CSE421: Design and Analysis of Algorithms	May 12, 2023
Homework 6	
Anup Rao	Due: May 21, 2023

Read the fine print[1]. Each problem is worth 10 points:

1. An edge of a flow network is called *critical* if decreasing the capacity of this edge results in a decrease in the maximum flow. Give an efficient algorithm that finds a critical edge in a network.

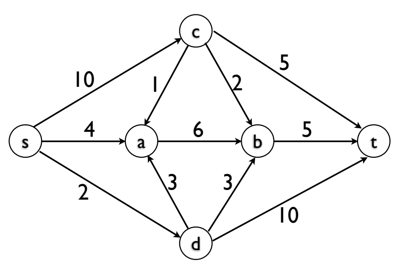
2. Suppose we are given a flow network, where instead of capacities on edges, each internal vertex has a capacity on the total flow that is allowed to pass through it. So for each vertex v, there is a non-negative integer c_v, and the flow must satisfy $f^{in}(v) \leq c_v$. Each edge can carry an arbitrary amount of flow. Give a polynomial time algorithm to find the maximum flow in such a network. (Hint: try to convert the problem into a flow network of the type we are used to.)

3. We all love vertex covers. The reason is that they are subsets of the vertices that touch every edge of an undirected graph. Give an algorithm to find a vertex cover of smallest size in a bipartite graph. Hints:

   (a) Construct a flow network from the input bipartite graph just as in the maximum matching algorithm.

   (b) Show that a "nice" min-cut in this flow network gives a vertex cover. Namely, if the graph has bipartitions A, B and X, Y are the corresponding components of the "nice" min-cut, show that $(A - X) \cup Y$ must be a vertex cover of smallest size.

4. Draw out a maximum $s - t$ flow for the graph below, and the corresponding residual graph G_f. What is the minimum cut that corresponds to this max flow?



[1] In solving the problem sets, you are allowed to collaborate with fellow students taking the class, but **each submission can have at most one author**. If you do collaborate in any way, you must acknowledge, for each problem, the people you worked with on that problem.