Consensus & Agreement

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Group Communication

- Unicast messages: from a single source to a single destination
- Multicast messages: from a single source to multiple destinations (designated as a group)
- Issues:
  - Fault tolerance: two kinds of faults in distributed systems
    - “Crash faults” (also known as fail-stop or benign faults): process fails and simply stops operating
    - “Byzantine faults”: process fails and acts in an arbitrary manner (or malicious agent is trying to bring down the system)
  - Ordering:
    - Achieve some kind of consistency in how messages of different multicasts are delivered to the processes

Basic Multicast

- Channels are assumed to be reliable (do not corrupt messages and deliver them exactly once)
- A straightforward way to implement B-multicast is to use a reliable one-to-one send operation:
  - B-multicast(g,m): for each process p in g, send (p,m).
  - receive(m): B-deliver(m) at p.
- A basic multicast primitive guarantees a correct process will eventually deliver the message, as long as the multicaster (sender) does not crash.

Reliable Multicast

- Desired properties:
  - Integrity: A *correct* (i.e., non-faulty) process *p delivers* a message m at most once.
  - Validity: If a correct process multicasts message *m*, then it will eventually deliver *m*. (Local liveness)
  - Agreement: If a correct process delivers message *m*, then all the other correct processes in *group(m)* will eventually deliver *m*.
  - Property of “all or nothing.”
  - Validity and agreement together ensure overall liveness
  - Question: how do you build reliable multicast using basic multicast?

Reliable multicast (contd.)

On initialization
  Received := {};

For process p to R-multicast message m to group g
  B-multicast(*g*, *m*);   // $p \in g$ is included as a destination

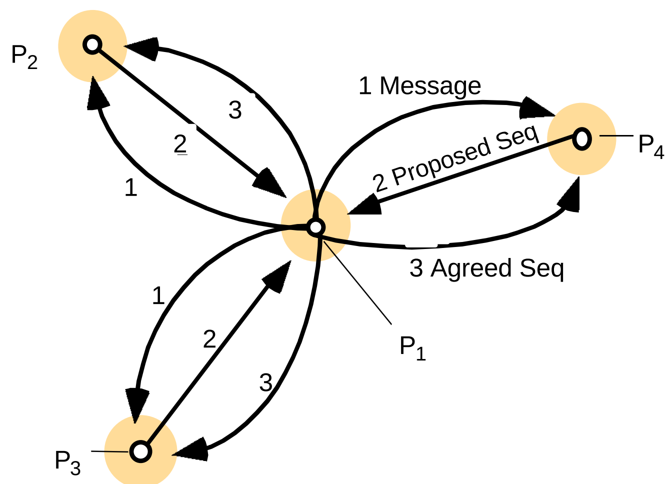
On B-deliver(*m*) *at process q with g* = *group*(*m*)
  if ($m \notin$ *Received*)
  then
    Received := *Received* $\cup$ {*m*};
    if ($q \neq p$) *then B-multicast*(*g*, *m*); *end if*
    R-deliver m;
  end if

Ordered Multicast

- Desirable ordering properties:
  - FIFO ordering: If a correct process issues *multicast(g,m)* and then *multicast(g,m')*, then every correct process that delivers *m'* will deliver *m* before *m'*.
  - Causal ordering: If *multicast(g,m)* → *multicast(g,m')* then any correct process that delivers *m'* will deliver m before *m'*.
  - Total ordering: If a correct process delivers message *m* before *m'*, then any other correct process that delivers *m'* will deliver *m* before *m'*.
- Causal ordering implies FIFO ordering
- Causal ordering does not imply total ordering
- Total ordering does not imply causal ordering

Implementing Total Ordering

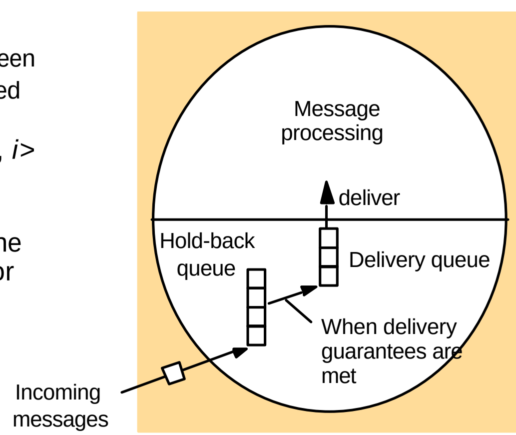
- Multicast a message, solicit sequence numbers from processes, multicast a sequence number that is computed based on solicited values



Implementing Total Ordering

- Each process, q keeps:
  - A^q_g the largest agreed sequence number it has seen
  - P^q_g its own largest proposed sequence number

1. Process p *B-multicasts* <*m*, *i*> to g, where i is a unique identifier for m.
2. Each process q replies to the sender p with a proposal for the message's agreed sequence number of
   - $P^q_g := \mathrm{Max}(A^q_g, P^q_g) + 1$.
   - places it in its hold-back queue
3. p collects all the proposed sequence numbers and selects the largest as the next agreed sequence number, a.
   It *B-multicasts* <*i*, *a*> to g.



4. Recipients set $A^q_g := \mathrm{Max}(A^q_g, a)$, attach a to the message and re-order hold-back queue.

Consensus

- Consensus: N Processes agree on a value.
  - For example, synchronized action (go / abort)
- Consensus may have to be reached in the presence of failure.
  - Process failure – process crash (fail-stop failure), arbitrary failure.
  - Communication failure – lost or corrupted messages.
- In a consensus algorithm:
  - All P_i start in an "undecided" state.
  - Each P_i proposes a value v_i from a set D and communicates it to some or all other processes.
  - A consensus is reached if all non-failed processes agree on the same value, d.
    - Each non-failed P_i sets its decision variable to d and changes its state to "decided."

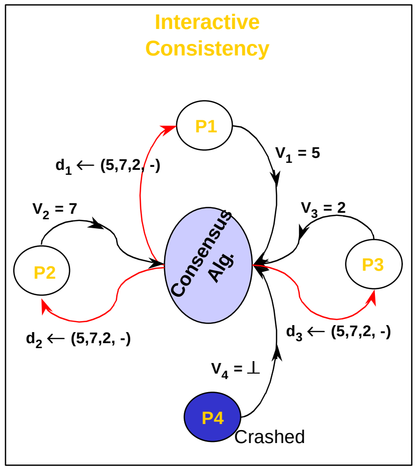
Consensus Requirements

- Termination:
  - Eventually each correct process sets its decision value.
- Agreement:
  - The decision value is the same for all correct processes, *i.e.,* if p_i and p_j are correct and have entered the decided state, then $d_i=d_j$
- Integrity:
  - If all correct processes P_i's propose the same value, d, then any correct process in the decided state has decision value = d.

- Rich problem space:
  - Synchronous vs. asynchronous systems
  - Fail-stop vs. byzantine failures
  - Process vs. message failures

Interactive Consistency Problem

- Interactive consistency is a special case of consensus where processes agree on a vector of values, one value for each process



Byzantine Generals Problem

- 3 or more generals need to agree to attack or to retreat.
- Problem
  - The commander issues the order.
  - One or more of the generals (including the commander) could be a traitor who'll give wrong information.
  - Each general sends his/her information to all others (assuming reliable communication).
  - Once each general has collected all values, it determines the right value (attack or retreat).
- The requirements are termination, agreement, and integrity.

Problem Equivalence

- Interactive consistency (IC) can be solved if there is a solution for Byzantine Generals (BG) problem:
  - Just run BG "n" times
- Consensus (C) can be solved if there is a solution for IC:
  - Run IC to produce a vector of values at each process
  - Then apply the majority function on the vector
  - Resulting value is the consensus value
  - If no majority, choose a "bottom" value
- BG is solvable if there is a solution to C:
  - Commander sends its proposed value to itself and each of the other generals
  - All processes run C with the values received
  - Resulting consensus value is the value required by BG

Consensus in a synchronous system

- For a system with at most f processes crashing, the algorithm proceeds in f+1 rounds, using basic multicast.
- $Values^r_i$: the set of proposed values known to P_i at the beginning of round r.
- Initially $Values^0_i = \{\}$; $Values^1_i = \{v_i\}$

for round = 1 to f+1 do

  B-multicast ($Values^r_i - Values^{r-1}_i$)

  $Values^{r+1}_i \leftarrow Values^r_i$

  for each V_j received

    $Values^{r+1}_i = Values^{r+1}_i \cup V_j$

  end

end

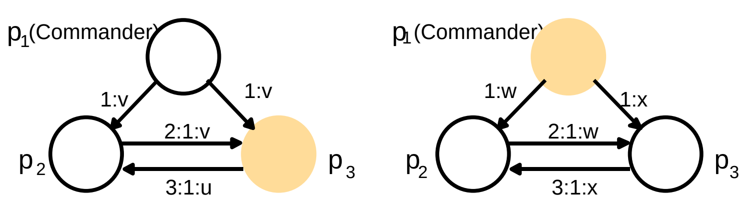
d_i = minimum($Values^{f+1}_i$)

Proof of correctness

- Proof by contradiction.
- Assume that two processes differ in their final set of values.
- Assume that p_i possesses a value v that p_j does not possess.
  - → A third process (p_k) sent v to p_i and crashed before sending v to p_j
  - → Any process sending v in the previous round must have crashed; otherwise, both p_k and p_j should have received v.
  - → Proceeding in this way, we infer at least one crash in each of the preceding rounds.
  - → But we have assumed at most f crashes can occur and there are $f+1$ rounds → contradiction.

Byzantine Generals in a synchronous system

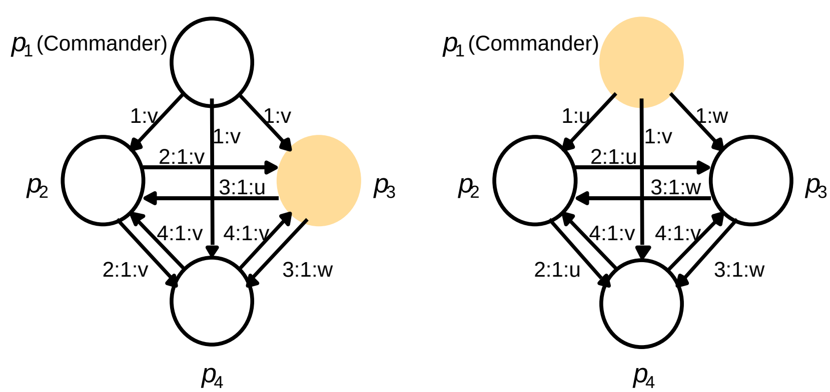
- A faulty process may send any message with any value at any time; or it may omit to send any message.
- In the case of arbitrary failure, no solution exists if $N<=3f$.



Solution

- To solve the Byzantine generals problem in a synchronous system, we require. *N>=3f+1*
- Consider *N=4, f=1*
  - In the first round, the commander sends a value to each of the other generals
  - In the second round, each of the other generals sends the value it received to its peers.
  - The correct generals need only apply a simple majority function on the set of values received.

Four generals, one fault

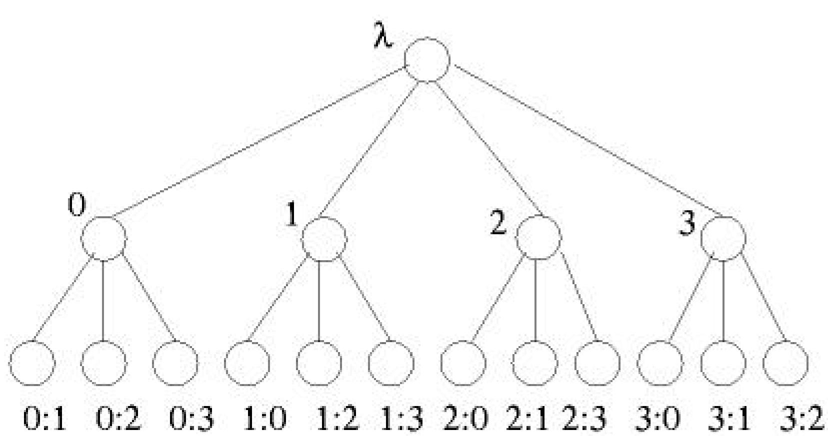


Consensus Algorithms for Byzantine Failures

- Minimum number of rounds is f + 1
- Exponential tree algorithm:
  - Each processor maintains a tree data structure in its local state
  - Each node of the tree is labeled with a sequence of processor indices with no repeats
    - Root's label is empty sequence
    - Root has n children labeled 0 through n-1
    - Child node labeled "i" has n-1 children labeled 0 through i-1 and i+1 through n-1
    - In general, node at level d with label v has n- d children skipping any index already present in v
    - Nodes at level f+1 are the leaves

Example of exponential tree

- Tree when n = 4 and f = 1



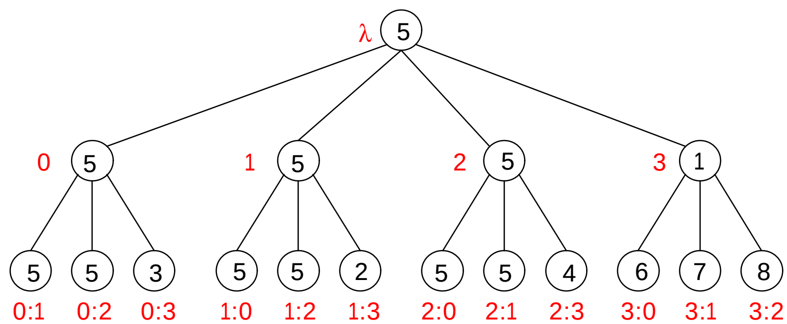
Exponential Tree Algorithm

- Each processor fills in the tree nodes with values as the rounds go by
- Initially, store your input in the root (level 0)
- Round 1: send level 0 of your tree (the root); store value received from p_j in node j (level 1)
- Round 2: send level 1 of your tree; store value received from p_j for node k in node "k:j" (level 2)
  - This is the "value that p_j told me that p_k told p_j"
- Continue for f + 1 rounds

Computing Decision Value

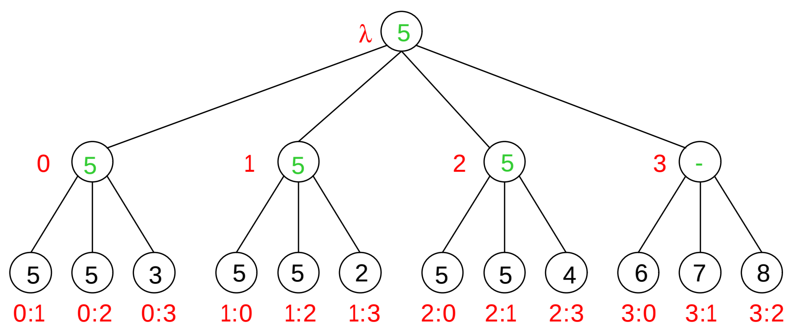
- In the last round, each processor uses the values in its tree to compute its decision
  - Decision is resolve(λ)
  - Where resolve(π) equals:
    - Value in tree node labeled "π" if it is a leaf
    - majority{resolve(π') : π' is a child of π}

Building Tree: top-down phase



- Assume that nodes 0, 1, and 2 are legitimate; they contribute value 5
- Assume that node 3 is byzantine

Resolving nodes



- Resolve a leaf node: return the value of the node
- Resolve an internal node: return the majority value of children
- Decision by processor: resolve the root

Proof of algorithm

- Resolve Lemma: Non-faulty processor p_i's resolved value for node $\pi = \pi' j$ equals what p_j has stored for π'.
- Proof: By induction on the height of π.

Basis: π is a leaf.

1) Then p_i stores in node π what p_j sends it for π' in the last round.
2) For leaves, the resolved value is the tree value.

Proof (contd.)

Induction: π is not a leaf.

By tree definition, π has at least n – f children

Since n > 3f, π has majority of non-faulty children

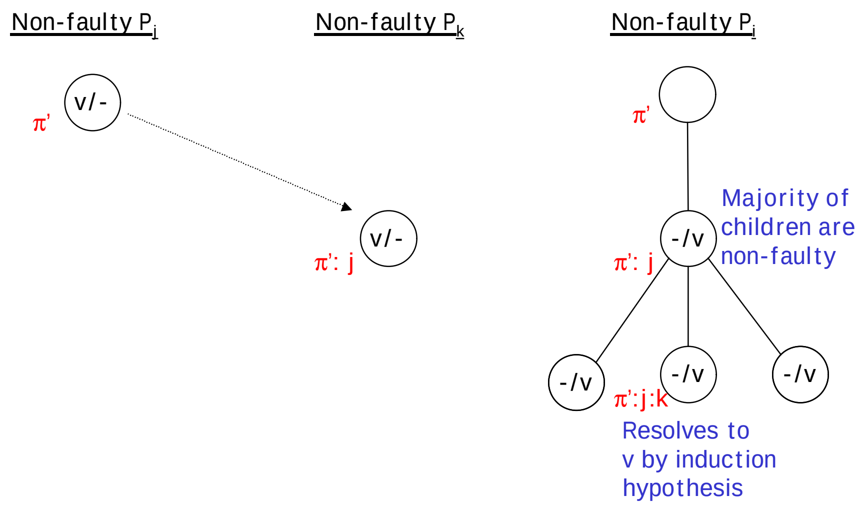
Let "π k" be a child of π such that p_k is non-faulty

Since p_j is non-faulty, p_j correctly reports to p_k that it has some value v in node π'; thus p_k stores v in node π = "π' j"

By induction, p_j's resolved value for "π k" equals the value v that p_k has in its tree node π

So all of π's non-faulty children resolve to v in p_j's tree, and thus π resolves to v in p_j's tree

Proof (contd.)



Proof of Validity

- Suppose all inputs are "v"
  - Non-faulty processor p_i decides on resolve(λ), which is the majority among resolve(j) (for all j from 0 to n-1)
  - The previous lemma implies that for each non-faulty p_j
    - resolve(j) for p_i = value stored at the root of p_j's tree
    - Value stored at the root is p_j's input = v
    - Thus p_i decides v

Proof of Agreement

- Show that all non-faulty processors resolve to the same value for their tree roots
- A node is common if all non-faulty processors resolve to the same value for it. (We will need to show that the root is common.)
- Strategy:
  - Show that every node with a certain property is common
  - Show that the root has the property
- Lemma: If every π-to-leaf path has a common node, then π is common.
- Proof by Induction:

  Basis: π is a leaf. Then every π-to-leaf path consists solely of π, and since the path is assumed to contain a common node, that node is π

Lemma (contd.)

- Induction Step:
  - π is not a leaf. Suppose in contradiction π is not common.
  - Then every child π' of π has the property that every π'-to-leaf path has a common node
  - Since the height of π' is smaller than the height of π, the inductive hypothesis implies that π' is common
  - Therefore, all non-faulty processors compute the same resolved value for π, and thus π is common

Prove that root has the property

- Show that every root-to-leaf path has a common node:
  - There are f+2 nodes on a root-to-leaf path
  - The label of each non-root node on a root-to-leaf path ends in a distinct processor index (the processor from which the value is to be received)
  - At least one of these indices is that of a non-faulty processor
  - “Resolve Lemma” implies that the node whose label ends with a non-faulty processor is a common node

Polynomial Algorithm for Byzantine Agreement

- Can reduce the message size with a simple algorithm that increases the number of processors to n > 4f and number of rounds to 2(f + 1)
- Phase King Algorithm: Uses f + 1 phases, each taking two rounds

Code for p_i

pref = my input

First round of phase k:

send pref to all

receive prefs of others

let “maj” be the value that occurs > n/2 times among all prefs (0 if none)

let “mult” be the number of times “maj” occurs

Algorithm (contd.)

Second round of phase k:

```
    if my_proc == k then send "maj"        // I am the phase king
    receive tie-breaker from p_k
    if mult > n/2 + f
            then pref = maj
            else pref = tie-breaker
    if k == f+1 then decide pref
```

Proof of Phase King Algorithm

- Lemma: If all non-faulty processors prefer v at start of phase k, then all do at end of phase k.
- Proof:
  - Each non-faulty processor receives at least $n - f$ preferences (including its own) for v in the first round of phase k
  - Since $n > 4f$:

    $n/2 > 2f$

    $(n - n/2) > f + f$

    $n - f > n/2 + f$.
  - Thus the processors still prefer v.

- Validity: follows from above lemma
  - All non-faulty processors start with the same value

Proof (contd.)

- Lemma: If the king of phase k is non-faulty, then all non-faulty processors have the same preference at the end of phase k.
- Proof:
  - Consider two non-faulty processors p_i and p_j
  - Case 1: p_i and p_j both use p_k's tie-breaker. Since p_k is non-faulty, they agree
  - Case 2: p_i uses its majority value and p_j uses the king's tie-breaker
    - p_i's majority value is v
    - p_i receives more than n/2 + f preferences for v
    - p_k receives more than n/2 preferences for v
    - p_k's tie-breaker is v

Proof (contd.)

  - Case 3: p_i and p_j both use their own majority values
    - p_i's majority value is v
    - p_i receives more than n/2 + f preferences for v
    - p_j receives more than n/2 preferences for v
    - p_j's majority value is also v

- Since there are f + 1 phases, at least one has a non-faulty king
- At the end of that phase, all non-faulty processors have the same preference
- From that phase onward, the non-faulty preferences stay the same
- Thus the decisions are the same.

Fischer-Lynch-Patterson (1985)

- No completely asynchronous consensus protocol can tolerate even a single unannounced process death

Assumptions

- Fail-stop failure:
  - Impossibility result holds for byzantine failure
- Reliable message system:
  - messages are delivered correctly and exactly once
- Asynchronous:
  - No assumptions regarding the relative speeds of processes or the delay time in delivering a message
  - No synchronized clock
    - Algorithms based on time-out can not be used
  - No ability to detect the death of a process

The weak consensus problem

- Initial state: 0 or 1 (input register)
- Decision state:
  - Non-faulty process decides on a value in {0, 1}
  - Stores the value in a write-once output register
- Requirement:
  - All non-faulty processes that make a decision must choose the same value.
  - For proof: assume that some processes eventually make a decision (weaker requirement)
- Trivial solution is ruled out
  - Cannot choose 0 arbitrarily
- Processes modeled by deterministic state machines

Notation

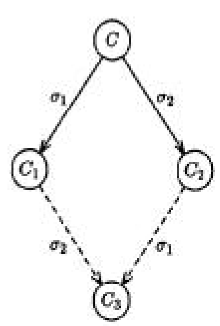
- A configuration consists of
  - All internal state of each process, the contents of message buffer
- Message system (think of the undelivered messages stored in a bag)
  - send(p, m)
  - receive(p) ➔ returns some message to be received by “p” or an empty message
- A step is a transition of one configuration C to another e(C), including 2 phases:
  - First, receive(p) to get a message m
  - Based on p’s internal state and m, p enters a new internal state and sends finite messages to other
- e = (p, m) is called an event and said e can be applied to C

Schedule, run, reachable and accessible

- A schedule from C
  - a finite or infinite sequence ó of events that can be applied, in turn, starting from C
  - The associated sequence of steps is called a run
  - ó(C) denotes the resulting configuration and is said to be reachable from C
- An accessible configuration C
  - If C is reachable from some initial configuration

Lemma 1

- Suppose that from some configuration C, the schedules ó$_1$ and ó$_2$ lead to configuration C_1 and C_2 respectively.
  - If the sets of processes taking steps in ó$_1$ and ó$_2$ respectively, are disjoint:
  - Then ó$_2$ can be applied to C_1 and ó$_1$ can be applied to C_2, and both lead to the same configuration.



Definitions

- A process is non-faulty
  - If it takes infinitely many steps
- A configuration C has decision value v if some process p is in a decision state with output register containing v.
- Deciding run
  - Some process reaches a decision state
- Admissible run
  - At most one process is faulty and all messages sent to non-faulty processes are eventually received

Bivalent, 0-valent/1-valent

- Let C be a configuration, V the set of decision values of configurations reachable from C
  - C is bivalent if $|V| = 2$.
  - C is univalent if $|V| = 1$.
- 0-valent or 1-valent according to the corresponding decision value.

Correctness

- A consensus protocol P is totally correct in spite of one fault:
  - No trivial solutions (there are some configurations that lead to result 0 and some that lead to result 1)
  - No accessible configuration has more than one decision value
  - Every admissible run is a deciding run

Theorem 1

- No consensus protocol is totally correct in spite of one fault.
- Proof strategy:
  - There must be some initial configuration that is bivalent
  - Consider some event e = (p, m) that is applicable to a bivalent configuration, C
    - Consider the set of configurations reachable from C w/o applying e (let this set be Σ)
    - Apply e to each one of these configurations to get the set D
    - Show that D contains a bivalent configuration
  - Construct an infinite sequence of stages where each stage starts with a bivalent configuration and ends with a bivalent configuration

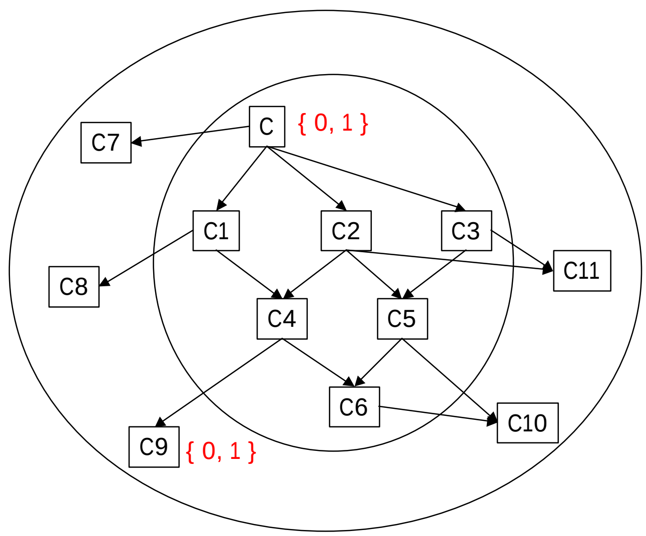
Lemma 2

- P has a bivalent initial configuration (Proof by contradiction)
- Consider configuration C1 = { 0, 0, 0, ..., 0 }
  - Every processor starts with input value 0
  - C1 is 0-valent
- Consider configuration C2 = { 1, 1, 1, ..., 1 }
  - C2 is 1-valent
- Transform C1 to C2 with at most one processor changing its input value
  - There must be two configurations C3 and C4:
    - C3 is 0-valent, C4 is 1-valent
    - Some processor p changed its value from 0 to 1
  - Consider some admissible deciding run from C3 involving no p-events.
    - Let σ be associated schedule.
    - Apply σ to C4. Clearly, resulting state should be 0.
    - Implies contradiction.

Lemma 3

- Let C be a bivalent configuration of P.
  - Let e = (p, m) be an event that is applicable to C.
  - Let Σ be the set of configurations reachable from C without applying e, and let $D = e(\Sigma) = \{e(E) \mid E \in \Sigma\}$.
  - Then, *D* contains a bivalent configuration.

Graphical Representation



Proof

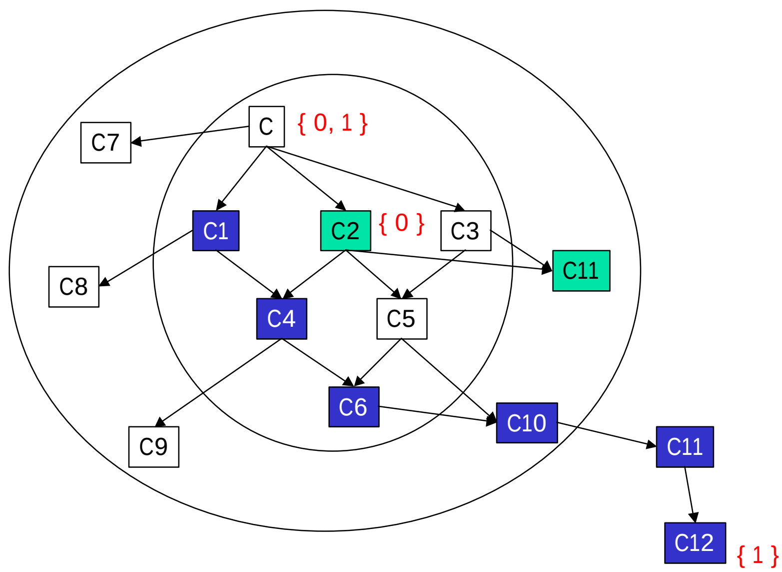
- There must be two states such that:

  C ~ E0 and C ~ E1

  where E0 is 0-valent and E1 is 1-valent

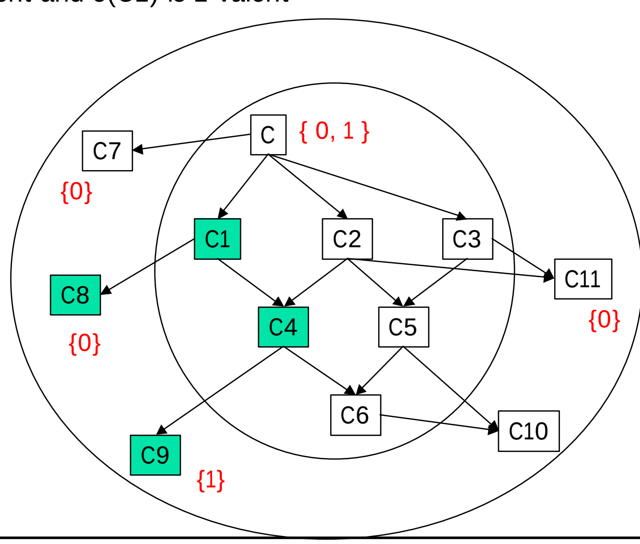
- Consider E0:
  - If E0 belongs to Σ, then e(E0) = F0 belongs to *D*
  - If E0 does not belong to Σ, then there is a F0:
    - Such that F0 belongs to *D*
    - F0 ~ E0
  - In either case, there is a F0 ∈ *D* and F0 is 0-valent
- Similarly there exists a F1 which is 1-valent and F1 ∈ *D*
- *D* contains 0-valent and 1-valent configurations

Two Cases

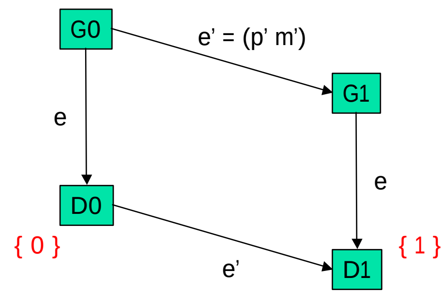


Proof (contd.)

- There exists two adjacent states G0 and G1 in Σ, such that e(G0) is 0-valent and e(G1) is 1-valent

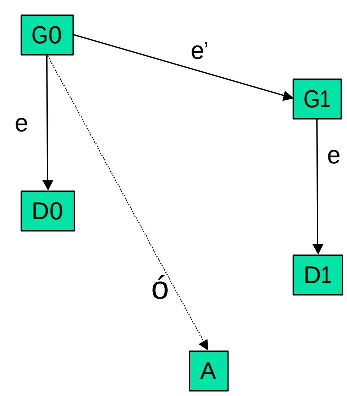


Proof (contd.)



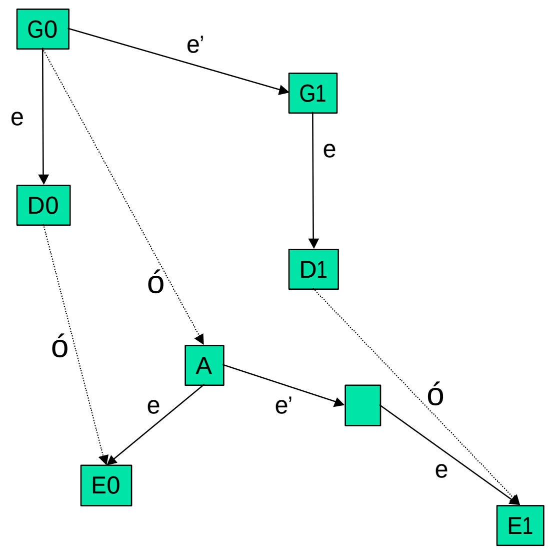
- Assume that the event that transforms G0 to G1 is e' = (p', m') and let p' != p
- Recall that p is the processor with the delayed message (and the delayed event e)
- e' is applicable to D0 and transforms D0 to D1 (commutativity lemma)
- What does this imply?

Proof (contd.)



- If p' is same as p: consider some configuration A that is reachable from G0 that involves no events to p, and is deciding. Let ó be the schedule.

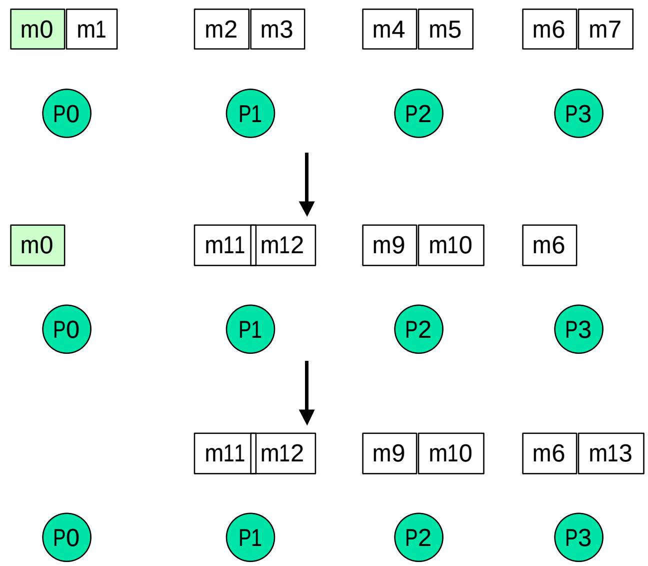
Proof (contd.)



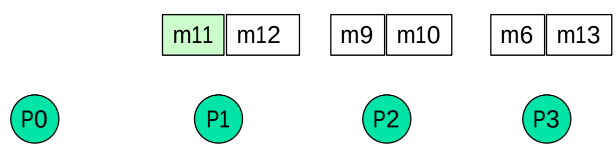
Proof Wrapup

- Goal is to construct an infinite sequence of events:
  - No processor fails
    - Each processor executes an infinite steps
    - All messages sent to a processor is delivered in finite time
  - Every configuration in the sequence is bivalent
- Previous theorem states that:
  - Start with a bivalent configuration
  - Delay some message
  - Can always find some other bivalent configuration that is reached by delivering the message

Construction of infinite run



Construction of infinite run (contd.)



- Block a message for the next processor, construct another possible bivalent configuration
- Construction can go on for ever:
  - No faults (infinite steps for each processor, messages delivered in finite time)
  - Always goes from one bivalent configuration to another bivalent configuration

Paxos Consensus

- Assume that a collection of processes that "can" propose values, choose a value
  - Only a value that has been proposed may be chosen
  - Only a single value is chosen
- Three classes of agents: proposers, acceptors, and learners
  - A single process may act as more than one agent
- Model:
  - Asynchronous messages
  - Agents operate at arbitrary speed, may fail by starting, and may restart. (If agents fail and restart, assume that there is non-volatile storage.)
  - Guarantee safety and not liveness

Simple solutions

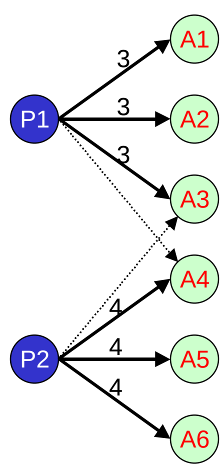
- Have a single acceptor agent
- Proposers send a proposal to the acceptor:
  - Acceptor chooses the first proposed value
  - Rejects all subsequent values
  - Failure of acceptor means no further progress
- Let's use multiple acceptor agents
  - Proposer sends a value to a large enough set of acceptors
  - What is large enough?
    - Some majority of acceptors, which implies that only one value will be chosen
    - Because any two majorities will have at least one common acceptor

Some Other Ground Rules

- There might be just one proposer
  - Number of proposers is unknown
- No liveness requirements:
  - If a proposal does not succeed, you can always restart a new proposal
- The three important actions in the system are:
  - Proposing a value
  - Accepting a value
  - Choosing a value (if a majority of acceptors accept a value)

Solutions that don't work

- There could be just one proposed value
  - An acceptor should accept the first value



Refinements

- Allow an acceptor to accept multiple proposals
  - Which implies that multiple proposals could be chosen

    P1: Have to make all of the chosen proposals be the same value!
  - Trivially satisfies the condition that only a single value is chosen
  - Requires coordination between proposers and acceptors

- Let proposals be ordered
  - One possibility: each proposal is a 2-tuple [proposal-number, processor-number]

- Ensure the following property:

  P2: If a proposal with value v is chosen, then every higher-numbered proposal that is *chosen* has the value v

  P2 ==> P1

More refinements

- Consider the following property:

  P3: If a proposal with value v is chosen, then every higher-numbered proposal that is *accepted* has the value v

  P3 ==> P2 ==> P1

- Consider an even stronger property:

  P4: If a proposal with value v is chosen, then every higher-numbered proposal that is *proposed* by any processor has value v

  P4 ==> P3 ==> P2 ==> P1

One more refinement

P5: For a proposal numbered n with value v:

- It is issued only if there is a set S consisting of a majority of acceptors such that either:
  - No acceptor in S has accepted any proposal numbered less than n, or
  - v is the value of the highest-numbered proposal among all proposals numbered less than n accepted by the acceptors in S

One can satisfy P4 by maintaining the invariant P5

How does one enforce P5?

Phase 1: prepare request

(1) A proposer chooses a new proposal version number *n* , and sends a *prepare request* (“prepare”, n) to a majority of acceptors:

(a) Can I make a proposal with number *n* ?

(b) if yes, do you suggest some value for my proposal?

Phase 1 (receive prepare request)

(2) If an acceptor receives a prepare request ("prepare", n) with n greater than that of any prepare request it has already responded, sends out ("ack", n, n', v') or ("ack", n, $\perp$, $\perp$)

(a) responds to the request with a promise not to accept any more proposals numbered less than n.

(b) suggest the value v of the highest-number proposal that it has accepted if any, otherwise $\perp$

Phase 2: accept request

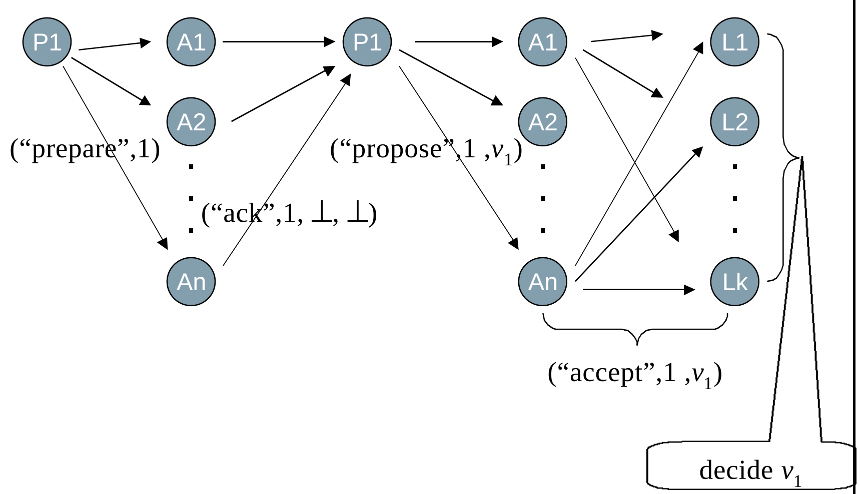
(3) If the proposer receives the requested responses from a majority of the acceptors, then it can issue a *propose request* ("propose", n , v) with number n and value v:

(a) n is the number that appears in the prepare request.

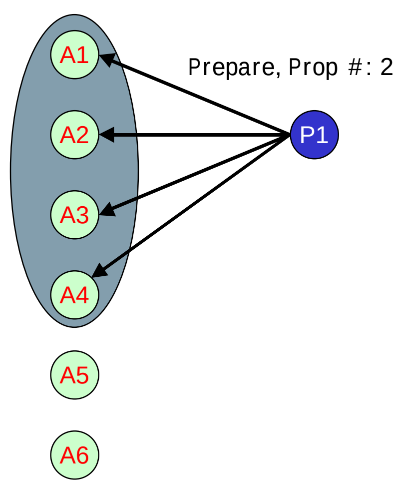
(b) v is the value of the highest-numbered proposal among the responses

(4) If the acceptor receives a request ("propose", n , v), it accepts the proposal *unless* it has already responded to a prepare request having a number greater than n.

In Well-Behaved Runs



Example



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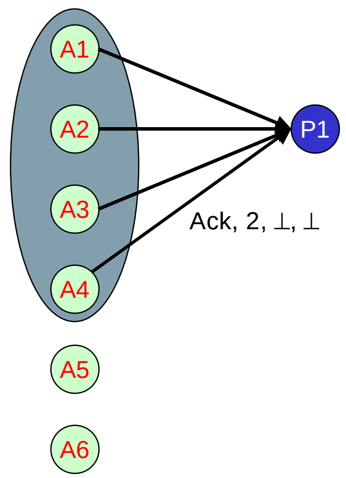

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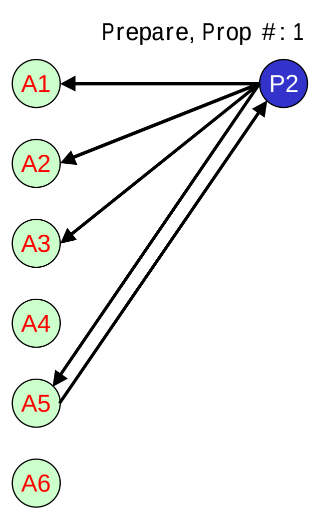

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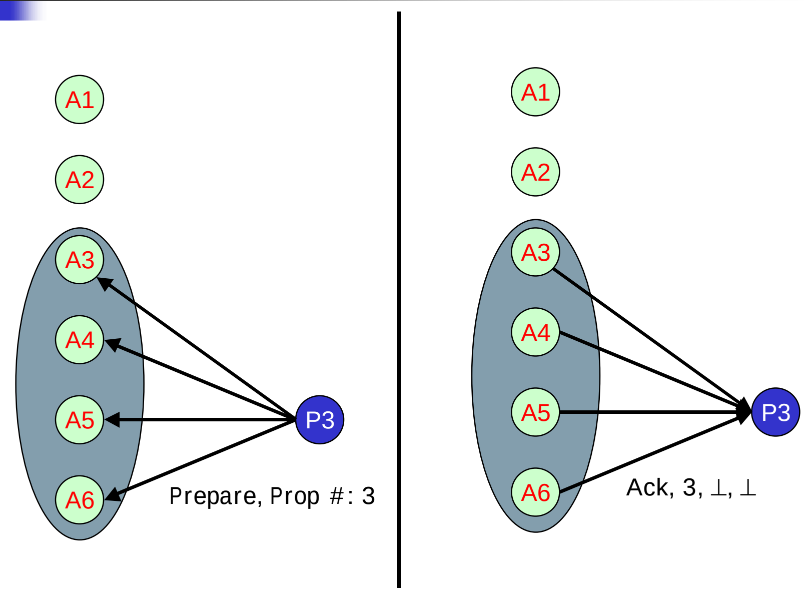

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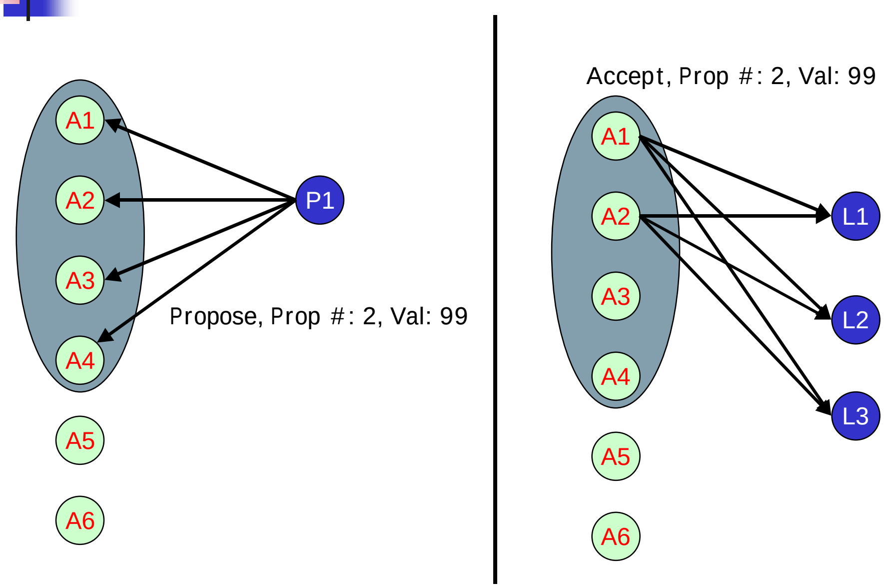

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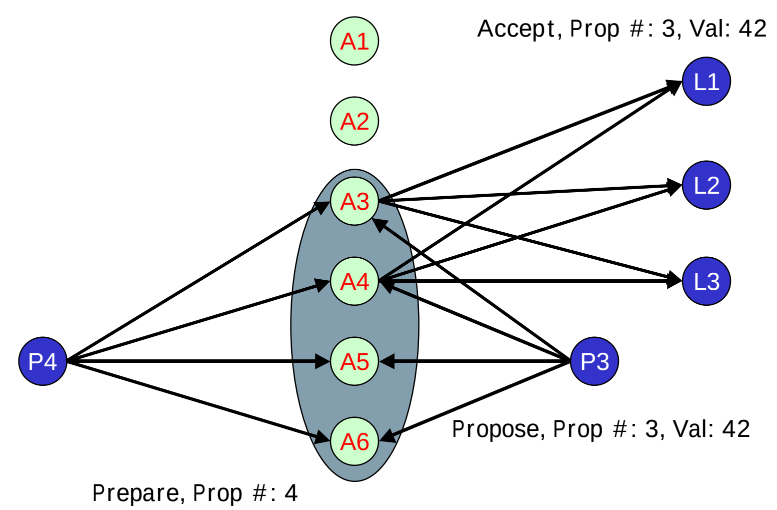

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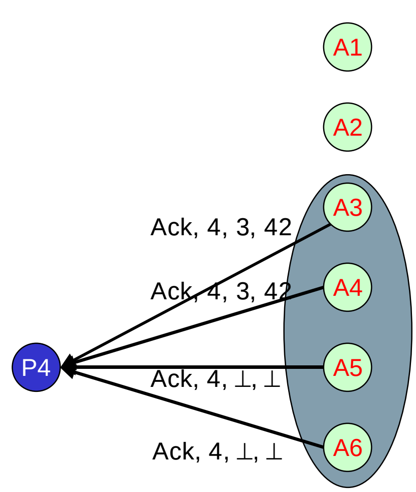

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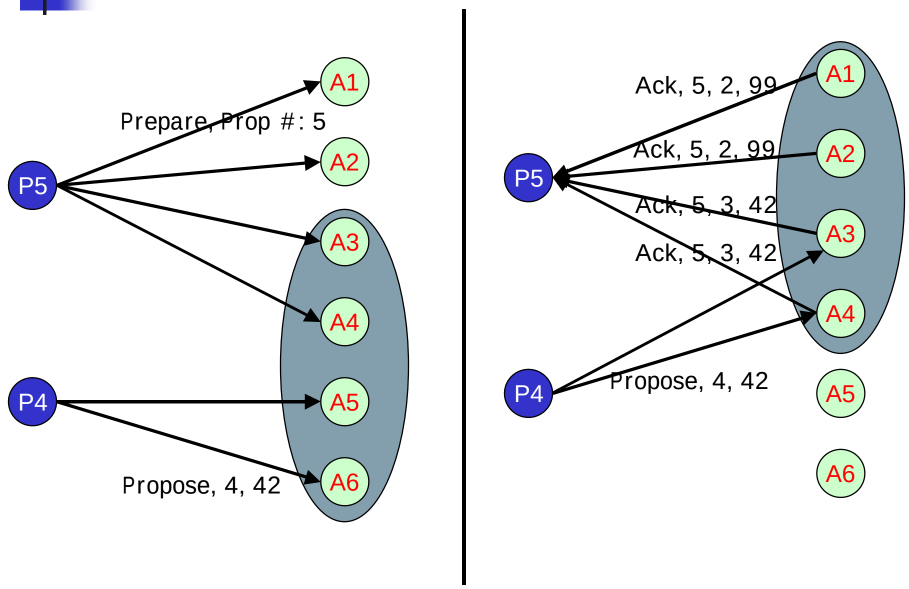

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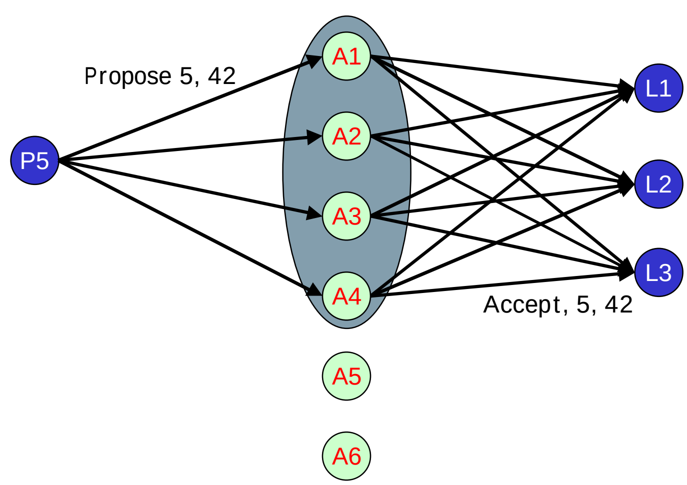

Paxos: other issues

- A proposer can make multiple proposals
  - It can abandon a proposal in the middle of the protocol at any time
  - Probably a good idea to abandon a proposal if some processor has begun trying to issue a higher-numbered one
- If an acceptor ignores a prepare or accept request because it has already received a prepare request with a higher number:
  - It should probably inform the proposer who should then abandon its proposal
- Persistent storage:
  - Each acceptor needs to remember the highest numbered proposal it has accepted and the highest numbered prepare request that it has acked.

Progress

- Easy to construct a scenario in which two proposers each keep issuing a sequence of proposals with increasing numbers
  - P completes phase 1 for a proposal numbered n1
  - Q completes phase 1 for a proposal numbered n2 > n1
  - P's accept requests in phase 2 are ignored by some of the processors
  - P begins a new proposal with a proposal number n3 > n2
  - And so on...

Announcements

- Class lecture notes updated
- Upcoming topics:
  - Secure routing (avoiding denial of service attacks)
  - Overlay/sensor networks
- Project checkpoint due