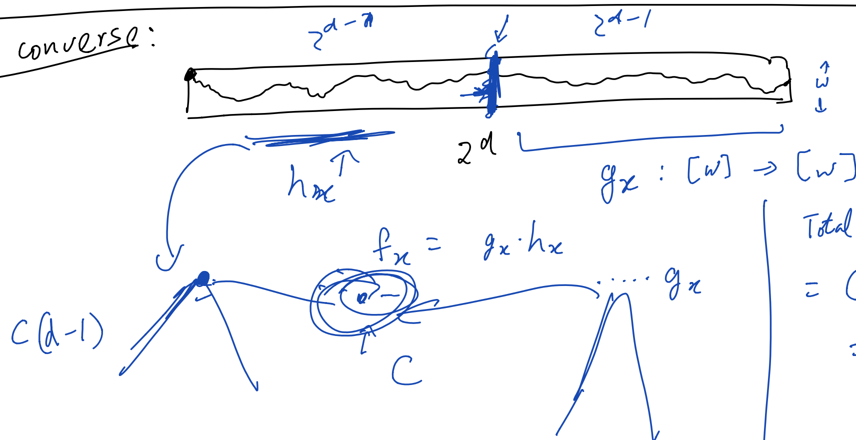
Thm [Barrington]: If f can be computed by a circuit of depth d, it can be computed by a branching program of width 5, length $O(4^d)$.

Converse: If f can be computed by a b.p. of width $O(1)$, length 2^d, then it can be computed by a circuit of depth $O(d) = Cd$

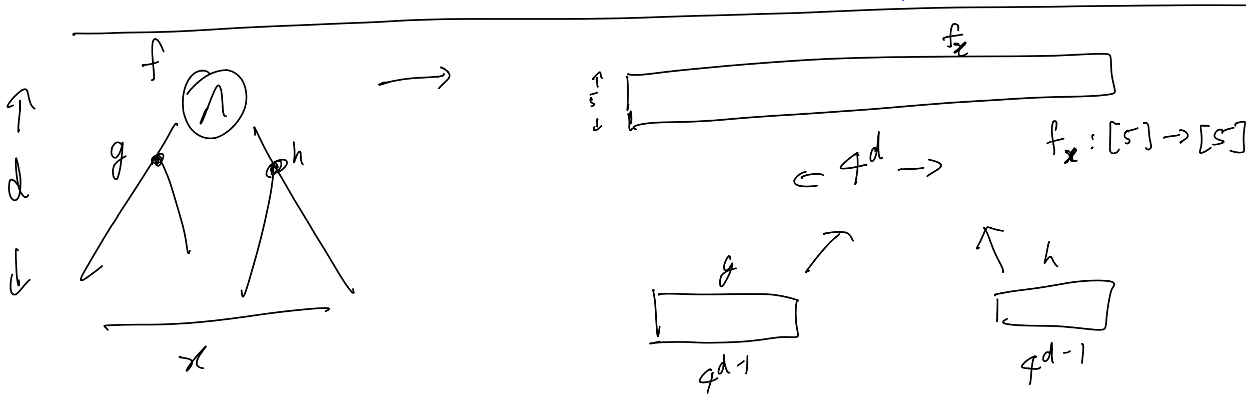
$f: \{0,1\}^n \to \{0,1\}$

$d = \log n$.

Pf of converse:



Total depth $= C + C \cdot (d-1)$
$= C(d)$.



- f_x, g_x, h_x are all permutations.
- f_x either identity or cyclic

Cyclic permutations

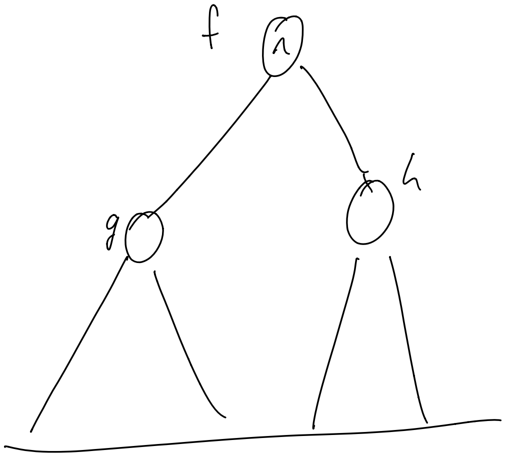
$1 \to 2 \to 3 \to 4 \to 5$ (back to 1) $\qquad 3 \to 4 \to 1 \to 5 \to 2$ (back to 3)

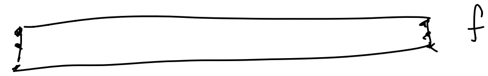
$1 \to 2 \to 3$ (back to 1), $4 \rightleftarrows 5$: not cyclic

- π cyclic $\iff$ π^{-1} cyclic
- Conjugation: If π, σ are cyclic then $\exists\ \tau$ s.t $\tau \pi \tau^{-1} = \sigma$
- Commutator: $\exists$ cyclic π, σ s.t $\pi \sigma \pi^{-1} \sigma^{-1}$ is cyclic.] does not hold for 4

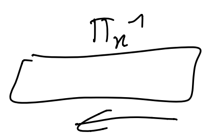
$\pi \cdot \sigma (x) = \pi(\sigma(x))$

$\longrightarrow \quad \gamma_x = \begin{cases} \text{cyclic} & \text{if } f(x) = 1 \\ \text{identity} & \text{if } f(x) = 0 \end{cases}$

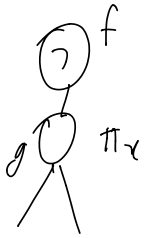




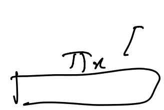
$\gamma_x = \pi_x \sigma_x \pi_x^{-1} \sigma_x^{-1}$

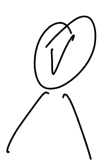


γ : the cyclic



$\longrightarrow$





$\sim \quad a \vee b = \neg(\neg a \wedge \neg b)$