if T still remains the minimum spanning tree of the graph. You may assume that all edge weights are distinct both before and after the update. HINT: If $e \in T$, consider the cut obtained by deleting e from T . If $e \notin T$, consider the cycle formed by adding e to T .
 - (b) Suppose T is no longer the minimum cost spanning tree. Give an $O(m)$ time algorithm to update the tree T to the new minimum cost spanning tree.
3. Suppose you are choosing between the following three algorithms:
 - Algorithm A solves the problem by dividing it into five subproblems of half the size, recursively solves each subproblem, and then combines the solution in linear time.
 - Algorithm B solves problems of size n by recursively solving two subproblems of size $n - 1$, and then combines the solution in constant time.
 - Algorithm C solves the problem by dividing it into nine subproblems of one third the size, recursively solves each subproblem, and then combines the solutions in quadratic time.

What are the running times of each of these algorithms?

4. You are given two sorted lists of integers of length m and n . Give an $O(\log m + \log n)$ time algorithm for computing the k 'th smallest integer in the union of the lists.

¹In solving the problem sets, you are allowed to collaborate with fellow students taking the class, but **each submission can have at most one author**. If you do collaborate in any way, you must acknowledge, for each problem, the people you worked with on that problem. The problems have been carefully chosen for their pedagogical value, and hence might be similar to those given in past offerings of this course at UW, or similar to other courses at other schools. Using any pre-existing solutions from these sources, for from the web, constitutes a violation of the academic integrity you are expected to exemplify, and is strictly prohibited. Most of the problems only require one or two key ideas for their solution. It will help you a lot to spell out these main ideas so that you can get most of the credit for a problem even if you err on the finer details. Please justify all answers. Some other guidelines for writing good solutions are here: <http://www.cs.washington.edu/education/courses/cse421/08wi/guidelines.pdf>.