

NAME: _____

CSE 531
Computational Complexity Theory
Final Exam, Autumn 2023

Anup Rao

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DIRECTIONS:

- Open book. Open notes.
- The exam consists entirely of True/False/Open questions. For full credit, you must write some kind of explanation (even it is just “we saw this in class”) for every question.
- The exam is due Sunday, December 17, at midnight.
- Good Luck!

For each of the following assertions:

- (3 points) State whether they are True, False, or Unknown to the best of your knowledge of complexity theory.
- (2 points) Briefly justify your answer.

1. (25 points, 5 each)

- i) There is a polynomial time algorithm that can take an integer x as input outputs a **TQBF** formula that is true if and only if x has a prime factor that is at most $x/10$.
- ii) There is a function $f : \{0, 1\}^* \rightarrow \{0, 1\}$ that is not computable in **BPP**.
- iii) There is a BPP algorithm for checking whether or not a given Boolean circuit computes the function which always outputs 0.
- iv) There is a BPP algorithm for checking whether or not a given arithmetic circuit computes a polynomial that is the 0 polynomial.
- v) Graph non-isomorphism is in **NP** as well as in **coNP**.

2. (25 points, 5 each)

- i) 3-SAT is in **IP**.
- ii) The class **RP** remains the same if the error probability is made $2/3$ instead of $1/3$.
- iii) In the definition of **IP**, if the verifier is restricted to being deterministic, then the class becomes the same as **NP**.
- iv) If $\mathbf{P} = \mathbf{NP}$, then $\mathbf{TQBF} \in \mathbf{P}$.
- v) $\mathbf{TQBF} \in \mathbf{L}$.

3. (25 points, 5 each)

- i) If the permanent can be computed in polynomial time, then $\mathbf{coNP} = \mathbf{NP}$.
- ii) There is an algorithm that can take an undirected graph and two vertices s, t as input and output whether or not there is a path between s and t in $O(\log^2 n)$ space.
- iii) $\mathbf{NL} = \mathbf{coNL}$.
- iv) A non-zero multivariate polynomial of degree d can have at most d roots.
- v) If $\mathbf{coNP} = \mathbf{NP}$, then since $\forall x, \phi(x)$ is equivalent to $\neg \exists x, (\neg \phi(x))$, $\mathbf{TQBF} \in \mathbf{NP}$.

4. (25 points, 5 each)

- i) If $f \in \mathbf{BPP}$, then for there is a constant c such that for every n , there is a circuit of size $O(n^c)$ that can compute f on n -bit inputs.
- ii) If $\mathbf{P} = \mathbf{NP}$, then $\mathbf{P} = \mathbf{PSPACE}$.
- iii) If $\mathbf{NP} = \mathbf{EXP}$, then 3-SAT does not have a polynomial time algorithm.
- iv) $\mathbf{BPP} \subseteq \mathbf{PSPACE}$.
- v) $\mathbf{BPP} \subseteq \mathbf{ZPP}$.