CSE531: Complexity Theory	January 27, 2022

Homework 2

Anup Rao Due: February 6, 2022

Read the fine print[1]. Each problem is worth 10 points:

1. Consider the following function $h : \{0,1\}^* \to \{0,1\}$.

$$h(\alpha) = \begin{cases} 1 & \text{if there is some } x \text{ such that } M_\alpha(x) \text{ halts with output 1,} \\ 0 & \text{else.} \end{cases}$$

   Someone claims to have a program that can compute h. Prove that their program must have a bug by showing that no turing machine can compute $h(\alpha)$ for every α.

2. Give an example of a function can be computed by polynomial sized circuits, but cannot be computed by a turing machine. HINT: use one of the functions that cannot be computed by Turing machines, and the fact that circuits can compute every function.

3. Show that if f is **NP**-complete and $f \in \mathsf{DTIME}(2^n)$, then $\mathbf{NP} \neq \mathbf{EXP}$.

4. Let HALT be the halting function we defined in class (i.e. $\mathsf{HALT}(\alpha, x) = 1$ iff $M_\alpha(x)$ halts). Show that HALT is **NP**-hard. Is it **NP**-complete?

[1] In solving the problem sets, you are allowed to collaborate with fellow students taking the class, but **each submission can have at most one author**. If you do collaborate in any way, you must acknowledge, for each problem, the people you worked with on that problem. The problems have been carefully chosen for their pedagogical value, and hence might be similar to those given in past offerings of this course at UW, or similar to other courses at other schools. Using any pre-existing solutions from these sources, for from the web, constitutes a violation of the academic integrity you are expected to exemplify, and is strictly prohibited. Most of the problems only require one or two key ideas for their solution. It will help you a lot to spell out these main ideas so that you can get most of the credit for a problem even if you err on the finer details. Please justify all answers. Some other guidelines for writing good solutions are here: `http://www.cs.washington.edu/education/courses/cse421/08wi/guidelines.pdf`.