CSE431: Complexity Theory April 11, 2024

Homework 2

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These are only the first two problems on the homework.
Read the fine print[1]. Each problem is worth 10 points:

1. Consider the following function $h : \{0,1\}^* \to \{0,1\}$.

$$h(\alpha) = \begin{cases} 1 & \text{if there is some } x \text{ such that } M_\alpha(x) \text{ halts with output 1,} \\ 0 & \text{else.} \end{cases}$$

   Someone claims to have a program that can compute h. Prove that their program must have a bug by showing that no turing machine can compute $h(\alpha)$ for every α.

2. If $f, g \in \mathbf{NP}$, discuss whether each of the following functions can be deduced to be in **NP** or not:

   - $h_1(x) = f(x) \wedge g(x)$.
   - $h_2(x) = f(x) \vee g(x)$.
   - $h_3(x) = 1 - f(x)$.

3. Give an example of a function can be computed by polynomial sized circuits, but cannot be computed by a turing machine. HINT: You already know that Halt cannot be computed by a Turing machines. Try to make a function on n bits that allows you to compute Halt on inputs of a smaller size, by padding the input with lots of 0's.

4. Given an integer $m \times n$ matrix A and an integer column vector b of length m, the 0-1 Integer Programming function is defined as follows:

$$\mathbf{IP}(A, b) = \begin{cases} 1 & \text{if there is a length } n \text{ binary vector } x \text{ that satisfies } Ax \geq b \\ 0 & \text{otherwise.} \end{cases}$$

   Here $Ax \geq b$ means that every coordinate of Ax is at least as large as the corresponding coordinate of b. Prove that **IP** is **NP**-complete, by reducing **3SAT** to it.

[1] In solving the problem sets, you are allowed to collaborate with fellow students taking the class, but **each submission can have at most one author**. If you do collaborate in any way, you must acknowledge, for each problem, the people you worked with on that problem. The problems have been carefully chosen for their pedagogical value, and hence might be similar to those given in past offerings of this course at UW, or similar to other courses at other schools. Using any pre-existing solutions from these sources, for from the web, constitutes a violation of the academic integrity you are expected to exemplify, and is strictly prohibited. Most of the problems only require one or two key ideas for their solution. It will help you a lot to spell out these main ideas so that you can get most of the credit for a problem even if you err on the finer details. Please justify all answers. Some other guidelines for writing good solutions are here: `http://www.cs.washington.edu/education/courses/cse421/08wi/guidelines.pdf`.