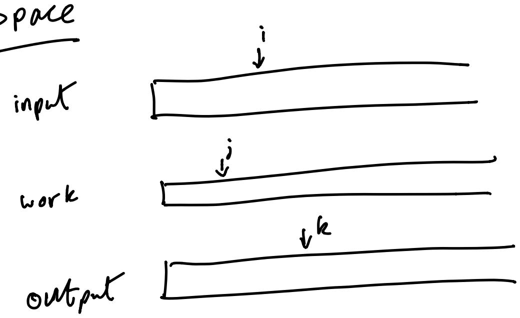
Space



space : max value of j.

$L \subseteq P \subseteq NP \subseteq PSPACE \subsetneq EXP$

$f : \{0,1\}^* \to \{0,1\}^*$ $\quad g : \{0,1\}^* \to \{0,1\}^*$

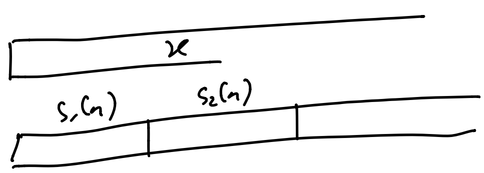
f : computable in space $s_1(n) \geq \log(n)$

g : computable in space $s_2(n) \geq \log(n)$

$f(g(x))$

Claim : $f(g(x))$ can be computed in space $O(s_1(n) + s_2(n))$.

Sketch : Simulate computing f.



Every time you need to read a bit of $g(x)$, compute it using space $s_2(n)$.

Savitch's Algorithm

Input : Directed graph G. (n vertices).

Goal : Is there a path from s to t ?

Breadth-first-search, Depth-first-search.

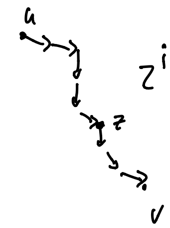
Both : $\Omega(n)$ space.

Savitch : $O(\log^2 n)$ space

OPEN : $O(\log n)$ space ?

$$A(u,v,i) = \begin{cases} 1 & \text{if there is a path of length} \le 2^i \text{ from } u \to v \\ 0 & \text{o.w.} \end{cases}$$

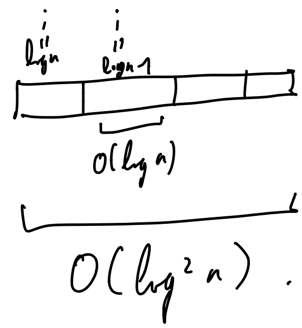
$A(u,v,i) = 1$ iff $\exists z$ s.t. $A(u,z,i-1) = 1$ and $A(z,v,i-1) = 1$



Algorithm for $A(u,v,i)$

If $i > 0$
- Run over all z
  - If $A(u,z,i-1) = A(z,v,i-1) = 1$
    - return 1.
- Else return 0.

If $i = 0$
- Return 1 if $u \to v$ is an edge.



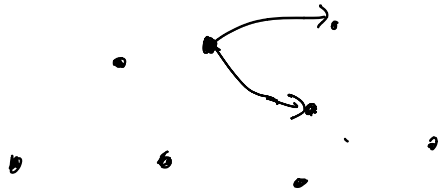
To compute if there is a path $s \to t$
- Set $i = \lceil \log n \rceil$
- Compute $A(s,t,i)$.

Configuration Graph

M: Turing machine (non-deterministic)

x: input

$G_{M,x}$: directed graph.



• $\hookrightarrow$ corresponds to a config

$s(n)$, n - location of all pointers to input, work tapes

$2^{s(n)}$ - value of work tape

$O(1)$ - line # of code

Lemma: If space $s(n) \geq \log(n)$ then # of config is at most $2^{O(s(n))}$.

Theorem: If M is in space $(s(n))$, then for every x, $G_{M,x}$ can be computed in space $O(s(n))$.

- Run over all config in $G_{M,\cancel{x}}$
- Simulate single step of $M(x)$

$$M, x \longrightarrow G_{M,x} \longrightarrow \text{output}$$

$O(s(n))$ space

$$O(\log^2(2^{O(s(n))}))$$
$$= O(s(n)^2)$$

Thm: $NSPACE(s(n)) \subseteq DSPACE(s(n)^2)$.

$$NPSPACE \subseteq PSPACE$$
$$n^c \qquad n^{2c}$$

$$NL \subseteq L^2$$
$$\shortparallel \qquad \shortparallel$$
$$NSPACE(\log n) \qquad DSPACE(\log^2 n)$$

Thm: If $s(n) \geq \log n$, $DSPACE(s(n)) \subseteq DTIME(2^{O(s(n))})$.

$$L \subseteq P$$
$$PSPACE \subseteq EXP$$

$NL = NSPACE(\log n) \subseteq L^2$.

$coNL = \{ g(x) : g(x) = 1 - f(x), f(x) \in NL \}$

NP

$coNP$

$$\boxed{\text{Is } NP = coNP \text{ ?}}$$

$\underline{NP}$

$f \in NP$ ... V

$f(x) = 1 \iff \exists w$ s.t $V(x, w) = 1$

<u>Thm:</u> $NL = coNL$ $(NL \subseteq L^2)$