CSE531: Complexity Theory May 23, 2022

Homework 4

Anup Rao Due: May 29, 2022

Read the fine print[1]. Each problem is worth 10 points:

1. Prove that **NP** $\neq$ co-**NP** implies that **BPP** $\neq$ **NP**.

2. Use Gaussian-Elimination to describe an arithmetic circuit of size $O(n^3)$ for computing the determinant of an $n \times n$ matrix.

3. In class we defined an artithmetic circuit to be one that has $+$ and $\times$ gates. Here we discuss the case where / (division) gates are allowed. If there are / gates, then the output of the circuit can be viewed as $p(X)/q(X)$, where p, q are polynomials.

   (a) Show that if there is a polynomial sized circuit computing $p(X)/q(X)$, then there is a polynomial sized circuit that simultaneously computes both $p(X)$ and $q(X)$, without using any division gates. Here we show that such a circuit can be simulated without division.

   (b) Suppose the original circuit computes a polynomial $f(X) = p(X)/q(X)$, so $q(X)$ is promised to divide $p(X)$. Assume that $f(X)$ is of degree n. Show that you can use the result of the previous step to compute $f(X)$ without division. To do this, observe that if $q(0) = 1$, then $(1 - q(X))$ and has no constant term, and we have

$$\frac{p(X)}{q(X)} = \frac{p(X)}{1 - (1 - q(X))} = \sum_{i=0}^{\infty} p(X) \cdot (1 - q(X))^i,$$

   and so the degree d homogenous part of $f(X)$ can be computed from this expression. Finally, use this formula to compute f without any division gates.

[1] In solving the problem sets, you are allowed to collaborate with fellow students taking the class, but **each submission can have at most one author**. If you do collaborate in any way, you must acknowledge, for each problem, the people you worked with on that problem. The problems have been carefully chosen for their pedagogical value, and hence might be similar to those given in past offerings of this course at UW, or similar to other courses at other schools. Using any pre-existing solutions from these sources, for from the web, constitutes a violation of the academic integrity you are expected to exemplify, and is strictly prohibited. Most of the problems only require one or two key ideas for their solution. It will help you a lot to spell out these main ideas so that you can get most of the credit for a problem even if you err on the finer details. Please justify all answers. Some other guidelines for writing good solutions are here: `http://www.cs.washington.edu/education/courses/cse421/08wi/guidelines.pdf`.