Complement Classes

$coP = \{f : 1-f \in P\} = P$

$coEXP = EXP$

$coL = L$

$coNP \overset{?}{=} NP$

$f \in NP \Rightarrow \boxed{f(x)=1 \Leftrightarrow \exists_w V(x,w)=1}$

$f \in coNP \Rightarrow f(x)=0 \Leftrightarrow \boxed{\exists w\ V(x,w)=1}$

If $P = NP$

$coNP = coP = P = NP$

Interesting problems in

$NP \cap coNP$

$f(x,k) =$ k^{th} bit of the factorization of x.

$f \in NP \cap coNP$

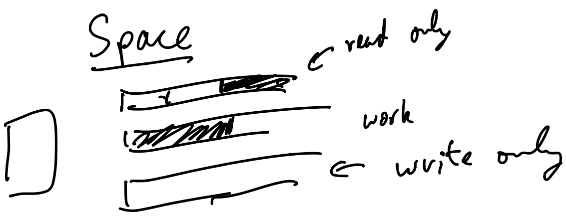
$NP \approx \exists w$ s.t. $V(x,w)=1$

$coNP \approx\ !\exists w$ s.t. $V(x,w)=1$

$\Rightarrow \forall w \quad V(x,w)=0$

$PH \approx \forall w_1 \exists w_2 \forall w_3 \cdots \exists w_k \quad V(x, w_1, w_2, \ldots, w_k) = 1$

Space



$L = DSPACE(\log n)$

PSPACE

NPSPACE

NL

Claim: f computable in space $s_1(n) \geq \log n$

g " " " $s_2(n) \geq \log n$

$\Rightarrow f(g(x))$ computable in space $s_1(n) + s_2(n)$.

Savitch's Algorithm

Input: Directed graph G, two special vertices s, t.

Goal: Is there a path from $s \to t$

Savitch: $O(\log^2 n)$ space is enough.

Open: Can this be done in $O(\log n)$ space?

Reingold: If graph is undirected, yes!

$$A(u,v,i) = \begin{cases} 1 & \text{if } \exists \text{ path } u \to v \text{ of length} \le 2^i \\ 0 & \text{otherwise} \end{cases}$$

To compute $A(u,v,i)$

1. Run over all vertices w

If $\exists w$ s.t.

$A(u,w,i-1) = 1$ and $A(w,v,i-1) = 1$

output 1

Otherwise output 0.

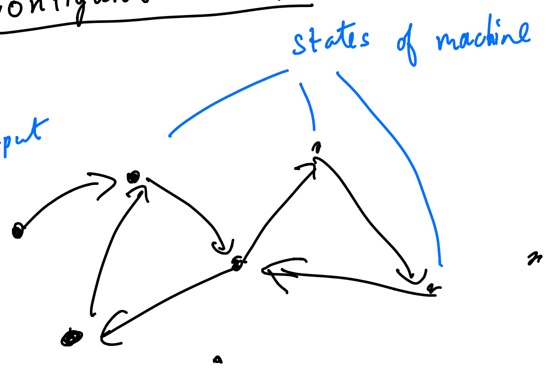
$\Rightarrow A(s,t,\log n)$ can be computed in $O(\log^2 n)$ space.

Configuration Graph

$G_{M,x}$

machine input

space $s(n)$



State

- value of input / work pointers
- contents of work tape
- line # of program

\# vertices

$\le O(n \cdot s(n) \cdot 2^{s(n)})$

$\le O(2^{\log n + s(n) + \log(s n)}) \le 2^{O(s(n))}$

when $s(n) \ge \log n$

Thm: If M has space $s(n) \ge \log n$, then $G_{M,x}$ can be computed in space $O(s(n))$.

Thm: If $s(n) \ge \log n$, $\mathrm{DSPACE}(s(n)) \subseteq \mathrm{DTIME}(2^{O(s(n))})$.

Corollary: $L \subseteq P$

Cor: $\mathrm{PSPACE} \subseteq \mathrm{EXP}$

<u>Thm (Savitch)</u>: For any space constructible $s(n)$,

$$NSPACE(s(n)) \subseteq DSPACE(s(n)^2).$$

$NL \subseteq DSPACE(\log^2 n) = L^2$ $\quad\quad$ $NL \subseteq L^2$

$NPSPACE \subseteq PSPACE$

RL : randomized log space algorithm.

<u>Thm:</u> (Saks-Zhou) $RL \subseteq L^{3/2}$.

A $n \times n$ matrix

approximate A^n in space $L^{3/2}$.

<u>Thm:</u> $NL = coNL$ (next time).