

Homework 2

*Anup Rao**Due: February 6, 2012*

Notes: You must work on the homework by yourself. Each problem is worth 10 points.

1. Suppose $f, g \in \mathbf{NP} \cap \mathit{coNP}$, then show that the exclusive or $f \oplus g \in \mathbf{NP} \cap \mathit{coNP}$.
2. Let **CONNECTED** denote the problem of deciding when a directed graph is strongly connected: namely deciding whether there is a path from u to v , for every pair of vertices u, v . Show that every boolean function in **NL** can be reduced to **CONNECTED** using log space.
3. Show that it is possible to determine whether an undirected graph is bipartite or not in **NL**.
4. Show that if f is in **NP**, then there is a polynomial p , and a machine M using $O(\log n)$ space such that for every x , $f(x) = 1$ if and only if there exists $w \in \{0, 1\}^{p(|x|)}$ such that $M(x, w) = 1$. (Thus, for the definition of **NL** it is crucial that the machine not be allowed to store its guesses for free). **HINT**: Appeal to the fact that every problem in **NP** can be reduced to **SAT** in log space.