CSE521: Algorithms December 2, 2022

Homework 4

Anup Rao Due: December 11, 2022

Read the fine print[1]. An algorithm is said to run in polynomial time if it runs in time $O(n^d)$ for some constant d on inputs of size n. Each problem is worth 10 points:

1. Show that if P=NP, then there is a polynomial time algorithm for factoring. Here you are given an n-bit number N, and you need to find a factor a that divides N, with $a \neq 1$, and $a \neq N$, if such an a exists.

2. Compute the dual of the following program:

$$\begin{array}{ll} \text{maximize} & x_1 - 3x_2 + 4x_3 \\ \text{subject to} & 5x_1 + 3x_2 \leq 0 \\ & 4x_1 - x_2 \leq 3 \\ & -x_2 + 3x_3 \leq 2 \\ & x_1 \geq 0 \\ & x_2 \geq 0 \end{array} .$$

3. You are given the following 4 points in the plane:

$$(a_1, b_1) = (1, 3), (a_2, b_2) = (2, 7), (a_3, b_3) = (3, 5), (a_4, b_4) = (4, -1).$$

   You want to find a line that approximately passes through these points. A line

$$\ell_{\alpha,\beta} = \{(x, y) : y = \alpha x + \beta\}$$

   is specified by the numbers α, β. The goal is to find the line that minimizes its error from the point farthest from it. Write a linear program to find the parameters α, β to minimize the error

$$\max_{i=1,2,3,4} |b_i - \alpha \cdot a_i - \beta|.$$

   The program need not be in standard form.

4. Consider a special version of the 3SAT problem, where every clause has exactly 3 literals, and each variable appears at most 3 times. Show that this version of 3SAT can be solved in polynomial time, by giving a polynomial time algorithm that finds a satisfying assignment.

[1] In solving the problem sets, you are allowed to collaborate with fellow students taking the class, but **each submission can have at most one author**. If you do collaborate in any way, you must acknowledge, for each problem, the people you worked with on that problem. The problems have been carefully chosen for their pedagogical value, and hence might be similar to those given in past offerings of this course at UW, or similar to other courses at other schools. Using any pre-existing solutions from these sources, for from the web, constitutes a violation of the academic integrity you are expected to exemplify, and is strictly prohibited. Most of the problems only require one or two key ideas for their solution. It will help you a lot to spell out these main ideas so that you can get most of the credit for a problem even if you err on the finer details. Please justify all answers. Some other guidelines for writing good solutions are here: `http://www.cs.washington.edu/education/courses/cse421/08wi/guidelines.pdf`.

HINT: Consider the bipartite graph with clauses on the left, and variables on the right. Connect a clause to a variable if the variable appears in the clause. Argue that this graph has a perfect matching. Then give an algorithm to find the perfect matching and find a satisfying assignment.