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CSE 431
Computational Complexity Theory
Sample Final Exam

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DIRECTIONS:

- The exam consists entirely of True/False/Open questions. If you get stuck on a particular question, move on and return to it later. For full credit, you must write some kind of explanation (even it is just “we saw this in class”) for every question.
- Good Luck!

For each of the following assertions:

- (3 points) State whether they are True, False, or Unknown to the best of your knowledge of complexity theory.
- (2 points) Briefly justify your answer.
- You will get full credit as long as you discuss all relevant facts we have talked about in class.

1. (25 points, 5 each)

   i) $\mathbf{P} \neq \mathbf{NP}$.

   ii) There is a function $f : \{0,1\}^* \to \{0,1\}$ that cannot be computed by any Turing machine.

   iii) For every n, there is a function $f : \{0,1\}^n \to \{0,1\}$ that cannot be computed by any boolean circuit.

   iv) For n large enough, it holds that there is a function $f : \{0,1\}^n \to \{0,1\}$ that can be computed by a circuit of size $2^{\sqrt{n}}$, but cannot be computed by a circuit of size n^2.

   v) If $\mathbf{P} \neq \mathbf{NP}$, then any algorithm for solving the independent set problem must take time at least 2^n on graphs with n vertices.

2. (25 points, 5 each)

i) Given the code of a Turing machine α and an input x, there is a machine $M(\alpha, x)$ that outputs 1 if the machine corresponding to α halts on input x within $2^{|x|}$ steps, and outputs 0 otherwise.

ii) Every function $f : \{0,1\}^n \to \{0,1\}$ can be computed by a circuit of size $O(2^n/n^2)$.

iii) If there is a deterministic logspace algorithm to check whether or not two vertices in a directed graph are connected, then $\mathbf{L} = \mathbf{NL}$.

iv) There is a non-deterministic logspace algorithm that takes as input directed graph, a vertex s and a number k, and either aborts, or outputs all the vertices at distance k from s in the graph.

v) There is a function in **BPP** that is not in **PSPACE**.

3. (25 points, 5 each)

i) $\mathbf{PSPACE} \subseteq \mathbf{EXP}$.

ii) $co\mathbf{NL} \neq \mathbf{PSPACE}$.

iii) If $\mathbf{P} = \mathbf{NP}$, then $\mathbf{P}^{\mathsf{3SAT}} \subseteq \mathbf{P}$.

iv) Every arithmetic circuit of size s computing a polynomial of degree d can be simulated by a circuit of depth $O((\log(s) + \log d) \log d)$.

v) The problem of determining whether or not a graph can be colored with 3 colors is **NP**-complete.

4. (25 points, 5 each)

   i) There is a randomized **RP** algorithm to check whether or not $AB = C$ for three n by n matrices A, B, C, that runs in time $O(n^2)$.

   ii) co**RP** $\subseteq$ co**NP**.

   iii) *co***RP** $\subseteq$ **BPP**.

   iv) The class **RP** remains the same if the error probability is made 2^{-n} in the definition. (Here, as usual, n is the length of the input.)

   v) Every function $f : \{0,1\}^* \to \{0,1\}$ that is computed by a Turing machine can also be computed by a polynomial sized family of circuits.

5. (25 points, 5 each)

   i) If the permanent can be computed in polynomial time, then $co\mathbf{NP} = \mathbf{NP}$.

   ii) There is an algorithm that can take an undirected graph and two vertices s, t as input and output whether or not there is a path between s and t in $O(\log^2 n)$ space.

   iii) $\mathbf{NL} = co\mathbf{NL}$.

   iv) A non-zero multivariate polynomial of degree d can have at most a finite number of roots.

   v) A non-zero univariate polynomial of degree d can have at most d roots.

6. (25 points, 5 each)

i) There is a function in **IP** which is not computable by polynomial sized circuits.

ii) If the permanent requires arithmetic circuits of size $2^{\Omega(n)}$, then $P \neq PSPACE$.

iii) In the definition of **IP**, if the verifier is restricted to being deterministic, then the class becomes the same as **NP**.

iv) For any oracle A, there is a function $f : \{0,1\}^* \to \{0,1\}$ that cannot be computed in $\mathbf{P}^A$.

v) If $\mathbf{P} = \mathbf{NP}$, then $\mathsf{TQBF} \in \mathbf{P}$.