NAME: _____

CSE 431
Complexity Theory
Sample Midterm Exam

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DIRECTIONS:

- Answer the problems on the exam paper.
- If you need extra space use the back of a page
- You have 80 minutes to complete the exam.
- Good Luck!

1	/50
2	/15
3	/15
Total	/90

1. (50 points, 5 each) For each of the following statements, say whether they are true, false or open according to our current state of knowledge. Briefly justify your answers. Don't worry too much about the difference between open and true or false. Just discuss what you know that is relevant to each statement.

   (a) $\mathsf{ISET} \leq_p \mathsf{3SAT}$ (here ISET is the independent set problem).

   (b) $\mathbf{P}^{\mathsf{3SAT}} = \mathbf{NP}$.

   (c) Either $\mathbf{P} = \mathbf{NP}$ or $\mathbf{NP} = \mathbf{EXP}$.

   (d) $\mathbf{L} \subseteq \mathbf{PSPACE}$.

(e) If $\mathbf{P} = \mathbf{PSPACE}$, then $\mathbf{NL}$ must be strictly smaller than $\mathbf{NP}$.

(f) If $\mathsf{3SAT} \in \mathsf{DTIME}(2^{\log^2 n})$, then $\mathbf{NP} \neq \mathbf{EXP}$.

(g) If $\mathbf{P} \neq \mathbf{NP}$, then for every oracle A, $\mathbf{P}^A \neq \mathbf{NP}^A$.

(h) If $f \in P$, then f can be computed with polynomial sized circuits.

(i) If 3SAT does not have polynomial sized circuits, then $\mathbf{P} \neq \mathbf{NP}$.

(j) There is a function $f : \{0,1\}^* \to \{0,1\}$ that cannot be computed by any Turing machine, yet for every n, $f(x)$ can be computed by a branching program of width n^{10} on inputs of length n.

2. (15 points) Give an example of a function that is computable in exponential space (space $2^{n^{O(1)}}$) yet does not have polynomial sized circuits. HINT: The space is enough to enumerate over all circuits of exponential size.

3. (15 points) Prove that for n large enough, there is a function that can be computed by a circuit of depth $n/2$, but cannot be computed by a circuit of depth $n/3$.