Computational Complexity

What is computation?

- step by step, input → output

$$f : D \longrightarrow R$$

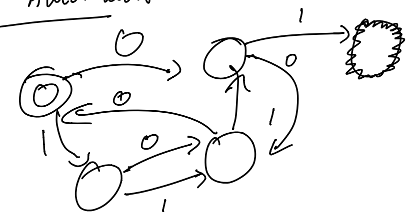
Two simplifications

D is either $\{0,1\}^*$ or $\{0,1\}^n$

R is $\{0,1\}$

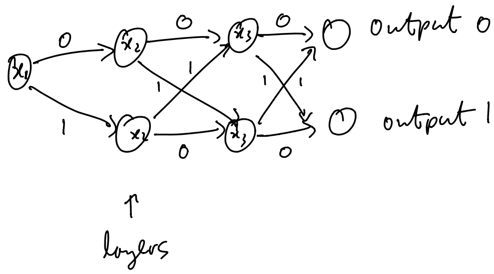
Models of Computation

Finite Automaton



$x = 01101$

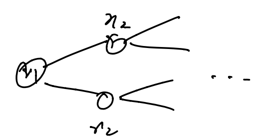
Branching Program $D = \{0,1\}^n$

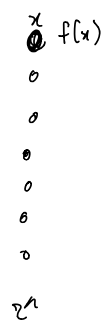


length : # layers

width : max # nodes in a layer.

Claim: Every $f : \{0,1\}^n \to \{0,1\}$ can be computed by a branching program of width 2^n and length n.



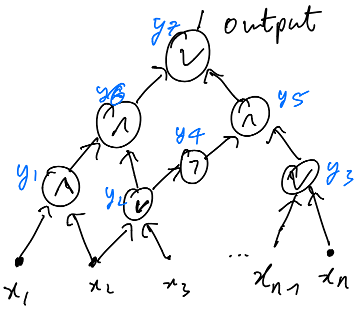


$x_1 \oplus x_2 \oplus \cdots \oplus x_n$

$w = 2$

length $= n$

Boolean Circuits



Size: # gates

depth: length of longest input → output path

Line Program

$y_1 = x_1 \wedge x_2$

$y_2 = x_2 \vee x_3$

$y_3 = x_{n-1} \vee x_n$

$y_4 = \neg y_2$

⋮

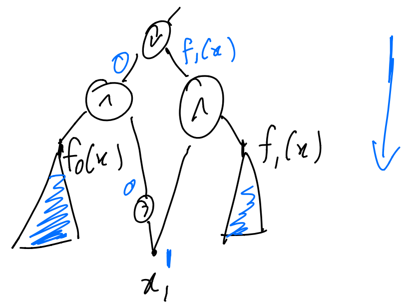
Formula: circuit that is a tree

Claim: Every $f: \{0,1\}^n \to \{0,1\}$ can be computed by a circuit of size $2^{O(n)}$ and depth $O(n)$.

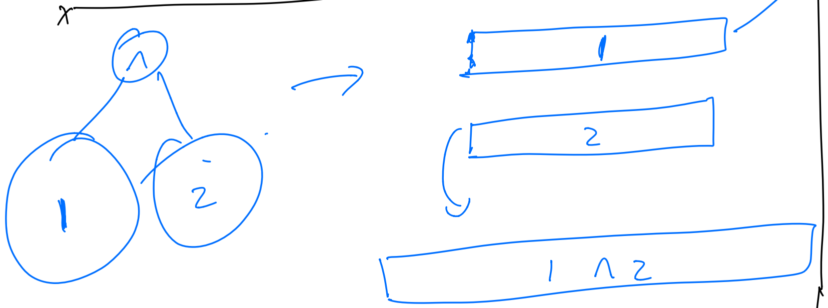
$f(x_1, \dots, x_n)$

$f_0(x_2, \dots, x_n) = f(0, x_2, \dots, x_n)$

$f_1(x_2, \dots, x_n) = f(1, x_2, \dots, x_n)$



Thm [Barrington]: If f can be computed by a circuit of depth d, it can be computed by a branching program of width 5, length $2^{O(d)}$.



computes permutation

$\pi: [5] \to [5]$

$\{1,2,3,4,5\}$