CSE431: Complexity Theory April 23, 2021

Homework 2

Anup Rao Due: April 30, 2021

Read the fine print[1]. Each problem is worth 10 points:

1. Consider the following function $h : \{0,1\}^* \to \{0,1\}$.

$$h(\alpha) = \begin{cases} 1 & \text{if there is some } x \text{ such that } M_\alpha(x) \text{ halts with output } 1, \\ 0 & \text{else.} \end{cases}$$

   Someone claims to have a program that can compute h. Prove that their program must have a bug by showing that no turing machine can compute $h(\alpha)$ for every α.

2. If $f, g \in \mathbf{NP}$, discuss whether each of the following functions can be deduced to be in **NP** or not (Warning: the answer may be "unknown"):

   - $h_1(x) = f(x) \wedge g(x)$.
   - $h_2(x) = f(x) \vee g(x)$.
   - $h_3(x) = 1 - f(x)$.

3. Let HALT be the halting function we defined in class (i.e. $\mathsf{HALT}(\alpha, x) = 1$ iff $M_\alpha(x)$ halts). Is HALT **NP**-hard? Is it **NP**-complete?

4. Given an integer $m \times n$ matrix A and an integer column vector b of length m, the 0-1 Integer Programming function is defined as follows:

$$\mathbf{IP}(A, b) = \begin{cases} 1 & \text{if there is a length } n \text{ binary vector } x \text{ that satisfies } Ax \geq b \\ 0 & \text{otherwise.} \end{cases}$$

   Here $Ax \geq b$ means that every coordinate of Ax is at least as large as the corresponding coordinate of b. Prove that **IP** is **NP**-complete, by reducing **3SAT** to it (7 points). Discuss what happens to the complexity of the problem if we remove the requirement that x is binary and allow its coordinates to be real valued (0 points for this part, if you are stuck look up Linear Programming). Discuss what happens if the constraints are m quadratic inequalities like $x_1^2 + x_1 + 5x_2 + x_8 \geq 9$, and the variables are allowed to be real valued (3 points). What can you say about the difficulty of computing whether the set of constraints is satisfiable? HINT: Try to use such quadratic constraints to enforce $x_1 \in \{0, 1\}$.

[1] In solving the problem sets, you are allowed to collaborate with fellow students taking the class, but **each submission can have at most one author**. If you do collaborate in any way, you must acknowledge, for each problem, the people you worked with on that problem. The problems have been carefully chosen for their pedagogical value, and hence might be similar to those given in past offerings of this course at UW, or similar to other courses at other schools. Using any pre-existing solutions from these sources, for from the web, constitutes a violation of the academic integrity you are expected to exemplify, and is strictly prohibited. Most of the problems only require one or two key ideas for their solution. It will help you a lot to spell out these main ideas so that you can get most of the credit for a problem even if you err on the finer details. Please justify all answers. Some other guidelines for writing good solutions are here: `http://www.cs.washington.edu/education/courses/cse421/08wi/guidelines.pdf`.