CSE531: Complexity Theory	November 3, 2023
Midterm	
Anup Rao	Due: November 12, 2023

For each of the following assertions:

- (3 points) State whether they are True, False, or Unknown to the best of your knowledge of complexity theory.
- (2 points) Briefly justify your answer.

1. (25 points, 5 each)
   (a) $\mathbf{P} \neq \mathbf{NP}$.
   (b) There is a function $f : \{0,1\}^* \to \{0,1\}$ that cannot be computed by any Turing machine.
   (c) For every n, there is a function $f : \{0,1\}^n \to \{0,1\}$ that cannot be computed by any boolean circuit.
   (d) For n large enough, it holds that there is a function $f : \{0,1\}^n \to \{0,1\}$ that can be computed by a circuit of size $2^{\sqrt{n}}$, but cannot be computed by a circuit of size n^2.
   (e) If $\mathbf{P} \neq \mathbf{NP}$, then any algorithm for solving the independent set problem must take time at least 2^n on graphs with n vertices.

2. (25 points, 5 each)
   (a) Given the code of a Turing machine α and an input x, there is a machine $M(\alpha, x)$ that outputs 1 if the machine corresponding to α halts on input x within $2^{|x|}$ steps, and outputs 0 otherwise.
   (b) Every function $f : \{0,1\}^n \to \{0,1\}$ can be computed by a circuit of size $O(2^n/n^2)$.
   (c) If there is a deterministic logspace algorithm to check whether or not two vertices in a directed graph are connected, then $\mathbf{L} = \mathbf{NL}$.
   (d) There is a non-deterministic logspace algorithm that takes as input directed graph, a vertex s and a number k, and either aborts, or outputs all the vertices at distance k from s in the graph.
   (e) There is a function in **BPP** that is not in **PSPACE**.

3. (25 points, 5 each)
   (a) $\mathbf{PSPACE} \subseteq \mathbf{EXP}$.
   (b) $co\mathbf{NL} \neq \mathbf{PSPACE}$.
   (c) **BPP** is equal to $\mathbf{RP} \cap co\mathbf{RP}$.
   (d) The problem of determining whether or not a graph can be colored with 3 colors is **NP**-complete.
   (e) There is an algorithm for 3SAT that takes n^2 time.

4. (25 points, 5 each)

   (a) If $\mathbf{NP} = co\mathbf{RP}$, then $\mathbf{ZPP} = \mathbf{RP}$.

   (b) $\mathbf{ZPP} \subseteq \mathbf{NP}$.

   (c) $co\mathbf{RP} \subseteq \mathbf{BPP}$.

   (d) The class **BPP** remains the same if the error probability is made 2^{-n} in the definition. (Here, as usual, n is the length of the input.)

   (e) Every function $f : \{0,1\}^* \to \{0,1\}$ that is computed by a Turing machine can also be computed by a polynomial sized family of circuits.