CSE521: Algorithms	November 23, 2022
Homework 3	
Anup Rao	Due: December 4, 2022

Read the fine print[1]. An algorithm is said to run in polynomial time if it runs in time $O(n^d)$ for some constant d on inputs of size n. Each problem is worth 10 points:

1. Given a sequence of characters $c_1, \ldots, c_n$, we say that a subsequence is a *palindrome* if it reads the same forwards and backwards. For example, "a,b,a,c,a,b,a" is a palindrome. Give an $O(n^2)$ time algorithm to find the longest palindrome subsequence in the input sequence $c_1, \ldots, c_n$. For example, in the sequence $c, l, m, a, l, f, d, c, a, f, m$, the longest palindrome subsequence is m, a, d, a, m. HINT: For $i < j$, let $p(i, j)$ denote the length of the longest palindrome in $x_i, \ldots, x_j$. Express $p(i, j)$ in terms of $p(i + 1, j), p(i, j - 1), p(i + 1, j - 1)$. Evaluate the values $p(i, j)$ in order of increasing $|i - j|$.

2. You are given a rectangular piece of cloth with dimensions $X \times Y$, where X and Y are positive integers, and a list of n products that can be made using the cloth. For each product i you know that a rectangle of cloth of dimensions $a_i \times b_i$ is needed and that the selling price of the product is c_i Assume the a_i, b_i and c_i are all positive integers. You have a machine that can cut any rectangular piece of cloth into two pieces either horizontally or vertically. Design an algorithm that runs in time that is polynomial in X, Y, n and determines the best return on the $X \times Y$ piece of cloth, that is, a strategy for cutting the cloth so that the products made from the resulting pieces give the maximum sum of selling prices. You are free to make as many copies of a given product as you wish, or none, if desired.

3. Give an algorithm to find a vertex cover of smallest size in a bipartite graph. Hints:

   (a) Construct a flow network from the input bipartite graph just as in the maximum matching algorithm.

   (b) Show that every min-cut in this flow network gives a vertex cover whose size is the same as the capacity of the cut.

   (c) Show that every minimum sized vertex cover in the bipartite graph gives a cut whose capacity is the same as the size of the vertex cover.

   (d) Conclude by giving an algorithm to find the smallest vertex cover.

4. You are running a truck business and need to fill a truck that can carry a total weight of 100 tons and volume 30 cubic meters. You can put three types of materials into the truck.

[1] In solving the problem sets, you are allowed to collaborate with fellow students taking the class, but **each submission can have at most one author**. If you do collaborate in any way, you must acknowledge, for each problem, the people you worked with on that problem. The problems have been carefully chosen for their pedagogical value, and hence might be similar to those given in past offerings of this course at UW, or similar to other courses at other schools. Using any pre-existing solutions from these sources, for from the web, constitutes a violation of the academic integrity you are expected to exemplify, and is strictly prohibited. Most of the problems only require one or two key ideas for their solution. It will help you a lot to spell out these main ideas so that you can get most of the credit for a problem even if you err on the finer details. Please justify all answers. Some other guidelines for writing good solutions are here: `http://www.cs.washington.edu/education/courses/cse421/08wi/guidelines.pdf`.

(a) Item 1 has density 2 tons per cubic meter, maximum available amount is 40 cubic meters and the revenue associated with it is 1000 dollars per cubic meter.

(b) Item 2 has density 5 tons per cubic meter, maximum available amount is 20 cubic meters and the revenue associated with it is 2000 dollars per cubic meter.

(c) Item 3 has density 7 tons per cubic meter, maximum available amount is 15 cubic meters and the revenue associated with it is 1500 dollars per cubic meter.

Write a linear program to calculate how much of each amount the truck should carry to maximize profits (no need to solve it).