CSE531: Computational Complexity Theory February 26, 2012

Homework 3

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Notes: You must work on the homework by yourself. Each problem is worth 10 points.

1. Consider the class of functions that have a single round interactive proof: the prover sends a single message, which the randomized verifier uses to verify that the value of the function is 1. Show that the set of functions verifiable with such proofs is contained in $\mathbf{NP}^{\mathsf{SAT}}$. HINT: Use the same method as the proof that shows that $\mathbf{BPP} \subseteq \mathbf{NP}^{\mathsf{SAT}}$.

2. Show that if $c(c+d) < 2$, then SAT cannot be computed by a Turing machine that uses n^c time and n^d space.

3. Recall that we defined an expander to be a constant degree graph for which for every subset S of at most $n/2$ vertices, $|\Gamma(S)| \geq (1 + \Omega(1))|S|$. We also defined the edge expansion of a constant degree graph to be

$$h(G) = \min_{S, |S| \leq n/2} \frac{\text{\# edges coming out of S}}{|S|}.$$

   Prove that the graph is an expander if and only if $h(G) = \Omega(1)$.

4. Suppose you are given a d-regular expander graph such that all eigenvalues of the normalized adjacency matrix except the largest one are at most λ in magnitude. Show that if S, T are two sets of vertices in the graph, and E is the number of edges going from S to T, then $|E - \frac{d|S||T|}{n}| < \lambda d \sqrt{|S||T|}$. HINT: Use the fact that if $1_S, 1_T$ are the corresponding indicator vectors, and A is the adjacency matrix, then $E = 1_S^t A 1_T$, and the Cauchy-Schwartz inequality.