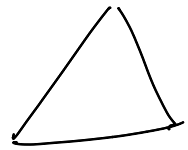
$f(x,y) = ?$

Alice		Bob
x	$\xrightarrow{m_1(x)}$	y
	$\xleftarrow{m_2(y, m_1)}$	
	$\longrightarrow$	

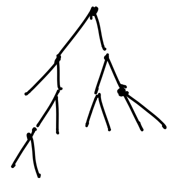
Complexity: # of bits exchanged.

Alice		Bob
$S \subseteq \{1, \ldots, 2n\}$	$\log \binom{2n}{n} \simeq \Omega(n)$	$y \in \{1, \ldots, 2n\}$
$\lvert S \rvert = n$	$\longrightarrow$	$x_2 = y$
$\{x_1, x_3, \ldots, x_{n+1}\} = S$	$x \in S$?	

One way: only Alice speaks.



Formula: underlying graph is a tree

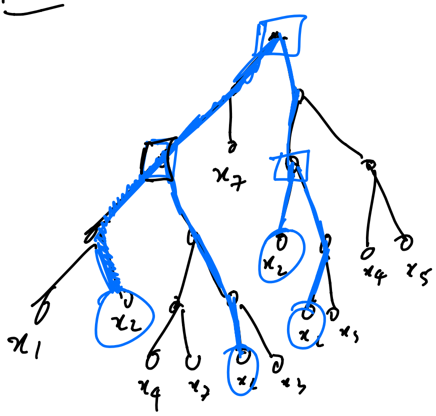


Neciporuk:

$x_1, \ldots, x_{n+1} \in \{1, 2, \ldots, 2n\}$

$$f(x_1, \ldots, x_{n+1}) = \begin{cases} 1 & \text{if } x_1 \ldots x_n \text{ are distinct} \\ 0 & \text{o.w.} \end{cases}$$

Thm: f requires formulas of size $\geq \Omega(n^2)$.



s gates

$\Rightarrow \exists\, i$ s.t. x_i occurs $\leq s/(n+1)$ times.

Claim: # of $\square$s is $O(s/n+1)$.

Alice sends 4 bits per $\square$.

$$\frac{s}{n+1} \geq \Omega(n)$$

$$\Rightarrow s \geq \Omega(n^2).$$

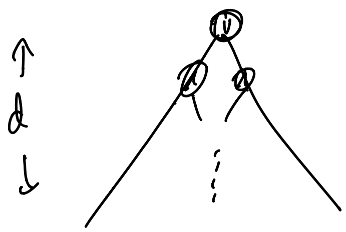
Monotone Depth.

G: graph on n vertices

Does G have a matching of size k?

k disjoint edges

Thm [KRW]: Any monotone circuit has depth $\geq \Omega(n)$.



Alice $\boxed{\text{randomness}}$ Bob

$X \subseteq \{1, .., n\}$ $Y \subseteq \{1, ..., n\}$

Are X, Y disjoint?

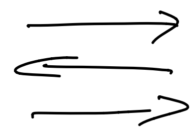
Thm: $\Omega(n)$ communication is reqd.

Alice

G: has a matching of size k

Bob

H: does not have matching of size k



d bits

$e \in G, \ e \notin H$.

Alice $X = \{ 1, 2, \}$

G: block of edges

$Y = \{ 2, 3 \}$

H= all pairs of vertices touching □

Idea: Randomly permute all vertices.
Run protocol from circuit.