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CSE 431
Computational Complexity Theory
Final Exam, Spring 2024

Anup Rao May 29, 2024

DIRECTIONS:

- Open book. Open notes. No discussion or using any resources outside of the class.
- The exam consists entirely of True/False/Open questions. For full credit, you must write some kind of explanation (even it is just "we saw this in class") for every question.
- The exam is due Wednesday, June 5, at midnight.
- Good Luck!

For each of the following assertions:

- (3 points) State whether they are True, False, or Unknown to the best of your knowledge of complexity theory.
- (2 points) Briefly justify your answer.

1. (25 points, 5 each)

   i) There is a polynomial time algorithm that can take an integer x as input outputs a **TQBF** formula that is true if and only if x has a prime factor that is at most $x/10$.

   ii) There is a function $f : \{0,1\}^* \to \{0,1\}$ that is not computable in **BPP**.

   iii) There is a BPP algorithm for checking whether or not a given Boolean circuit computes the function which always outputs 0.

   iv) There is a BPP algorithm for checking whether or not a given arithmetic circuit computes a polynomial that is the 0 polynomial.

   v) Graph non-isomorphism is in **NP** as well as in **coNP**.

2. (25 points, 5 each)

   i) 3-SAT is in **IP**.

   ii) The class **RP** remains the same if the error probability is made 2/3 instead of 1/3.

   iii) In the definition of **IP**, if the verifier is restricted to being deterministic, then the class becomes the same as **NP**.

   iv) For any oracle A, there is a function $f : \{0,1\}^* \to \{0,1\}$ that cannot be computed in $\mathbf{P}^A$.

   v) $\mathbf{TQBF} \in \mathbf{L}$.

3. (25 points, 5 each)

   i) If $\mathbf{NP} = co\mathbf{RP}$, then $\mathbf{ZPP} = \mathbf{RP}$.

   ii) $\mathbf{ZPP} \subseteq \mathbf{NP}$.

   iii) $co\mathbf{RP} \subseteq \mathbf{BPP}$.

   iv) The class **BPP** remains the same if the error probability is made 2^{-n} in the definition. (Here, as usual, n is the length of the input.)

   v) Every function $f : \{0,1\}^* \to \{0,1\}$ that is computed by a Turing machine can also be computed by a polynomial sized family of circuits.

4. (25 points, 5 each)

   i) $\mathbf{PSPACE} \subseteq \mathbf{EXP}$.

   ii) $co\mathbf{NL} \neq \mathbf{PSPACE}$.

   iii) If $\mathbf{P} = \mathbf{NP}$, then $\mathbf{P}^{\mathsf{3SAT}} \subseteq \mathbf{P}$.

   iv) **BPP** is equal to $\mathbf{RP} \cap co\mathbf{RP}$.

   v) The problem of determining whether or not a graph can be colored with 3 colors is **NP**-complete.

5. (25 points, 5 each)

   i) If the permanent can be computed in polynomial time, then $co\mathbf{NP} = \mathbf{NP}$.

   ii) There is an algorithm that can take an undirected graph and two vertices s, t as input and output whether or not there is a path between s and t in $O(\log^2 n)$ space.

   iii) $\mathbf{NL} = co\mathbf{NL}$.

   iv) A non-zero multivariate polynomial of degree d can have at most d roots.

   v) If $co\mathbf{NP} = \mathbf{NP}$, then since $\forall x, \phi(x)$ is equivalent to $\neg\exists x, (\neg\phi(x))$, $\mathsf{TQBF} \in \mathbf{NP}$.

6. (25 points, 5 each)

   i) If $f \in BPP$, then for there is a constant c such that for every n, there is a circuit of size $O(n^c)$ that can compute f on n-bit inputs.

   ii) If $\mathbf{P} = \mathbf{NP}$, then $\mathbf{P} = \mathbf{PSPACE}$.

   iii) If $\mathbf{NP} = \mathbf{EXP}$, then 3-SAT does not have a polynomial time algorithm.

   iv) $\mathbf{BPP} \subseteq \mathbf{PSPACE}$.

   v) $\mathbf{BPP} \subseteq \mathbf{ZPP}$.