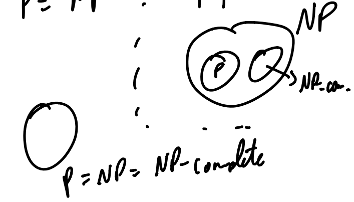
NP-complete

f s.t $f \in NP$ $\forall$ $g \in NP$ $g \leq_p f$.

FACT: If f is NP-complete then $f \in P \iff P = NP$.

$P = NP$. $P \neq NP$



Thm: Circuit-SAT is NP-complete.

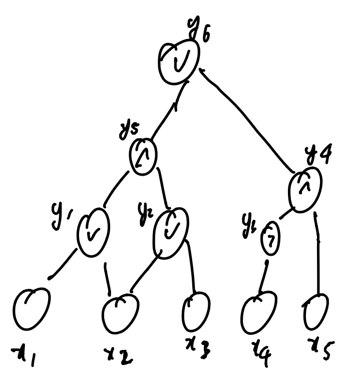
3-SAT

3-CNF (conjunctive normal form)

$$\phi = (x_1 \vee \bar{x}_2 \vee x_3) \wedge (x_5 \vee \bar{x}_1 \vee x_2) \wedge \dots$$

$$\text{3-SAT}(\phi) = \begin{cases} 1 & \text{if } \exists x \text{ s.t } \phi(x) = 1 \\ 0 & \text{o.w.} \end{cases}$$

Goal: Show Circuit-SAT $\leq_p$ 3-SAT.



$\iff$

$y_1 = x_1 \vee x_2$
$y_2 = x_2 \vee x_3$
$y_3 = \neg x_4$
$\vdots$

$\iff$

$\phi_1 \wedge \phi_2 \wedge \phi_3 \cdots \wedge \phi_6 \wedge (y_6)$

ϕ_1 is satisfiable iff $y_1 = x_1 \vee x_2$

ϕ_2 is sat iff $y_2 = x_2 \vee x_3$
$\vdots$

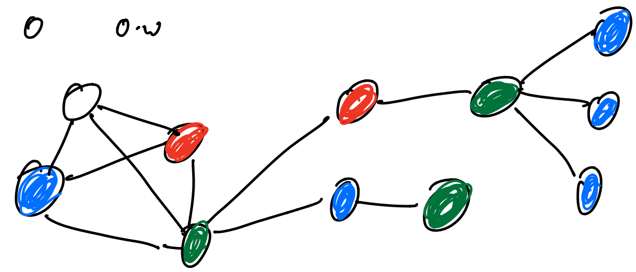
$\phi_3(y_3, x_4, z)$ sat iff $y_3 = \neg x_4$

y_1	x_1	x_2	f
0	0	0	1
1	0	0	0
0	0	1	0
1	0	1	1
0	1	0	0
1	1	0	⋮
0	1	1	
1	1	1	

$(\bar{y}_1 \vee x_1 \vee x_2)$
$\wedge (y_1 \vee x_1 \vee \bar{x}_2)$
$\wedge \dots$

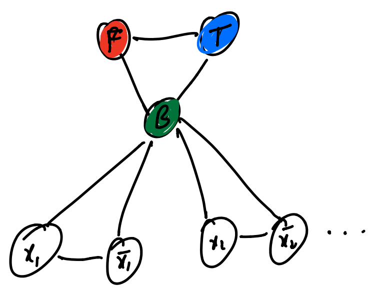
$$3\text{-col}(G) = \begin{cases} 1 & \text{if } G \text{ is 3-colorable} \\ 0 & \text{o.w} \end{cases}$$

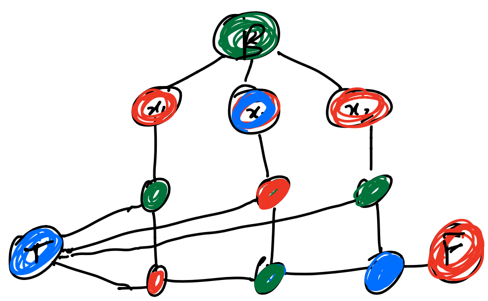
undirected graph



Thm: 3-SAT $\leq_p$ 3-col

$$\phi = (x_1 \vee \bar{x}_2 \vee x_3) \wedge (\bar{x}_5 \vee x_7 \vee x_8) \wedge \ldots$$





$x_1 \vee \bar{x}_2 \vee x_3$

$\forall f \in NP$

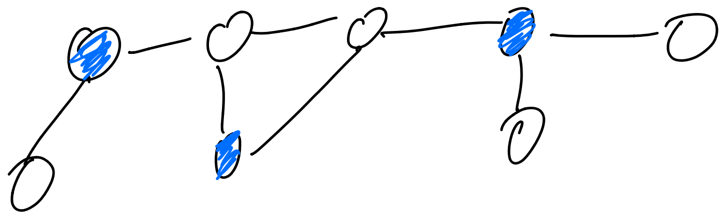
$$f \leq_p \text{Circuit-SAT} \leq_p \text{3-SAT} \leq_p \text{3-col}$$

$f(x) = \text{Circuit-SAT}(h(x))$

$\text{Circuit-SAT}(y) = \text{3-SAT}(h'(y))$

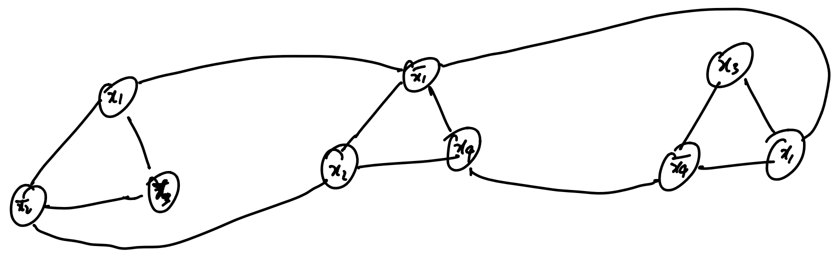
$$\Rightarrow f(x) = \text{3-SAT}(h'(h(x))).$$

$$ISET(G,k) = \begin{cases} 1 & \text{if } G \text{ has an ind set of size } \geq k. \\ 0 & \text{o.w.} \end{cases}$$



3-SAT $\leq_p$ ISET

$(x_1 \vee \bar{x}_2 \vee x_3) \wedge (\bar{x}_1 \vee x_2 \vee x_4) \wedge (x_3 \vee \bar{x}_4 \vee x_1) \ldots$



<u>Hamiltonian Path</u>

$$HPATH(G) = \begin{cases} 1 & \text{if } G \text{ has a path that visits every vertex exactly once} \\ 0 & \text{o.w.} \end{cases}$$

$(x_1 \vee \bar{x}_2 \vee x_3) \wedge \ldots$