

NAME: \_\_\_\_\_

CSE 531  
Computational Complexity Theory  
Final Exam, Autumn 2023

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December 9, 2023

DIRECTIONS:

- Open book. Open notes.
- The exam consists entirely of True/False/Open questions. For full credit, you must write some kind of explanation (even it is just “we saw this in class”) for every question.
- The exam is due Sunday, December 17, at midnight.
- Good Luck!

For each of the following assertions:

- (3 points) State whether they are True, False, or Unknown to the best of your knowledge of complexity theory.
- (2 points) Briefly justify your answer.

1. (25 points, 5 each)

- i) There is a polynomial time algorithm that can take an integer  $x$  as input outputs a **TQBF** formula that is true if and only if  $x$  has a prime factor that is at most  $x/10$ .
- ii) There is a function  $f : \{0, 1\}^* \rightarrow \{0, 1\}$  that is not computable in **BPP**.
- iii) There is a BPP algorithm for checking whether or not a given Boolean circuit computes the function which always outputs 0.
- iv) There is a BPP algorithm for checking whether or not a given arithmetic circuit computes a polynomial that is the 0 polynomial.
- v) Graph non-isomorphism is in **NP** as well as in **coNP**.

2. (25 points, 5 each)

- i) 3-SAT is in **IP**.
- ii) The class **RP** remains the same if the error probability is made  $2/3$  instead of  $1/3$ .
- iii) In the definition of **IP**, if the verifier is restricted to being deterministic, then the class becomes the same as **NP**.
- iv) If  $\mathbf{P} = \mathbf{NP}$ , then  $\mathbf{TQBF} \in \mathbf{P}$ .
- v)  $\mathbf{TQBF} \in \mathbf{L}$ .

3. (25 points, 5 each)

- i) If the permanent can be computed in polynomial time, then  $\mathbf{coNP} = \mathbf{NP}$ .
- ii) There is an algorithm that can take an undirected graph and two vertices  $s, t$  as input and output whether or not there is a path between  $s$  and  $t$  in  $O(\log^2 n)$  space.
- iii)  $\mathbf{NL} = \mathbf{coNL}$ .
- iv) A non-zero multivariate polynomial of degree  $d$  can have at most  $d$  roots.
- v) If  $\mathbf{coNP} = \mathbf{NP}$ , then since  $\forall x, \phi(x)$  is equivalent to  $\neg \exists x, (\neg \phi(x))$ ,  $\mathbf{TQBF} \in \mathbf{NP}$ .

4. (25 points, 5 each)

- i) If  $f \in \mathbf{BPP}$ , then for there is a constant  $c$  such that for every  $n$ , there is a circuit of size  $O(n^c)$  that can compute  $f$  on  $n$ -bit inputs.
- ii) If  $\mathbf{P} = \mathbf{NP}$ , then  $\mathbf{P} = \mathbf{PSPACE}$ .
- iii) If  $\mathbf{NP} = \mathbf{EXP}$ , then 3-SAT does not have a polynomial time algorithm.
- iv)  $\mathbf{BPP} \subseteq \mathbf{PSPACE}$ .
- v)  $\mathbf{BPP} \subseteq \mathbf{ZPP}$ .