NP-Complete Problems

Thm: Circuit-SAT is NP-complete

Thm: 3-SAT is NP-complete

$\phi = \underline{(x_1 \vee x_2 \vee \bar{x}_3) \wedge (\bar{x}_2 \vee \bar{x}_1 \vee x_5) \cdots}$
$\wedge (\bar{x}_5 \vee x_3 \vee x_1) \cdots$

INDSET $\leq_p$

$\underset{\wedge \atop NP}{X} \leq_p$ 3-SAT $\leq_p$ INDEP. SET

Thm: If ϕ is k-SAT formula each clause shares variables with at most $O(2^{k/2})$ clauses $\Rightarrow$ satisfiable

(Moser). (Constructive Lovasz local-Lemma)

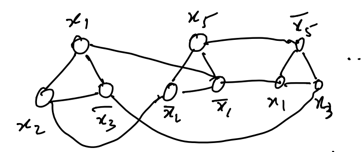
Independent Set

Input: G graph, k

Does G have an independent set of size k?

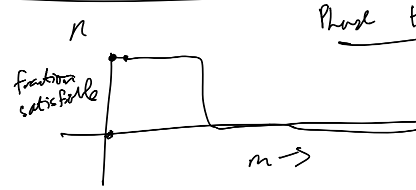
set of vertices with no edges

Thm: Independent set is NP-complete.



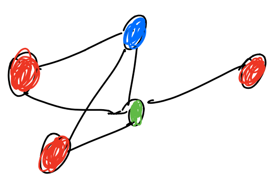
m $\wedge$'s, $k = m$.

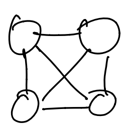
Phase transitions



Graph Coloring

A graph is 3-colorable if vertices can be colored with 3 colors s.t. every edge gets 2 colors.



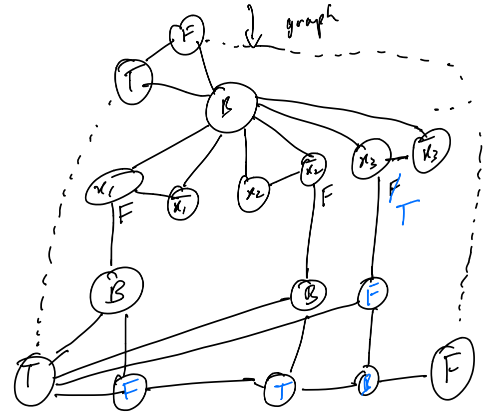


Input: graph G

Is G 3-colorable?

$(x_1 \vee \bar{x}_2 \vee x_3) \wedge (x_4 \vee x_2 \vee \bar{x}_3) \wedge \ldots$

graph



Directed Hamiltonian Cycle

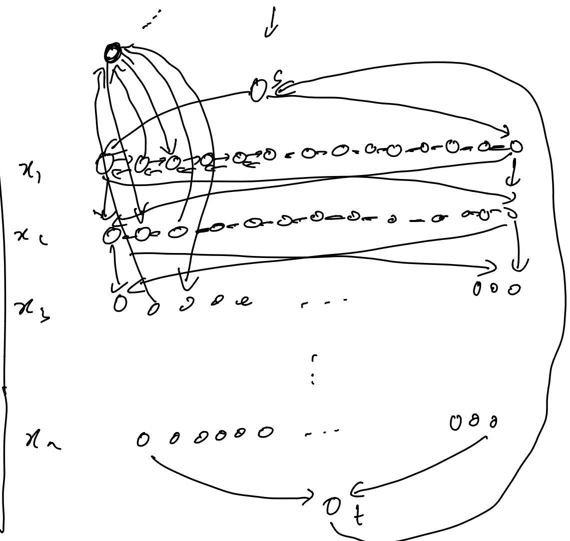
G : directed graph

Hamiltonian cycle : cycle that visits every vertex exactly once.

Input : G

Goal : Is there a Hamiltonian cycle.

$(x_1 \vee \bar{x}_2 \vee x_3) \wedge \cdots$



Subset Sum

Input : $a_1, a_2, \ldots, a_k$, t

Goal : Is there set S

$$\sum_{i \in S} a_i = t?$$

$(x_1 \vee \bar{x}_2 \vee x_3) \wedge \cdots (\bar{x}_1 \vee x_3 \vee x_2)$

t_1	1	0	0	1	0
f_1	1	0	0	0	1
t_2	0	1	0	0	1
f_2	0	1	0	1	0
t_3	0	0	1	1	1
f_3	0	0	1	0	0
b_{11}	0	0	0	1	0
b_{12}	0	0	0	1	0
b_{21}	0	0	0	0	1
b_{22}	0	0	0	0	1
t	1	1	1	3	3

...