- Bounded alternations · formulas
- Monotone circuits
- "Formal Correctness"

$$x_1^2 - x_1 + x_2 \quad \text{vs} \quad x_2$$

$$\mathrm{Perm}(M) = \sum_{\sigma} \prod_{i=1}^{n} M_{i\sigma(i)}$$

$$\left.\mathrm{Det}(M) = \sum_{\sigma} \mathrm{sign}(\sigma) \prod_{i=1}^{n} M_{i\sigma(i)} \right] O(n^3) \text{ time.}$$

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1. If no negative constants then Perm requires exponential size.
2. Every arithmetic circuit of size s, computing poly of degree d can be simulated by size poly(s), depth $O(\log s \cdot \log d)$.

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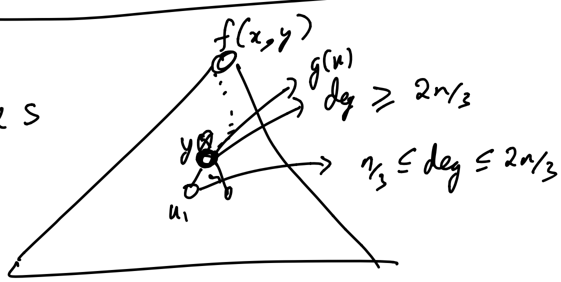
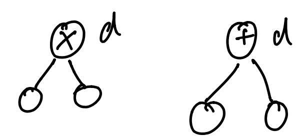
① size s

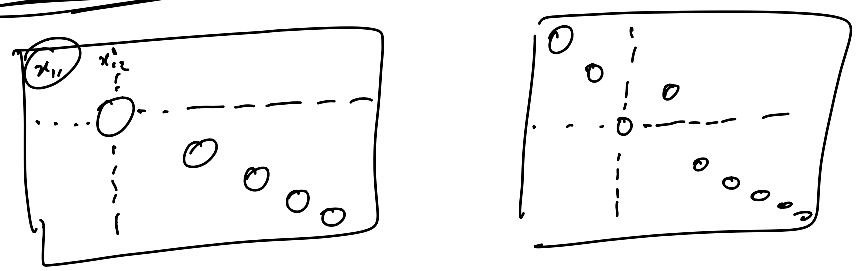




$$\mathrm{Perm}(x) = f(x, g(x)) \qquad a(x) + u_1(x) \cdot v_1(x)$$

Claim: $f(x,y) = a(x) + b(x) \cdot y$.

Claim: $\mathrm{Perm}(x) = \sum_{i=1}^{s} u_i(x) \cdot v_i(x)$ where $n/3 \le \deg(u_i) \le 2n/3$.



Observation: $\deg(u_i) + \deg(v_i) = n$

A = set of rows that contrib variable to $u_i(X)$

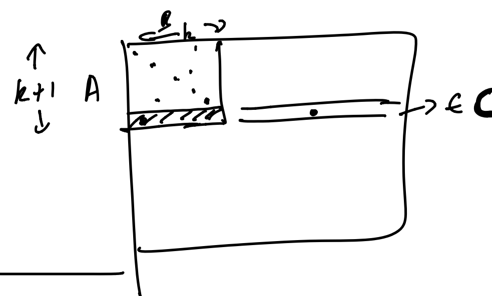
B = " " col " " " " $u_i(X)$

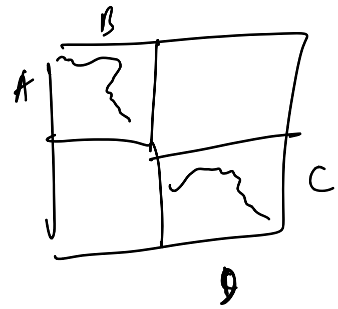
C = " " rows " " " " $v_i(x)$

D = " " col " " " " $v_i(x)$

FACT: $A \cap C = \phi$, $B \cap D = \phi$

FACT: $|A| = |B|$, $|C| = |D|$,





Say $\deg(u_i) = k$ $\quad n/3 \le k \le 2n/3$

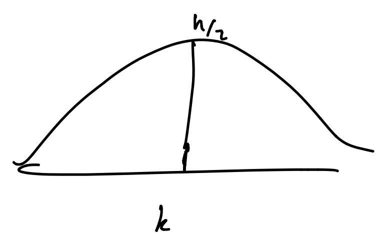
$u_i v_i$ can generate at most

$(k!)(n-k)!$ monomials

$$= \frac{n!}{\binom{n}{k}}$$

$$\le \frac{n!}{\binom{n}{n/3}}$$

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$



So $s \ge \binom{n}{n/3} = 2^{\Omega(n)}$.