

Homework 3

Anup Rao

Due: May 17, 2024

Read the fine print¹. Each problem is worth 10 points:

1. Show that if $\mathbf{P} = \mathbf{NP}$, then there is a polynomial time algorithm that given a 3-CNF, ϕ , finds a satisfying assignment x with $\phi(x) = 1$, if such an assignment exists. HINT: Try to fix $x_1 = 1$ and run the algorithm for 3-SAT on the restricted formula, and then fix $x_1 = 0$ and run the 3-SAT algorithm on the restricted formula again.
2. Show that:
 - (a) if O is an oracle that is in \mathbf{BPP} , then $\mathbf{BPP}^O = \mathbf{BPP}$.
 - (b) if for any oracle O , $\mathbf{BPP}^O = \mathbf{NP}$, then $\mathbf{NP} = \text{co-NP}$.
3. Consider the following game between two players: Given a directed graph $G = (V, E)$, and a start vertex s , the players (starting with Player 1) alternately choose an outgoing edge incident to the current vertex to reach a vertex that was not previously visited. If one of the players cannot choose a next vertex, he loses. Let $\mathbf{GAME}(G)$ be the function that is 1 if and only if Player 1 has a strategy that ensures that she always wins no matter what Player 2 does.

Show that \mathbf{GAME} is in \mathbf{PSPACE} .

4. Suppose \mathbf{TQBF} is also \mathbf{PSPACE} -complete under log-space reductions—meaning that for every $f \in \mathbf{PSPACE}$, there is a logspace computable function h such that $f(x) = \mathbf{TQBF}(h(x))$. Prove that this implies that $\mathbf{TQBF} \notin \mathbf{NL}$. Hint: Use Savitch's theorem and one of the hierarchy theorems.

¹In solving the problem sets, you are allowed to collaborate with fellow students taking the class, but **each submission can have at most one author**. If you do collaborate in any way, you must acknowledge, for each problem, the people you worked with on that problem. The problems have been carefully chosen for their pedagogical value, and hence might be similar to those given in past offerings of this course at UW, or similar to other courses at other schools. Using any pre-existing solutions from these sources, for from the web, constitutes a violation of the academic integrity you are expected to exemplify, and is strictly prohibited. Most of the problems only require one or two key ideas for their solution. It will help you a lot to spell out these main ideas so that you can get most of the credit for a problem even if you err on the finer details. Please justify all answers. Some other guidelines for writing good solutions are here: <http://www.cs.washington.edu/education/courses/cse421/08wi/guidelines.pdf>.